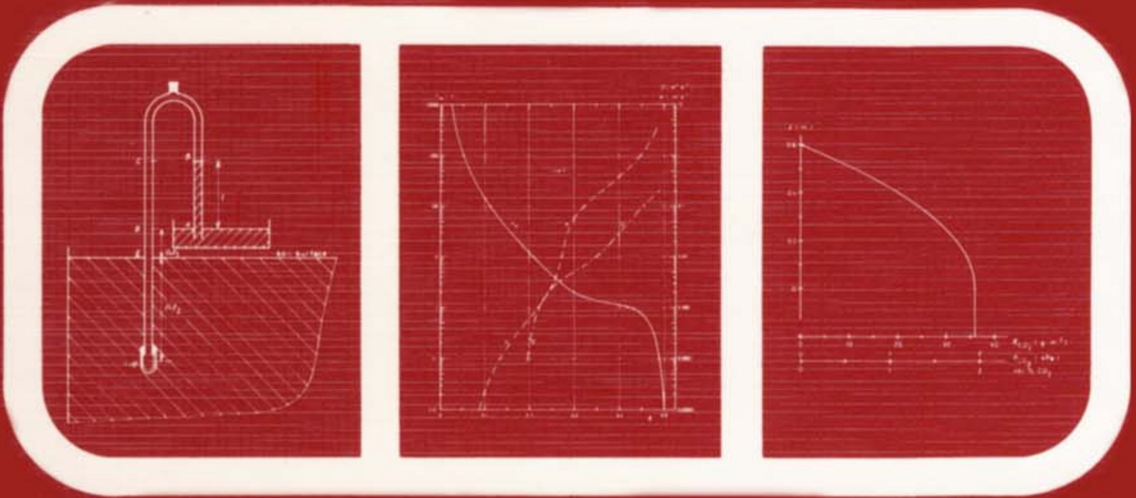
P. KOOREVAAR, G. MENELIK AND C. DIRKSEN

ELEMENTS OF SOIL PHYSICS



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Developments in Soil Science 13

ELEMENTS OF SOIL PHYSICS

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Preface

This book is not meant to be 'just another textbook' on Soil Physics. Instead, it presents a different approach to teaching students the basic principles of Soil Physics. The main idea is that students should be forced to think through and apply immediately what they read. To achieve this goal, the text is written mostly in rather short paragraphs and is alternated with questions. Many questions are an integral part of the transfer of knowledge; these, in particular, should be answered by the student before he continues. The student can check his answers with those given at the end of each chapter.

The origin of the present book goes back to lectures notes by G. H. Bolt, which eventually resulted in two lecture syllabi written in the early sixties by G. H. Bolt, A. R. P. Janse and F. F. R. Koenigs for internal use at the Department of Soil Science and Plant Nutrition of the Agricultural University, Wageningen, The Netherlands. One syllabus was on Soil Chemistry, the other on Soil Physics; both had to be studied by the students as one unit. In the course of time, several reasons developed for revising these syllabi. First, the c.g.s. system of units needed to be replaced by the new S.I.-system. Secondly, in 1970 a new study program was introduced at the Agricultural University, for which some students needed to study only Soil Physics. Unfortunately, some subjects that were needed for Soil Physics were discussed only in the Soil Chemistry syllabus, e.g., the behaviour of clay particles and the diffuse electric double layer. Obviously, it is desirable to incorporate all subjects of Soil Physics in one syllabus. A third reason for revision was the need to introduce a consistent set of definitions, symbols, etc., based on a rigorous, lucid development from first principles, in agreement with the latest internationally accepted conventions.

The final and most elaborate, if not most important reason for revision was the need to seek ways to improve the examination results of the students. To this end, we first handed out questions at the lectures, some of which were prepared by L. Stroosnijder. Once a week the 'homework' was discussed in small groups of about 15 students and evaluated by the teacher. This system was used with about 150 students, and thus was very time-consuming for the teaching staff. For the students, the gain in examination results was marginal, in spite of their larger time investment. This led eventually to the idea to integrate the questions and answers with the subject matter in the syllabus and develop some kind of 'half-programmed' text for selfstudy. During a year of experimenting with a first draft of such a text, in which about half of the subjects were given in this new form, the students liked the new approach, obtained better results at the examinations, and even needed less time to master the material than with the old method. For the teaching staff also, the new method was far more efficient. Lectures were no longer needed and were replaced by sessions in which students could ask questions about the material studied. On the average, only about ten percent of the students made use of these sessions. From 1974 on, the new method has been used with good results.

The first author played a major role in the development and implementation of the elementary study program in Soil Physics, along the lines described above. He was assisted in this by G. Muggen of the Bureau for Development and Research of Education (B.O.O.) of the Agricultural University. This effort resulted, in 1975, in a new syllabus entitled 'Bodemnatuurkunde. Zelfinstructieve leertekst'. A. Kamphorst translated this text in English and added some material for use in a M.Sc-course on Soil Science and Water Management at the Agricultural University for students from developing countries. This English version, the experience gathered during about eight years of use with Dutch and foreign students, and many years of experience of the third author in a research environment in the USA formed the basis for the present, much expanded and revised book. G. H. Bolt wrote the epilogue.

The book is meant to be a first introduction in Soil Physics, but students who already have some knowledge in this area may also benefit from working through it. It presents a rigorous, complete and consistent treatment of the definitions and relationships for the fundamental physical processes occurring in soil. In addition, it gives detailed descriptions and solutions of most elementary problems encountered in the realm of Soil Physics. The treatment normally involves many simplifications, such as homogeneous soil, isothermal conditions, isotropy, etc. Because of its elementary level, we deemed it not necessary to include references each time a new concept was introduced. Instead, at the end of each chapter we present a list of recommended literature for further study. This literature, in turn, gives references to more specific or advanced literature, which should be adequate for starting a more thorough study of a particular subject. The teacher may set a time-table for studying the various subjects and accompany the students by means of question sessions. Students could be tested only at the end of the course, but we have had good experience with giving two or three tests at regular intervals, followed by a final examination.

The S.I.-system is used rigorously throughout the text. A few other units which are used quite often in Soil Physics and related disciplines, and are accepted – at least temporarily – alongside the S.I.-system, are also included, e.g., bar, mbar, day, hour, minute, degree Celsius, hectare. The generally used phrase 'per unit' was avoided, because it is dimensionally incorrect. For instance, bulk density, usually defined as 'mass per unit volume', is here defined as 'mass divided by volume', or 'mass per volume'. As is standard practice, symbols for 'quantities' are printed in italics, while units are printed in roman characters. Failure to recognize this may, in some instances, lead to erroneous results (see, e.g., Answer 3.24).

Finally, we wish to thank G. H. Bolt and M. G. M. Bruggenwert for their efforts which helped to bring this rather eventful undertaking to a succesful conclusion. We also want to thank Mrs. I. E. Diraouï-van Rijsinge for typing the manuscript and P. A. C. Raats and J. H. Dane for commenting on various parts of the manuscript.

Wageningen, June 1983 P. K., G. M. and C. D.

CONTENTS

List of symbols

Symbol	Description	Unit
$\boldsymbol{a}$	acceleration	$m\,s^{-2}$
a	length of clay platelet (chapter 1)	m
	mathematical coefficient	–
A	area	m^2
	coefficient in infiltration equation	$m\,s^{-1}$
b	width of clay platelet (chapter 1)	m
	mathematical coefficient	–
B	coefficient in infiltration equation	$m\,s^{-\frac{3}{2}}$
$\boldsymbol{c}$	electrostatic field strength	$N\,C^{-1}$
c	concentration in solution (chapter 1)	$mol\,m^{-3}$
	differential water capacity (chapters 4 to 7)	m^{-1}
C	differential water capacity	Pa^{-1}
C_h	differential heat capacity	$J\,m^{-3}\,K^{-1}$
d	diameter	m
D	soil water diffusivity	$m^2\,s^{-1}$
D_0	diffusion coefficient of gas in free air	$m^2\,s^{-1}$
D_g	diffusion coefficient of gas in soil	$m^2\,s^{-1}$
D_h	heat diffusivity	$m^2\,s^{-1}$
E	energy	J
f	mass fraction	–
f_h	heat flux density	$W\,m^{-2}$
f_i	flux density of gas component *i*	$kg\,m^{-2}\,s^{-1}$
f_w	water flux density	$kg\,m^{-2}\,s^{-1}$
f_n	net radiation flux density at the soil surface	$W\,m^{-2}$
f_a	part of the net radiation flux density used for heating of air (chapter 7)	$W\,m^{-2}$
f_e	part of the net radiation flux density used for evaporation	$W\,m^{-2}$
f_s	part of the net radiation flux density used for heating of soil	$W\,m^{-2}$
f_t	part of the net radiation flux density used for transpiration	$W\,m^{-2}$
$\boldsymbol{F}$	force	N
F	magnitude of force	N
$\boldsymbol{g}$	gravitational field strength	$N\,kg^{-1}$
g	magnitude of gravitational field strength	$N\,kg^{-1}$
G	universal gravitational constant (chapter 2)	$N\,m^{-2}\,kg^{-2}$
G_i	mass of gas component *i* per volume of soil	$kg\,m^{-3}$

Symbol	Description	Unit
h	pressure head of soil water	m
h_a	pneumatic head of soil water	m
h_m	matric head of soil water	m
h_o	osmotic head of soil water	m
H	hydraulic head of soil water	m
i	infiltration rate of water	m s^{-1}
I	cumulative infiltration	m
$\mathbf{k}$	unit vector in vertical direction	–
k	hydraulic conductivity	m s^{-1}
K	hydraulic conductivity	m^2 Pa^{-1} s^{-1}
K_i	intrinsic permeability	m^2
l	length	m
m	mass	kg
M	molar mass	kg mol^{-1}
n	number of moles	mol
n_i	number of pores of size class i per area	m^{-2}
p	absolute pressure	Pa
p'	pressure relative to atmospheric pressure	Pa
p_a	pressure equivalent of pneumatic potential ($\approx$ excess gas pressure)	Pa
p_h	pressure equivalent of hydraulic potential	Pa
p_m	pressure equivalent of matric potential ($=$ matric pressure)	Pa
p_t	tensiometer pressure	Pa
pF	$^{10}\log(-h_m/\mathrm{cm})$	–
q	volume flux density of water	m s^{-1}
Q	volume flux of water (rate of discharge)	m^3 s^{-1}
	electrical charge (chapter 2)	C
	heat content per volume of soil (chapter 4 and 7)	J m^{-3}
r	radius	m
R	radius of curvature of gas-liquid interface	m
	molar gas constant ($= 8.314$ J mol^{-1} K^{-1})	J mol^{-1} K^{-1}
s	specific surface area (chapter 1)	m^2 kg^{-1}
	space coordinate (in direction of movement)	m
S	sorptivity	m s$^{-\frac{1}{2}}$
t	time	s
T	absolute temperature	K
u	potential extent of the double layer	m
v	velocity	m s^{-1}

Symbol	Description	Unit
V	volume	$\mathrm{m^3}$
w	wetness (gravimetric water content)	$(\mathrm{kg\,kg^{-1}})$
x	horizontal space coordinate	m
y	horizontal space coordinate	m
z	vertical space coordinate (height)	m
	gravitational head	m
z_c	height of capillary rise	m

Greek

α	production term in continuity equation	$\mathrm{kg\,m^{-3}\,s^{-1}}$ or $\mathrm{m^3\,m^{-3}\,s^{-1}}$
β	dummy variable	–
γ	surface energy of solid-liquid interface	$\mathrm{J\,m^{-2}}$
δ	thickness of clay platelet	m
η	viscosity (dynamic)	$\mathrm{Pa\,s}$
θ	volume fraction of liquid ($=\phi_l$)	–
λ	heat conductivity (chapter 4 and 7)	$\mathrm{W\,m^{-1}\,K^{-1}}$
	Boltzmann variable (chapter 5)	$\mathrm{m\,s^{-\frac{1}{2}}}$
Π	osmotic pressure	Pa
ρ_α	density of phase α	$\mathrm{kg\,m^{-3}}$
σ	surface energy of gas-liquid interface	$\mathrm{J\,m^{-2}}$
σ_s	soil pressure	Pa
σ_g	intergranular pressure	Pa
σ_w	water pressure	Pa
τ	tortuosity factor	–
φ	angle	rad
ϕ	porosity	–
ϕ_α	volume fraction of phase α	–
χ_i	mass concentration of gas component i	$\mathrm{kg\,m^{-3}}$
ψ	potential of soil water	$\mathrm{J\,kg^{-1}}$
ψ_a	pneumatic potential of soil water	$\mathrm{J\,kg^{-1}}$
ψ_g	gravitational potential of soil water	$\mathrm{J\,kg^{-1}}$
ψ_h	hydraulic potential of soil water	$\mathrm{J\,kg^{-1}}$
ψ_m	matric potential of soil water	$\mathrm{J\,kg^{-1}}$
ψ_o	osmotic potential of soil water	$\mathrm{J\,kg^{-1}}$

Symbol	Description	Unit
ψ_p	tensiometer pressure potential of soil water	$J\ kg^{-1}$
ψ_t	total potential of soil water	$J\ kg^{-1}$
ψ_α	adsorption potential of water in double layer	$J\ kg^{-1}$
ψ_δ	pressure potential of water in double layer	$J\ kg^{-1}$
ψ_ω	excess osmotic potential of water in double layer	$J\ kg^{-1}$
ω	angular frequency	s^{-1}

1 Composition and Physical properties of soils

1.1. COMPOSITION OF SOILS

1.1.1. Components and phases

Soil is the natural material that covers the land surface of the earth. It is the product of mechanical, chemical and biological weathering of parent material. Its four constituent fractions or components are

- mineral matter (both primary and secondary minerals)
- organic matter
- water solution
- soil air.

Soils are very complex in their composition and quite variable in their occurrence and properties. Their complex nature, changing behaviour upon use, and variable spatial distribution are the result of nonhomogeneous mixing and interacting of all their components.

In spite of this intricate nature, soils can be handled and studied systematically. Their components, like all other matter in nature, exist in the three states of matter, i.e. solid, liquid and gas. With some exceptions, all three states occur in soils side by side. Hence, these states are called 'phases' of the soil. Strictly speaking, the term 'phase' is not used properly here, since in chemistry it is reserved for a portion of a system having definite geometrical boundaries and uniform properties. The latter is not true here. Nevertheless, we will adhere to this terminology and discuss, successively,

- the solid phase
- the liquid phase
- the gas phase.

Question 1.1 Suggest some situations where only two of the three phases occur in soil.

Question 1.2 Is it possible for a soil to consist of only one phase?

Solid phase

The solid phase is composed of mineral and/or organic matter. The mineral portion consists of particles of various shapes, sizes and chemical composition. Minerals such as quartz, feldspar and colloidal silicates form the major part of the mineral particles. Their density (for definition, see Section *1.1.4*) varies between 2600 and 2850 kg m^{-3}. Heavier minerals, such as magnetite, garnet or hornblendes, also occur, but in small amounts. Generally, one can take 2700 kg m^{-3} as the average density of the mineral matter.

The organic matter consists of plant and animal residues. It also includes living organisms. This portion has densities of 1200 to 1500 kg m^{-3} (on the average 1400 kg m^{-3}). The presence of organic matter may highly influence the average density of the solid phase. For instance, some mineral soils with high organic matter contents have densities less than 2400 kg m^{-3}. For the same reason topsoils have lower densities than subsoils. Except for precise scientific work, 2650 kg m^{-3} can be taken as the average density of the solid phase of mineral soils. When the organic matter content exceeds 20%, soils are usually called organic rather than mineral. Their average density is usually hard to estimate and can better be measured.

Question 1.3 What would be your estimate for the density of a stone found in the soil?

The solid phase forms the matrix or 'skeleton' of the soil. The soil matrix, however, is not always as rigid as the word 'solid' suggests. Certain soils swell and shrink, i.e. the solid phase particles move with respect to one another. In other situations, part of the soil matrix may detach itself and be carried along by moving liquid. This is referred to as particle migration.

The spaces between the solid particles are called soil pores (Figure 1.1). Soil pores are cavities of different shape and size determined by the shape, size and arrangement of the solid particles. Normally the pores form a continuous space throughout the soil. Although larger pores are connected by narrow channels, there are strictly speaking no discrete pores.

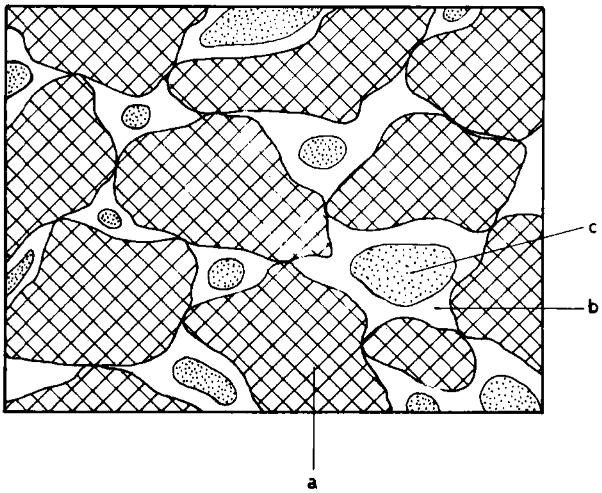

Figure 1.1 Cross-section of soil. Soil particles (a) forming soil pores, partly filled with liquid (b) and gas (c).

Within the pores, the liquid and gas phases reside and move. The portion of the pore space not occupied by the liquid phase is filled by the gas phase.

Question 1.4 Do isolated pores occur in soils?

Liquid phase

The liquid phase, also called soil solution, is an aqueous solution of various electrolytes. Ions commonly found in solution include Na^+, K^+, Ca^{2+}, Mg^{2+}, Cl^-, NO_3^- and SO_4^{2-}. Most of them are plant macro- and micro-nutrients. The liquid phase differs in composition, amount and mobility from pore to pore.

The soil solution may fill the soil pores either completely or partly. In the former case, the soil is said to be 'saturated'. In the latter 'unsaturated' case, the soil solution occurs as thin films along particle surfaces, as annular wedges around contact points of particles and as isolated bodies in narrow pore passages.

Though the density of the liquid phase depends on the composition of the electrolytes in solution, the difference from 1000 kg m^{-3} (the density of pure water at about 4 °C) is of no concern except in the most precise studies. Hereafter, in order to avoid confusion, we shall use the term 'pure water' for the pure chemical constituent H_2O. The terms liquid phase, soil water, water and aqueous solution will be used interchangeably for soil solution.

Question 1.5 Will a fragment of soil, when gently immersed in water, become completely saturated with water?

Gas phase

The gas phase, also known as soil air or soil atmosphere, is composed mainly of N_2, O_2, water vapour and CO_2, with traces of other gases. The soil air may vary in composition with prevailing conditions. Presence of organic matter, activity of plant roots and soil micro-organisms, soil aeration and chemical reactions all may alter the concentration of the gas components. Soil aeration is the gas exchange between soil and atmosphere. Even in a well-aerated soil the gas phase contains a higher CO_2 concentration and has a higher relative humidity – nearly vapour saturated – than atmospheric air. In the absence of aeration chemical reduction takes place, producing gases like methane, hydrogen sulphide and nitrous oxide. Ignoring trace gases, we can say that the volume fraction of N_2 is about 0.8 (the same as in the atmosphere) and that of CO_2 and O_2 together is about 0.2.

Because of variations in the composition, the density of soil air fluctuates from 1.0 to 1.5 kg m^{-3}. For calculations, an average of 1.3 kg m^{-3} may be taken.

Question 1.6 What would be the volume fraction of CO_2, if there were no aeration (and no chemical reduction)?

Question 1.7 A surface soil in good condition for plant growth may have the following volume fractions: mineral matter 0.45, organic matter 0.05, soil solution 0.2–0.3 and soil air 0.3–0.2.

a. What is the total volume fraction of the pores in this surface soil?
b. What is the volume fraction of liquid phase required to saturate this soil?

Question 1.8 Consider 1 m^3 of the surface soil of Question 1.7.

a. What is the mass of water required to saturate it?
b. What is the mass of air in it when completely dry?
c. What is the mass of the solid phase?
d. What is the mass of the soil when it is saturated? And when dry?
e. What is the mass of the soil when the volume fraction of liquid phase is only 0.25?

Question 1.9 Convert the average densities of the solid, liquid and gas phases given in the text to g cm^{-3}.

1.1.2 Texture and structure

The solid phase of soils consists of discrete units, called primary soil particles. These particles may vary widely in size, shape and composition. The particle-size distribution or texture of mineral soils determines to a large extent the physical and chemical behaviour. Since these distributions vary little with time, they are used to classify soils.

There are several classification systems in use, which differ only slightly in dividing the spectrum of particle sizes into ranges, called soil separates. The soil separates have been given names. Those recognized by the International Society of Soil Science (ISSS), and their equivalent diameters, are

coarse sand	2000–200 μm
fine sand	200– 20 μm
silt	20– 2 μm
clay	< 2 μm

The determination of the amounts of the various soil separates in a soil sample is called particle-size analysis. For particles larger than 50 μm this is done with sieves with circular holes or wire-mesh screens. Particle sizes smaller than 50 μm are determined by sedimentation, assuming the particles to be spherical (Section *2.1.4*). Of course, primary soil particles are not spherical. Sand and silt particles are isometric or isodiametric, while clay particles are shaped like plates, flakes, planks, or even rods. For this reason, the results of the sieving and sedimentation procedures are expressed in equivalent diameters.

The above soil separates are often divided further into subgroups. The most significant of these is the fine or colloidal clay with equivalent diameters between 1 nm and 0.2 μm. Others are the various grades of sand. Only particles with equivalent diameters smaller than 2 mm, sometimes called fine earth, are included in the particle-size distribution. Coarser soil particles are usually divided into gravel, with equivalent diameters between 2 and 75 mm, and stones which are larger than 75 mm.

Natural soils exhibit infinite combinations of mass fractions in the various soil separates. Soils are given soil textural class names according to the weight percentages of sand, silt and clay, as shown in Figure 1.2. Note that the boundary between sand and silt in this classification system is chosen at 50 μm, rather than 20 μm, All possible combinations are represented in the triangle and the soil textural classes are demarcated by heavy lines. To find the composition of the soil depicted by X, draw a line parallel to the sand axis to find the clay percentage, and parallel to the clay axis to find the silt percentage. The rest is sand. Soil X has weight percentages of 30, 30 and 40, respectively.

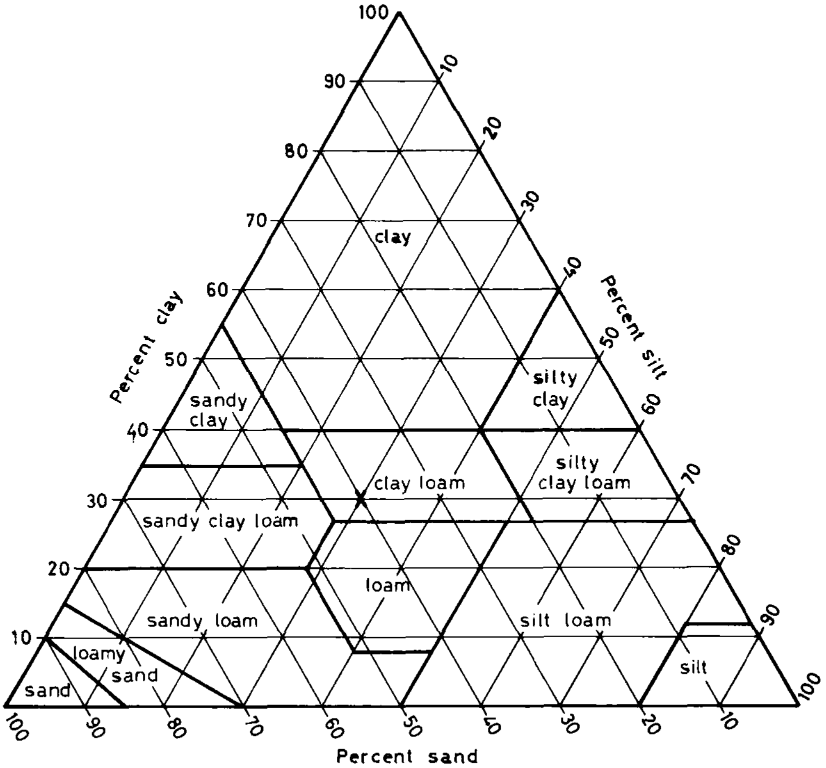

Figure 1.2 Textural soil classes determined by percentages of clay (< 2 μm), silt (2–50 μm) and sand (50–2000 μm).

Another physical property of soils is the soil structure (see also Section *1.4.4*). This is the spatial arrangement of the primary soil particles into compound particles, aggregates or peds and the stability of these units. Common names for structural types are platy, prismatic, columnar, blocky, granular and crumb. Both soil texture and soil structure directly influence the shape, size and volume fraction of soil pores. Soil structure governs characteristics of major concern for plant growth: aeration, temperature, movement of soil solution, microbial activity and root penetration.

1.1.3 Porosity

An adequate supply of soil solution and soil air, especially oxygen, to plant roots is essential for plant growth and crop production. Soil solution and soil air are stored and transported within the soil pores. Also plant roots exist in the soil pores. Thus, the volume fraction of soil pores, ϕ, commonly called porosity, is a very important property of soils. It is defined as

$$\phi = \frac{\text{total volume of pores}}{\text{volume of soil}} \qquad (1.1)$$

where volume of soil represents volume of pores plus volume of solids. The volume of soil is also referred to as bulk volume of soil.

Question 1.10 100 cm^3 of a dry soil requires 40 g of water to saturate it. Find its porosity.

Question 1.11 Assume that the soil in Question 1.10 is located on an irrigated farm. The farmer wants to saturate it down to 25 cm depth.

a. Calculate the volume of water he has to apply per cm^2 of soil surface.

b. If this were to be applied as a single application, what would be the depth of flooding?

Soils differ strongly in porosity. The following ranges have been found in cultivated surface soils:

sandy soils	0.35–0.60
clay soils	0.30–0.70
silty soils	0.30–0.60
peat soils	0.80–0.85
loamy soils	0.30–0.60

Storage, availability and transport of soil solution and soil air are not nearly as much dependent on the porosity as such, as on how the total pore space is partitioned, the so-called pore-size distribution. Soil pores differ in size and shape as a result of textural and structural arrangement (Section *1.1.2*). Based on the diameter at the narrowest point, pores may be classified as follows:

macropores	> 100 μm
mesopores	30 – 100 μm
micropores	< 30 μm

Although these limits are arbitrary, the functioning of these classes is roughly the following. Macropores conduct water only during flooding, ponding rain, etc. They soon drain after cessation of such water supply. Thus they affect aeration and drainage. Mesopores are effective in conducting water also after the macropores have become empty, such as during non-ponding rain, redistribution of water, etc. The remaining soil solution is retained or moves very slowly within the micropores. Part of this water can be taken up by plant roots.

Question 1.12 What is meant by 'drainage' here?

Sandy soils have large pores, whereas clay soils have small pores with or without larger fractures depending on shrinkage. Clay soils retain water more strongly than sandy soils (Section *3.1.5*). Generally, clay soils have a larger porosity and can retain more water than sandy soils, but this does not necessarily mean that more water is available for uptake by plant roots.

Soil porosity consists of gas-filled porosity or aeration porosity, ϕ_g, and liquid-filled porosity, ϕ_l, which is equal to the volume fraction of liquid, θ. For optimum plant growth the aeration porosity should be at least 0.10–0.12 within two to three days after irrigation or heavy rainfall.

1.1.4 Density

In Section *1.1.1* the densities of the three phases of soils were discussed without defining density. Density, also called volumic mass, is the mass of matter divided by its volume. It is expressed in kg m^{-3}. Accordingly, phase density, ρ_α, is the mass of phase α divided by the volume of phase α.

The phase density of the solid phase, also called particle density, is written as ρ_s, where s stands for solid. Those of the liquid and gas phases are written as ρ_l and ρ_g, respectively. In the same manner, the densities of the mineral and organic matter can be written as ρ_{min} and ρ_{org}, respectively. In Section *1.1.1* average densities of the solid, liquid and gas phase of soils were given. They were 2650, 1000 and 1.3 kg m^{-3}, respectively. The average solid-phase density can be calculated if the phase densities and volume fractions of all the constituents of the solid phase are known. Alternatively, it can be determined in the laboratory by measuring the volume of water displaced by a known mass of soil particles.

Question 1.13 To determine ρ_s, a surveyor took 82 g of powdered dry soil at random, put it in a glass bottle slightly larger than 100 cm^3, and added deaerated water at room temperature. After ascertaining that all the air in the soil was driven out, he filled the bottle to exactly 100 cm^3. (A bottle with which this can be done with great accuracy is called a pycnometer.) The mass of the water and soil was 151 g.

a. Calculate ρ_s.

b. Why was it necessary to drive out all the air?

The mass of soil divided by the total or bulk volume of soil is called bulk density, ${}^b\rho$, of the soil, where b stands for bulk. The bulk density is the volume-weighted mean density of the three phases:

$$ {}^b\rho = \phi_s \rho_s + \phi_l \rho_l + \phi_g \rho_g \tag{1.2} $$

where ϕ_α is the volume fraction of phase α:

$$ \phi_\alpha = \frac{\text{volume phase } \alpha}{\text{bulk volume}} \tag{1.3} $$

Question 1.14 What is the sum of the volume fractions of the three phases in any soil?

Question 1.15 Calculate the bulk density of a soil with volume fractions of solid, liquid and gas of 0.5, 0.3 and 0.2, respectively.

Question 1.16 a. Compare the contribution of each of the three phases to the bulk density in Question 1.15.

b. If only the contributions of the liquid and solid phases were taken, what would be the bulk density? Express the difference in density as a fraction of the former bulk density.

c. What do you conclude from Answers 1.16a and 1.16b?

Question 1.17 Suppose the soil in Question 1.15 were dry.

a. What would be its bulk density?

b. Compare the contribution of each of the phases to the bulk density.

c. What do you conclude from Answer 1.17b?

These answers illustrate that a distinction must be made between the wet bulk density, ${}^b\rho_w$, and the dry bulk density, ${}^b\rho_d$, of a soil sample. The former is the bulk density of the original wet sample; the latter is the bulk density of the sample after drying in an oven at 105 °C to a constant mass, based on the original volume.

Question 1.18 a. Derive an equation for dry soil analogous to Equation 1.2.

b. Express the difference between ${}^b\rho_w$ and ${}^b\rho_d$ as an equation.

Answers 1.16 and 1.17 show that the density of the gas phase is so low that the contribution of that phase to the bulk density can be neglected, regardless of the water content. Thus, the equations in Answer 1.18 can be simplified to

$$ {}^b\rho_w \approx \phi_s \rho_s + \phi_l \rho_l \tag{1.4} $$

$$ {}^b\rho_d \approx \phi_s \rho_s \tag{1.5} $$

and ${}^{\mathrm{b}}\rho_{\mathrm{w}} - {}^{\mathrm{b}}\rho_{\mathrm{d}} \approx \phi_{\mathrm{l}}\rho_{\mathrm{l}}$ (1.6)

Bulk densities can be determined by the core method, and by the clod or paraffin method. With the core method the determinations are made on undisturbed samples obtained from natural soil in a cylinder of known volume, generally 100 cm^3. With the clod method, no special effort is needed to obtain a sample of known volume. Instead, a soil clod is coated with paraffin wax and its volume determined by means of its submerged weight. This is discussed further in Chapter 2 (**Answer 2.13**). The core method is used whenever possible. In soils that swell and shrink, special precautions must be taken.

Question 1.19 A 100 cm^3 undisturbed soil sample has a mass of 162.5 g. After drying at 105 °C, its mass is reduced to 132.5 g.

a. Find ${}^{\mathrm{b}}\rho_{\mathrm{w}}$ and ${}^{\mathrm{b}}\rho_{\mathrm{d}}$.
b. Find ϕ_{s}, ϕ_{l} and ϕ_{g}.
c. Is the soil well-aerated?
d. Are the obtained values of ϕ_{l}, ϕ_{g} and ϕ_{s} true for every part of the soil sample?

A high dry bulk density may mean that the porosity is small and that the soil particles are closely packed or compacted. Soil compaction affects aeration, air and water permeability, infiltration rate, internal drainage, root growth and seedling emergence. Thus, soil compaction hinders crop production. Average dry bulk densities found in surface soils are

1600 kg m^{-3} for sandy soils
1100 kg m^{-3} for clay soils
1100 kg m^{-3} for loamy soils
250 kg m^{-3} for peat soils.

Dry bulk densities are needed for the conversion of gravimetric to volumetric water contents (Equation 1.7). Wet bulk densities together with dry bulk densities are used to estimate the volume fraction of air-filled pore space. Without specification, the term 'bulk density' is normally used for 'dry bulk density'.

1.1.5 Water content

It is difficult to measure soil water contents in situ. In addition, there is the problem of spatial variation. To overcome both obstacles, it is common practice to collect replicate soil samples in the field and to determine the water contents in the laboratory. The water content of a soil sample is, by convention, the amount of water removed by drying at 105 °C. This can be expressed as the mass ratio of water to solid phase, w, commonly called wetness or gravimetric water content. The water content can also be expressed as the volume fraction of water in bulk soil, θ, (Section *1.1.3*) commonly called volumetric water content (volumic water content would be a better term). The volume fraction of water is more difficult to determine than the wetness, but is more useful for calculations of water transport in soils, irrigation management, etc. We will use the term 'water content' only in a general, qualitative meaning. If specific water contents are meant, we will use volume fraction of water for θ and wetness for w.

Question 1.20 Express w in terms of ${}^{\mathrm{b}}\rho_{\mathrm{w}}$ and ${}^{\mathrm{b}}\rho_{\mathrm{d}}$.
Question 1.21 Express θ in terms of ${}^{\mathrm{b}}\rho_{\mathrm{w}}$ and ${}^{\mathrm{b}}\rho_{\mathrm{d}}$.

From Answers 1.20 and 1.21 follows immediately the often used relationship between θ and w:

$$\theta = \frac{{}^{\mathrm{b}}\rho_{\mathrm{d}}}{\rho_{\mathrm{l}}} w \tag{1.7}$$

Question 1.22 For a soil sample ${}^{\mathrm{b}}\rho_{\mathrm{w}} = 1716\ \mathrm{kg\,m^{-3}}$ and ${}^{\mathrm{b}}\rho_{\mathrm{d}} = 1430\ \mathrm{kg\,m^{-3}}$.
Calculate:
a. the volume fraction of water, θ
b. the wetness, w
c. the gas-filled porosity, ϕ_{g}.

1.1.6 Organic matter

Organic matter consists of plant and animal residues (Section *1.1.1*). Soils of bogs, marshes and swamps are chiefly composed of organic matter. Surface layers of mineral soils have mass fractions of organic matter of only 0.01 to 0.1; those of peat soils, however, can be as high as 0.80 to 0.95.

Despite the small amounts, organic matter has a profound influence on the physical and chemical properties of mineral soils. It promotes the formation and stability of soil structure and increases water retention, infiltration and drainage. In addition, it provides a source of nitrogen, phosphorus and sulphur, which are essential plant nutrients.

Although organic matter is continuously reworked and decomposed by micro-organisms, its resistant components only break down to finer materials. These finer materials, together with proteins synthesized by soil organisms, form humus.

Humus, like the mineral component clay, is colloidal. Its capacity to hold water and nutrient ions exceeds that of clay. When it is plentiful, it keeps the soil open and loose. Clay is generally present in larger amounts and its contribution to the properties of soil usually equals or exceeds that of humus. Both humus and clay are characterized by a large specific surface area (i.e. surface area divided by mass) and surface charge.

When the surface area of any body is very large in relation to its volume or mass, the special properties of the surface molecules (higher energy and capacity to attract other molecules which approach them) influence the properties of the material. Many properties are proportional to specific surface area. If particles are so small that surface effects dominate, they are called colloids. Phenomena effected by soil colloids are dispersion, swelling, shrinkage, flocculation, cohesion and plasticity (Section 1.3 and 1.4).

Organic matter is usually specified as a separate volume fraction, ϕ_{org}. Together with ϕ_{min}, the volume fraction of mineral matter, it constitutes the volume fraction of the solid phase, ϕ_s. The mass fractions of each of these components can be obtained from the loss of weight of an oven-dry sample upon complete oxidation of the organic matter at about 900 °C.

Question 1.23 a. Calculate ${}^{b}\rho_w$, ${}^{b}\rho_d$, w, θ, ϕ_{min}, ϕ_{org}, ϕ_s, ϕ, ϕ_g and ρ_s of a certain soil sample, if

volume of core sampler = 90 cm³
mass of core sampler = 120 g
mass of sampler and moist soil = 243 g
mass of sampler and oven-dry soil = 207 g
mass of sampler and soil after heating at 900 °C = 167 g

b. What is the significance of ρ_s here?

A soil sample should be as representative as possible for the field or natural soil body from which it is taken. If, for instance, a stone found in a clod or core sample is deemed unrepresentative of the field, a correction should be made for the volume and mass of the stone to obtain the bulk density and other parameters of the remaining part of the sample.

Question 1.24 Correct the results of Question 1.23 for a stone found in the core sample. The stone has a mass of 13 g and a density of 2800 kg m^{-3}.

1.2 SPECIFIC SURFACE AREA

Many of the soil properties are attributable to surface phenomena that take place at the interface between the liquid and the solid phase. These properties can be correlated – they are even approximately proportional – to the specific surface area of the solid phase, s, which is defined as the surface area of particles divided by their mass.

For a homogeneous granular material, the specific surface area can be calculated from the density, size and shape of the particles. If the soil is heterogeneous, the contribution of each component must be evaluated separately, and the total specific surface area is then the mass-weighted mean specific surface area of the components.

There are many methods of estimating the specific surface area of soils. It can be derived from the particle geometry, from adsorption characteristics, heat of immersion, etc. The following is an introduction to the geometrical method.

1.2.1 Regularly shaped particles

Consider a cube of side l. Its total surface area is $6l^2$, its volume l^3, and its mass $\rho_s l^3$. Therefore, its specific surface area is

$$s = \frac{6l^2}{\rho_s l^3} = \frac{6}{\rho_s l} \tag{1.8a}$$

Thus, s is inversely proportional to l.

Question 1.25 Calculate s for cubical particles with a density of 2650 kg m^{-3} and a side of 2 cm, 200 μm and 2 μm, respectively.

For a sphere of diameter d, the surface area and volume are πd^2 and $\frac{1}{6}\pi d^3$, respectively. Its mass is $\frac{1}{6}\rho_s \pi d^3$. Hence,

$$s = \frac{\pi d^2}{\frac{1}{6}\rho_s \pi d^3} = \frac{6}{\rho_s d} \tag{1.8b}$$

This result is identical to that of Equation 1.8a, except that l is replaced by d.

Consider a regular polyhedron with n polyhedral planes and $\frac{d}{2}$ the normal from the centre of the polyhedron to each of the polyhedral planes. Subdivide the regular polyhedron into n polyhedrons, each polyhedron with apex at the centre of the regular polyhedron and with one of the regular polyhedral planes as a base. Now, each polyhedron has a height of $\frac{d}{2}$ and a volume of $\frac{1}{3} \times A \times \frac{d}{2}$, where A is the area of each polyhedral plane. Therefore, the volume of the regular polyhedron is $n \times \frac{1}{3} \times A \times \frac{d}{2} = \frac{1}{6}nAd$, its surface area is nA and its mass is $\rho_s \frac{nAd}{6}$. Hence,

$$s = \frac{nA}{\rho_s \frac{nAd}{6}} = \frac{6}{\rho_s d} \tag{1.8c}$$

Equation 1.8a to 1.8c show for all these regularly shaped particles the same relationship between the specific surface area and a characteristic length. The smaller the particles, the larger is the specific surface area.

1.2.2 Plate-shaped particles

So far, we have derived equations for specific surface area of particles characterized by equal edges, e.g., cubes or regular polyhedrons. Such particles do not occur in nature, except in sand which has more or less spheroid particles.

Consider a cube of side l, sliced parallel to one of its sides into n plates, each of thickness $\delta = l/n$. The original cube has a surface area of $6l^2$; each time a slice is cut off, the total surface area increases with $2l^2$. Thus, after cutting $n - 1$ times to obtain n plates, the total surface area becomes $6l^2 + 2l^2(n-1)$.

Question 1.26 a. Using this result, derive an equation for s in n, δ and ρ_s.

b. Give an approximation of s, for n is large.

Consider a plate-shaped, rectangular particle with δ as thickness and a and b as length and width. Its volume is $ab\delta$, its mass $\rho_s ab\delta$, and its total surface area $2(ab + \delta a + \delta b)$. Thus, the specific surface area is

$$s = \frac{2(ab + \delta a + \delta b)}{\rho_s \delta ab} = \frac{2}{\rho_s \delta}\left(1 + \frac{\delta}{b} + \frac{\delta}{a}\right) \tag{1.9a}$$

or

$$s = \frac{2}{\rho_s}\left(\frac{1}{\delta} + \frac{1}{b} + \frac{1}{a}\right) \tag{1.9b}$$

If $\delta = a = b = l$, Equation 1.9a reduces to Equation 1.8a. If a and b are kept constant, and δ is reduced, s increases (Equation 1.9b). If $\delta \ll a$ and $\delta \ll b$, which is true for thin plates, Equation 1.9a reduces to

$$s \approx \frac{2}{\rho_s \delta} \tag{1.9c}$$

This is the same result as obtained in Answer 1.26. If $\delta \ll a$, but $\delta = b$, i.e. a rod-shaped particle, Equation 1.9a reduces to

$$s \approx \frac{4}{\rho_s \delta} \tag{1.9d}$$

Equations 1.9 do not apply to objects with rough or bulging shapes.

Question 1.27 a. Which equations are appropriate for estimating specific surface area of sand, silt and clay particles?

b. Calculate the maximum specific surface area of sand and silt. $\rho_s = 2650\ \mathrm{kg\ m^{-3}}$

c. What do you expect about the specific surface area of clay?

Question 1.28 What general characteristic does one observe in Equations 1.8a to 1.9d?

Question 1.29 a. Calculate the specific surface area of plate-shaped clay particles with $\delta = 1$ nm, $a = b = 20$ nm and $\rho_s = 2750\ \mathrm{kg\ m^{-3}}$.

b. Give the result also in $\mathrm{km^2\ kg^{-1}}$ and $\mathrm{ha\ kg^{-1}}$.

c. Find the number of particles of the size specified in Question 1.29a that would make a total mass of 1 kg.

Question 1.30 a. Calculate $\frac{\delta}{a}$ for the data in Question 1.29.

b. Does s differ significantly from that in Answer 1.29a if one uses Equation 1.9c instead of Equation 1.9b?

Question 1.31 Assuming $a = b$, calculate the percent error in s when Equation 1.9c is used instead of Equation 1.9a for the following values of $\frac{\delta}{a}$: 0.05, 0.02, 0.01, 0.005, 0.002, 0.001 and 0.0005.

Question 1.32 If $\delta = b = 1$ nm and $a = 20$ nm (a rod-shaped particle), is s significantly different if calculated with Equations 1.9c and 1.9d instead of with Equation 1.9a?

Question 1.33 If clay particles would have circular bases of radius r, what would the use of Equation 1.9c imply?

1.2.3 Soils

The specific surface area of a soil of which the particle-size distribution is known, can be calculated by the summation equation

$$s = \sum_i f_i s_i \tag{1.10}$$

where f_i is the mass fraction and s_i is the specific surface area for particle size group i.

Question 1.34 Calculate the specific surface area of a soil that has the following particle-size distribution:

80% fine sand	(average d = 0.1 mm)
10% silt	(average d = 10 μm)
10% illite clay	(average $a = b = 200$ nm and $\delta = 5$ nm).

The density of the solid phase for sand and silt is 2660 kg m^{-3}, and for illite clay 2750 kg m^{-3}.

Question 1.35 a. What is the contribution of each of the textural separates to the specific surface area of the soil in Question 1.34?

b. What is the textural class name of this soil?

c. To which component can most of the physical and chemical properties of such a soil be attributed?

In summary, Equations 1.8a and 1.8b show that a sphere and a cube have the same specific surface area when the diameter and the edge length are equal. If one side of a cube is reduced while the base remains the same, Equation 1.9c shows that the specific surface area increases. In other words, the thinner the particle, the larger the specific surface area. Clay consists of very thin, plate-shaped particles and thus has a much larger specific surface area than sand or silt, which consist of isodiametric particles of relatively large size. The larger the specific surface area, the larger the surface activity or colloidal activity.

1.3 INTERACTION BETWEEN SOLID AND LIQUID PHASES

Soil particles range in size from slightly larger than molecular size to stones (Section *1.1.2*). The larger particles form the framework or skeleton of the soil, while the smaller ones fill the space around the contact points and cover the surfaces of the larger particles. The liquid phase in the soil pores is in physical contact with the solid phase. At the contact surface, forces of electrical nature cause phenomena such as swelling, shrinkage, aggregation, flocculation and dispersion, which in turn influence transport of air and soil solution. They are typical of soils containing clay and humus (Section *1.1.6*).

1.3.1. Clay minerals

The clay fraction (Section *1.1.2*) consists, by definition, of solid particles less than 2 μm in equivalent diameter. It differs from the coarser sand and silt fractions in that it is composed mainly of minerals which are newly constituted from the products of weathering, which are not encountered in unweathered rocks. The so-called clay minerals constitute most, but not necessarily all, of the subfraction finer than 0.2 μm, known as colloidal clay. Clay minerals are complex in nature. Only a few essentials will be mentioned here.

Clay minerals are built up of silica and alumina sheets and are classified into types 2:1 and 1:1. The former consist of two sheets of silica with one sheet of alumina between them; the latter are formed from one sheet of silica and one sheet of alumina. The structural elements of the sheets are the silicon tetrahedron and aluminum octahedron. The silicon tetrahedron is made up of one silicon atom surrounded by four oxygen atoms; the aluminum octahedron consists of an aluminum atom surrounded by six hydroxyl groups or oxygen atoms. The tetrahydral and octahydral sheets have similar dimensions and are held together with covalent bonds, forming clay mineral layers.

Examples of clay minerals are kaolinite, various micas, illite, chlorite, vermiculite, montmorillonite and a group of allophanes. The most significant of these is montmorillonite; it is characterized by a great degree of swelling and shrinkage (Section *1.3.4*).

The specific surface area of clay minerals, which governs many soil properties, varies from one mineral to another. It is 700–800 $m^2 g^{-1}$ for montmorillonite, 300–500 $m^2 g^{-1}$ for vermiculite, 100–300 $m^2 g^{-1}$ for mica and 5–100 $m^2 g^{-1}$ for kaolinite.

The thickness of clay mineral layers varies from mineral to mineral. It is about 1 nm for montmorillonite, illite and mica, 1.4 nm for vermiculite, and 0.7 nm for kaolinite.

The planar extensions (i.e. length and width) of clay mineral layers varies from 100 nm to 2 μm. The ratio thickness/planar extension varies between about 0.5×10^{-3} and 10×10^{-3}, which means that Equation 1.9c may be applied for clay minerals.

Question 1.36 What are the symbols in Section *1.2.2* corresponding to the planar extensions and thickness of clay mineral layers?

Question 1.37 Can you guess why clay particles have different specific surface areas? (Assume that they have equal planar extensions.)

Question 1.38 a. Which clay mineral has the specific surface area calculated in Answer 1.29a?

b. For constant planar extensions as in Question 1.29a, calculate the specific surface area if δ is 1, 2, 3, 4, 5 or 10 nm.

1.3.2 Surface charge of clay minerals

During the formation of clay minerals the tetrahedral and octahedral sheets do not meet as already finished sheets. Rather, they grow together, unit by unit. During growth, Si, Al and Mg are hardly ever present in the ideal ratios required for the clay mineral. Therefore, if Al^{3+}-ions are present in excess, they may occupy Si^{4+}-sites in the crystal lattice. Similarly, Mg^{2+}-ions may occupy sites of Al^{3+}-ions if there is a shortage of Al^{3+}-ions. This process leads to an excess negative charge in the crystal lattice, which manifests itself as specific surface charge density.

The replacement of one cation by another of similar size (e.g. Si^{4+} by Al^{3+} and Al^{3+} by Mg^{2+}) is known as isomorphous substitution. It is named isomorphous because these substitutions hardly affect the shape of the crystal lattice. Only cations that fit in the lattice without making it unstable, can be substituted. These substitutions take place only while clay minerals are being formed. Once a clay particle is formed, extraction of the substituting cation is hardly possible without destroying the lattice.

In some clay minerals there is no isomorphous substitution. Then the clay particles remain electrically neutral, except that there is another source of charge. This is a result of incomplete charge compensation of the ions at crystal lattice edges. Below pH = 7, the edges of clay particles have a positive electrical charge.

1.3.3 Adsorption and dissociation of counterions

Since overall electrical neutrality is always maintained in nature, the net negative charge of a clay platelet is compensated by an equivalent positive charge of cations, such as Na^+, K^+, H^+, Mg^{2+} and Ca^{2+}, which are either adsorbed at the platelet surface or swarm in the surrounding solution. These cations counteract the negative electrical charge of the clay platelet towards its surroundings, and are, therefore, also called counterions.

Question 1.39 What is the main binding force between adsorbed cations and a clay platelet?

Question 1.40 Which influence does a clay platelet have on anions?

Question 1.41 What is the difference in electrostatic attraction by a clay platelet on Na^+-ions and Ca^{2+}-ions?

Question 1.42 How does the electrostatic attractive force change, qualitatively, with distance from a clay platelet?

Question 1.43 What is the electrical status of a clay platelet dried out from a soil solution?

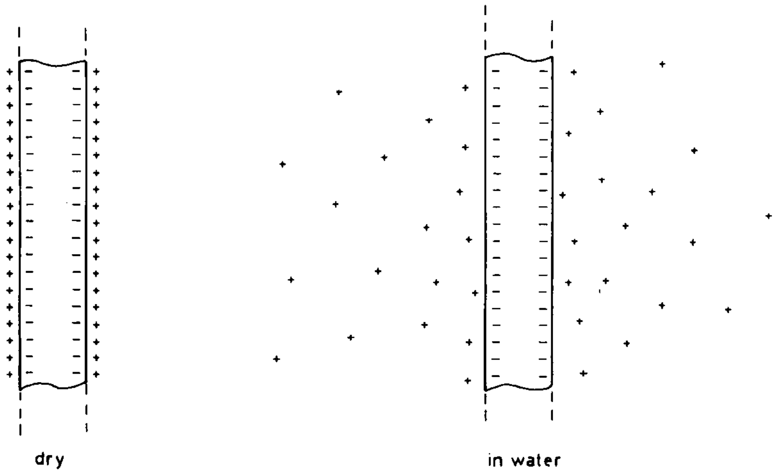

Figure 1.3 Dissociation of cations upon wetting of a dry clay platelet.

Upon wetting a dry clay platelet with water, loosely adsorbed cations dissociate from the surface of the platelet (Figure 1.3), while strongly bound cations form a thin layer on the surface of the platelet. The tendency of the cations to dissociate is due, in part, to the high value of the permittivity (dielectric constant) of water caused by its dipolar nature. Due to their thermal motion (kinetic energy), the dissociated cations have a strong tendency to diffuse away from the clay particle and attain the same concentration everywhere. This represents the condition of maximum entropy of the system (Figure 1.4b). However, the cations remain electrically attracted to the negatively charged clay particle. This tends to drive the system towards the condition of lowest energy (Figure 1.4a). The net result of these opposed influences is the condition of minimum free energy, in which the concentration of cations is very high near the surface of the clay platelet, but decreases with distance from the platelet, at first fast, and then gradually (Figure 1.4c). The portion of the soil solution where the cationic concentration is uniform, is called the bulk solution.

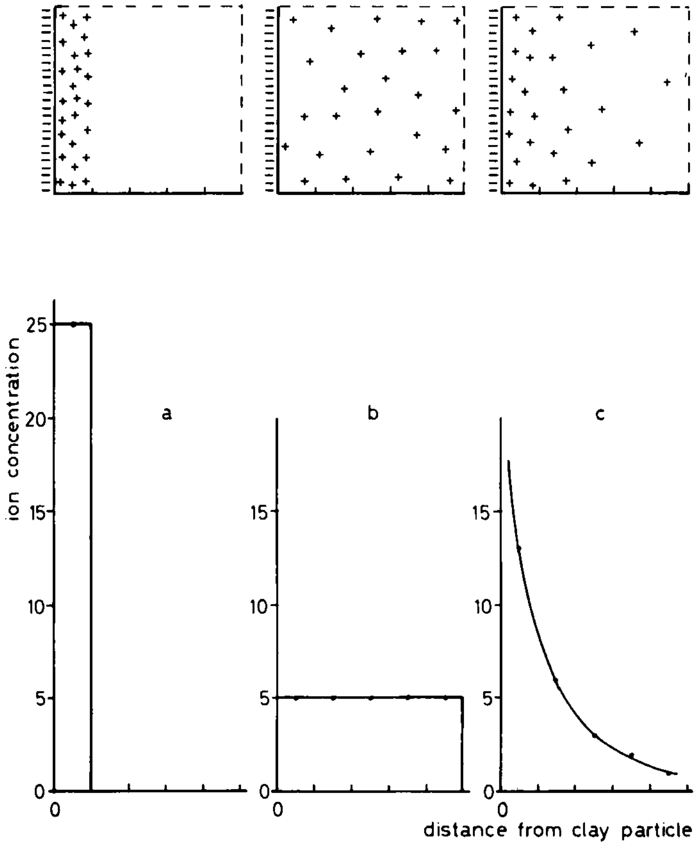

Figure 1.4 Concentration of counterions in the double layer. a: condition of minimum energy; b: condition of maximum entropy; c: condition of minimum free energy.

The charged surface of a clay platelet with its swarm of counterions is called a diffuse electrical double layer. The term double layer refers to the spatial separation of the negatively charged clay platelet and the positively charged counterions. The latter can be divided into two layers: a fixed layer, also called Stern layer, and a diffuse layer. The fixed layer consists of cations strongly adsorbed at the surface of the clay platelet, whereas the diffuse layer is the adjacent zone of more loosely bound cations. The thickness of the fixed layer is little more than the diameter of a single hydrated cation.

Question 1.44 What causes part of a double layer to be diffuse?

The extent of the double layer, *u*, is defined as the distance from the clay surface at which the cation concentration reaches a uniform value or a minimum. It is also the distance over which the electrical influence of the clay platelet on its surroundings vanishes.

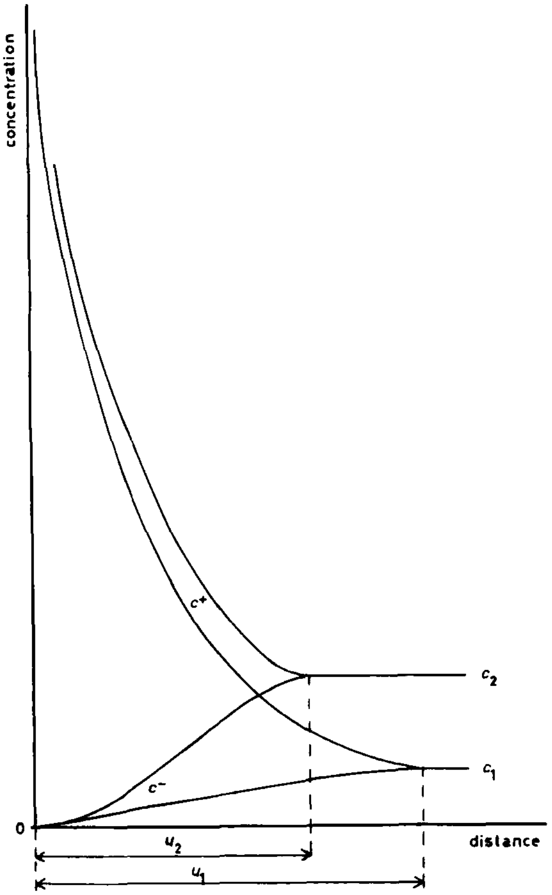

Figure 1.5 Concentration of cations and anions in the double layer of a clay particle for two different concentrations of the bulk solution, c_1 and c_2.

Since the bulk solution is electrically neutral, it contains positive and negative ions in equal concentrations of ionic equivalent. In the double layer, however, the anion concentration decreases towards the clay surface (Figure 1.5). The difference between the total positive electrical charge of the cations and the total negative charge of the anions in the double layer is equal to the negative charge of the clay platelet.

Question 1.45 Why does the anion concentration increase with distance from a clay platelet?

Question 1.46 What will be the influence of an increase in salt concentration in the bulk solution on the extent of a double layer?

Question 1.47 Suppose the cations in the diffuse electric double layer of Figure 1.5 are Na^+-ions and are replaced by an equal ionic equivalent of Ca^{2+}-ions. (Take only the lower concentration of the bulk solution.)

Describe, qualitatively, how the equilibrium distribution of ionic equivalents in the diffuse layer will change. Use a sketch.

1.3.4 Swelling and shrinkage

At low soil water contents, the soil solution around clay particles is present as thin films. The double layer is then prevented from attaining its full (potential) extent. Such a double layer is referred to as a truncated double layer. Figure 1.6 shows schematically an extended and a truncated diffuse double layer.

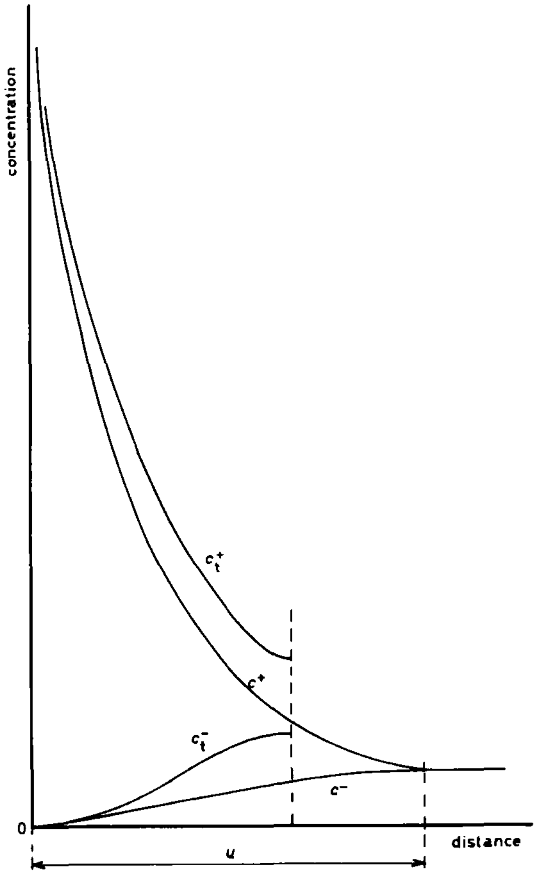

Figure 1.6 Concentration distribution in a fully extended (c^+, c^-) and in a truncated double layer (c_t^+, c_t^-).

A truncated double layer has a higher cation concentration than an extended one. Thus it exhibits the capacity to absorb water by osmosis. If the particles can move unhindered, they are pushed apart until repulsion becomes negligible. If the particles are not free to move, the repulsion force expresses itself as a swelling pressure.

The amount of swelling depends upon the potential extent of the double layer. This potential extent depends on the clay type, the types of counterions and the salt concentration of the bulk solution. For instance, montmorillonite exhibits more swelling than other clay minerals under the same conditions. Upon withdrawal of water, e.g. due to root uptake, evaporation, etc., the extent of the double layer decreases and the soil shrinks.

Swelling involves separation of the smallest, primary soil particles. This often occurs at the expense of the larger pores and cracks, without significant increase in the total porosity. As a result of swelling the permeability of a clay soil for water can be reduced by many orders of magnitude (Section *5.2.1*). Thus swelling hampers the growth of plants because it impedes the movement of soil solution and soil air. Shrinkage, on the other hand, forms cracks between soil aggregates, which increase the permeability for rain and irrigation water of clay soils which otherwise would be impermeable.

Question 1.48 Fill out 'large' or 'small' ('high' of 'low'):

A soil will swell more strongly if

a. its specific surface area is ...

b. its potential extent, u, of the double layer is ...

c. its actual extent of the double layer is ...

d. its clay content is ...

e. its dry bulk density is ...

f. its content of monovalent ions is ...

g. its concentration of salt in the soil solution is ...

Question 1.49 A non-saline alkali soil ($u = 20$ nm) contains 33.3% illitic clay ($s = 90$ m^2 g^{-1}), while $^{b}\rho_w = 1700$ kg m^{-3} and $^{b}\rho_d = 1300$ kg m^{-3}.

a. Calculate the specific surface area of the soil.

b. Calculate the volume fraction of water.

c. Calculate the average thickness of the water films around the particles.

d. How will this soil respond to rain?

1.4 INTERACTIONS BETWEEN SOLID PHASE PARTICLES

1.4.1 Plate condensation

If the distance between adjoining clay platelets is strongly reduced by removal of water, the counterions in the double layers will be forced into the central plane between them. The positive charges of the counterions and the negative charges of the clay surfaces may then approach each other close enough to exert a very large mutual

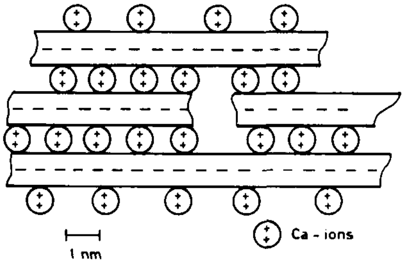

Figure 1.7 Plate-condensation of Ca-clay.

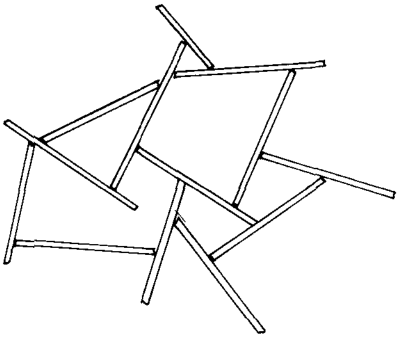

Figure 1.8 'Card-house' structure of floccules.

attraction. In clays with a high percentage of Ca^{2+}-ions (or other polyvalent ions) this attraction may be strong enough to lead to a stable condition called plate condensation (Figure 1.7), which prevents reabsorption of water. In Na-clays the attraction is, generally, not strong enough to prevent swelling. This is partly due to the higher osmotic pressure.

Question 1.50 Why is the osmotic pressure in the double layer higher for Na-clay than for Ca-clay?

Question 1.51 How is the stability of plate condensation influenced by the salt concentration of the soil solution?

Question 1.52 Can you suggest some sources of salts in soil?

1.4.2 Flocculation

The edges of clay platelets have a positive electrical charge at pH below 7. If the extent of the double layer is small, the positively charged edges may approach the negatively charged surfaces of the platelets sufficiently to form weakly bonded floccules. This process is called flocculation. Floccules are loose combinations of clay particles, similar to 'card-house' arrangements (Figure 1.8). Since the stability of these arrangements decreases with increasing extent of the double layer, a Ca-clay will flocculate at a lower salt concentration of the soil solution than a Na-clay.

Clays suspensions probably initially flocculate in card-house arrangements. With slow drying, the floccules tend to orient and stick together in the same manner as with plate-condensation.

Peptisation, dispersion or deflocculaltion is the reverse process of flocculation, namely the separation into primary particles. This can be achieved either chemically or mechanically.

When a flocculated clay is dried, it forms a sediment with extensive cracks. It also tends to be crumbly. When this sediment is rewetted, the crumbs as well as the sediment as a whole are stable. When a dispersed clay is dried, it forms a hard crust. If the wetting and drying cycle is repeated, hard, large clods are formed. When the sediment is rewetted, it forms a sticky mud without structure. This is typical of Na-clays, but not of Ca-clays.

Question 1.53 Explain briefly how plate condensation and card-house type flocculation take place in a clay suspension.

Question 1.54 a. How can one deflocculate a flocculated clay?
b. Which laboratory determination already mentioned requires this treatment?

1.4.3 Cementing agents

The primary soil particles within an aggregate may be bound together by certain substances called cementing agents. The main cementing agents are organic matter, silicate clays, lime and sesquioxides.

Humus, which is colloidal organic matter like clay, adsorbs cations. If humus contains a high proportion of Ca^{2+}- and other divalent cations, its long polymer chains can form bonds with each other and with the mineral components of the solid phase. It also binds clay domains to quartz, which is the primary mineral component of silt and sand. In this way, a stable 'clay-humus complex' is formed, resulting in aggregates. In soils rich in Na^{+}- (alkali soils) or H^{+}-ions (acid soils), the bonds are unstable and the humus dissolves.

Silicate clays, too, may cement particles together, but their binding effect is much smaller than that of humus. The degree of cementing and the kind of aggregates formed, vary with the electrical charge, its distribution on the clay particles and the type of clay involved.

Other cementing agents for sand and silt are lime (mainly $CaCO_3$) and sesquioxides (Al- and Fe-oxides). Lime, when it precipitates around contact points between soil particles, acts as a cement. The binding effect of iron oxide is doubtful, but aluminum oxide is probably effective. Finally, soil organisms also may keep soil particles together by their activity and sometimes by their byproducts.

Question 1.55 How can one destroy the binding action of the different cementing agents for particle-size analysis?

1.4.4 Soil structure and structural stability

The spatial arrangement or clustering of primary soil particles into secondary units, called aggregates or peds, is known as soil structure (Section *1.1.2*). Soil structure influences water transport, soil temperature, air transport and mechanical impedance of soil to seedling emergence and root penetration. It is nearly impossible to measure soil structure directly. The spatial arrangement and orientation of soil particles and soil pores as seen in thin sections, probably represents soil structure the closest. Soil structure is often, somewhat arbitrarily, described by the sizes and shapes of aggregates. It is also characterized by porosity and pore-size distribution (Section *1.1.3*). These are an indication of the soil's ability to retain water, to allow water to infiltrate and to make water available to crops. One can also measure parameters such as soil air permeability, infiltration rate, bulk density, aeration porosity and penetration resistance. Though indirectly, these measurements all render information about soil structure.

Question 1.56 Why is soil structure important for plant growth?
Question 1.57 Why is structure only partly defined by porosity and pore-size distribution (volume fractions of pore size classes)?

Processes such as plate condensation and flocculation of clays and humus enhance structure formation, but are by themselves not sufficient. Structure and its stability depend upon the processes of formation, as well as the presence of adequate binding agents. If sufficient amounts of cementing agents are present, even sandy soils can form stable fabrics with many large and small pores produced by soil fauna, roots or frost.

Structural stability of aggregates is as relevant as structure itself. It is the resistance of soil structure to mechanical and physico-chemical destructive forces. This resistance is determined by the attractive forces between the soil constituents under various conditions. For instance, lack of stability of soil structure may seal the soil surface and reduce water infiltration, hamper aeration and impede emergence of seedlings. The process by which dry soil aggregates disintegrate upon wetting is called slaking. This may be aggravated by air locked inside aggregates, while a dry soil is being flooded or infiltrated. The pressure of occluded air may then build up such that the aggregates explode. When the disintegrated aggregates dry, a soil crust may be formed.

Other physico-chemical forces include swelling, shrinkage, flocculation and dispersion. Frost action may also improve soil structure. Growing ice lenses displace soil particles and form large pores. The water being accumulated in the ice lenses causes decreasing water contents elsewhere, often leading to plate condensation, flocculation, etc.

Examples of mechanical, external forces are tillage, movement of farm machines, treading by animals and raindrop splash. Tillage loosens the soil during seedbed preparation. Farm machines compact the soil as they move over the field for various purposes. Animals compact the soil by treading or puddle the soil when it is wet. Raindrops disperse the surface soil by impact; splashed particles are detached and displaced or carried away.

Question 1.58 a. Do you know a tillage procedure where the surface soil is dispersed on purpose?
b. Why is this done?

1.5 SUGGESTED LITERATURE

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1.6 ANSWERS CHAPTER 1

Answer 1.1 The only combinations of two phases that occur together in soil are:

a. the solid and liquid phase;
b. the solid and gas phase.

The former occurs when the soil pores are completely filled with the liquid phase. The latter occurs when the soil is absolutely dry.

Answer 1.2 The only possibility for a soil to consist of one phase would be solid rock, and this is not considered to be a soil.

Answer 1.3 A good estimate for the density of a stone is 2700 kg m^{-3}, unless there are a great deal of clearly recognizable heavy minerals.

Answer 1.4 Isolated pores may occur in soil. Since they are not part of the continuous pore space, they do not contribute to the transport of the liquid and gas phases. In cultivated agricultural soils, they hardly exist.

Answer 1.5 In general, the fragment of soil will not become completely saturated. There usually are pores from which air escapes only partially, or not at all, during wetting. Even though the soil is often called saturated, this is strictly speaking not correct, as long as such pores exist. The air in such pores is known as entrapped air or occluded air. This occurs, in practice, during infiltration of water, and to a lesser extent when the ground water is rising.

Answer 1.6 If there is no aeration, all the O_2 would be consumed and replaced by CO_2 produced by biochemical reactions. Thus, the volume fraction of CO_2 would be up to 20%.

Answer 1.7 a. Since the solid phase, i.e. organic and mineral matter, occupies 50% by volume, the rest (50%) is pore space.

b. When the soil is saturated, the volume of the liquid equals that of the pore space. Hence, it is 50%.

Answer 1.8 a. When saturated, 50% by volume of the soil is filled with water. This amounts to 0.5 m^3 of 500 kg water.

b. When completely dry, there is 0.5 m^3 air in the soil. For an average density of 1.3 kg m^{-3}, this is 0.65 kg air.

c. The mineral matter occupies 45% of the volume and the organic matter 5%, or 0.45 m^3 and 0.05 m^3, respectively. Since the densities are approximately 2700 kg m^{-3} and 1400 kg m^{-3}, the mass contributions are 1215 kg and 70 kg, respectively. Thus the mass of the solid phase is 1285 kg.

d. When the soil is saturated, its mass is 1785 kg (Answers 1.8a and 1.8c). When it is dry, its mass is 1285.65 kg (Answers 1.8b and 1.8c).

e. The mass of the liquid phase is 0.25×1000 kg = 250 kg. The mass of the air is 0.25×1.3 kg = 0.325 kg. Thus, the total mass is $(1285 + 250 + 0.325)$ kg = 1535.325 kg.

Of course, this can be rounded off to 1535 kg (see Answer 1.16).

Answer 1.9 The average density of the solid phase is:

$$2650\ \mathrm{kg\,m^{-3}} = \frac{2650 \times 10^3\ \mathrm{g}}{10^6\ \mathrm{cm^3}} = 2.65\ \mathrm{g\,cm^{-3}}$$

The average density of the liquid phase is:

$$1000\ \mathrm{kg\,m^{-3}} = \frac{1000 \times 10^3\ \mathrm{g}}{10^6\ \mathrm{cm^3}} = 1.0\ \mathrm{g\,cm^{-3}}$$

The average density of the gas phase is:

$$1.3\ \mathrm{kg\,m^{-3}} = \frac{1.3 \times 10^3\ \mathrm{g}}{10^6\ \mathrm{cm^3}} = 0.0013\ \mathrm{g\,cm^{-3}}$$

Answer 1.10 If the density of the water is 1.0 g $\mathrm{cm^{-3}}$, 40 $\mathrm{cm^3}$ is needed to completely fill the pore space. Hence, per definition, the porosity is 40 $\mathrm{cm^3}$/100 $\mathrm{cm^3}$ = 0.40.

Answer 1.11 a. The volume of soil to be saturated under every $\mathrm{cm^2}$ of soil surface is 25 $\mathrm{cm^3}$. Since the porosity is 0.40, the water in this volume is 0.40 × 25 $\mathrm{cm^3}$ = 10 $\mathrm{cm^3}$. Therefore, he has to apply 10 $\mathrm{cm^3}$ per $\mathrm{cm^2}$ surface area.

b. The depth of flooding would be 10 $\mathrm{cm^3}$/1 $\mathrm{cm^2}$ = 10 cm.

Answer 1.12 By 'drainage' is meant the loss of excess water from the root zone to the lower regions of the soil profile, under the influence of gravity. Lack of drainage impedes the normal functioning of roots because they cannot obtain enough oxygen. Excess drainage causes plants to suffer from lack of water.

Answer 1.13 a. Volume of pycnometer = 100 $\mathrm{cm^3}$

mass of solids and water = 151 g

mass of soil solids = 82 g

mass of water only = (151−82) g = 69 g

$$\text{volume of water} = \frac{69\ \mathrm{g}}{1\ \mathrm{g\,cm^{-3}}} = 69\ \mathrm{cm^3}$$

volume of soil solids = (100−69) $\mathrm{cm^3}$ = 31 $\mathrm{cm^3}$

$$\rho_s = \frac{\text{mass of solids}}{\text{volume of solids}} = \frac{82\ \mathrm{g}}{31\ \mathrm{cm^3}} \approx 2.65\ \mathrm{g\,cm^{-3}} = 2650\ \mathrm{kg\,m^{-3}}$$

b. If all the air in the soil is not driven out, the mass of solids and water in the pycnometer will be too small. As a result, the calculated volume of the solid is larger than what it actually is. This results in a value for ρ_s which is too small.

Answer 1.14 A volume fraction is defined as the ratio of the volume of a phase and the total or bulk volume of soil. Since the sum of the volumes of the soil phases is equal to the total volume of the soil, the sum of the volume fractions equals unity, i.e. $\phi_s + \phi_l + \phi_g = 1$.

Answer 1.15 ${}^{b}\rho = \phi_s\rho_s + \phi_l\rho_l + \phi_g\rho_g = (0.5 \times 2650 + 0.3 \times 1000 + 0.2 \times 1.3)$ kg $\mathrm{m^{-3}}$ = 1625.26 kg $\mathrm{m^{-3}}$.

Answer 1.16 a. The contributions of the three phases are 1325 kg, 300 kg and 0.26 kg for the solid, liquid and gas phases, respectively. Thus the contributions of the liquid and solid phases are each more than 1000 times that of the gas phase.

b. It would be 1625.0 kg $\mathrm{m^{-3}}$.

The deviation is $\frac{(1625.26 - 1625.0)\,\text{kg m}^{-3}}{1625.26\,\text{kg m}^{-3}} \approx 1.6 \times 10^{-4}$

c. The contribution of the gas phase is negligible compared to the contributions of the other two phases. Therefore, it can safely be neglected.

Answer 1.17 a. Since the soil is dry, the volume of the liquid phase is taken over by the gas phase. Therefore, the gas phase occupies 50% of the bulk volume.

$^{b}\rho = \phi_s \rho_s + \phi_g \rho_g = (0.5 \times 2650 + 0.5 \times 1.3)\,\text{kg m}^{-3} = 1325.65\,\text{kg m}^{-3}$

b. The solid phase contributes 1325 kg m^{-3}, while the gas phase only 0.65 kg m^{-3}.

Thus, the contribution of the solid phase is at least 2000 times that of the gas phase.

c. Despite the fact that the gas phase now occupies half of the volume of the soil, its contribution is still negligible compared to the contribution of the solid phase.

Answer 1.18 a. Since the soil is dry, we have only gas and solid masses. Thus

$^{b}\rho_d = \phi_s \rho_s + \phi \rho_g$

b. $^{b}\rho_w = \phi_s \rho_s + \phi_l \rho_l + \phi_g \rho_g$

$^{b}\rho_d = \phi_s \rho_s + \phi \rho_g$

$^{b}\rho_w - {}^{b}\rho_d = \phi_l \rho_l - \rho_g(\phi - \phi_g)$

Answer 1.19 a. $^{b}\rho_w = \frac{162.5\,\text{g}}{100\,\text{cm}^3} = 1625\,\text{kg m}^{-3}$

$^{b}\rho_d = \frac{132.5\,\text{g}}{100\,\text{cm}^3} = 1325\,\text{kg m}^{-3}$

b. The mass of the solid phase is 132.5 g. Since $\rho_s = 2650\,\text{kg m}^{-3}$, this mass occupies $\frac{0.1325\,\text{kg}}{2650\,\text{kg m}^{-3}} = 50\,\text{cm}^3$. Thus $\phi_s = \frac{50}{100} = 0.50$.

The difference in mass before and after drying is 30 g. Since $\rho_l = 1000\,\text{kg m}^{-3} = 1\,\text{g cm}^{-3}$, this water occupied 30 cm^3.

Thus $\phi_l = \frac{30}{100} = 0.30$.

$\phi_s + \phi_l + \phi_g = 1$. Since $\phi_s = 0.50$ and $\phi_l = 0.30$, $\phi_g = 0.20$.

c. The soil is well-aerated, because ϕ_g is more than 0.12 (Section *1.1.3*).

d. The obtained values are only average values for the whole sample.

Answer 1.20 For an undisturbed soil sample of known volume:

$$w = \frac{\text{mass liquid}}{\text{mass solid}} = \frac{\phi_l \rho_l \times \text{volume of soil}}{\phi_s \rho_s \times \text{volume of soil}} = \frac{\phi_l \rho_l}{\phi_s \rho_s}$$

Substitution of Equation 1.6 in the numerator and Equation 1.5 in the denominator yields:

$$w = \frac{\phi_l \rho_l}{\phi_s \rho_s} \approx \frac{^{b}\rho_w - {}^{b}\rho_d}{^{b}\rho_d}$$

Answer 1.21 Since $\phi_l = \theta$, it follows from Equation 1.6:

$$\theta \approx \frac{{}^{\mathrm{b}}\rho_{\mathrm{w}} - {}^{\mathrm{b}}\rho_{\mathrm{d}}}{\rho_{\mathrm{l}}}$$

Answer 1.22 a. $\theta = \phi_{\mathrm{l}} \approx \frac{{}^{\mathrm{b}}\rho_{\mathrm{w}} - {}^{\mathrm{b}}\rho_{\mathrm{d}}}{\rho_{\mathrm{l}}} = 0.286$

b. $w = \frac{{}^{\mathrm{b}}\rho_{\mathrm{w}} - {}^{\mathrm{b}}\rho_{\mathrm{d}}}{{}^{\mathrm{b}}\rho_{\mathrm{d}}} = 0.200$

c. $\phi_{\mathrm{g}} = 1 - (\phi_{\mathrm{s}} + \phi_{\mathrm{l}}) \approx 1 - (\frac{{}^{\mathrm{b}}\rho_{\mathrm{d}}}{\rho_{\mathrm{s}}} + \phi_{\mathrm{l}}) = 1 - (\frac{1430}{2650} + 0.286) \approx 0.174$

Answer 1.23 a. Moist soil = (243 – 120) g = 123 g

Ovendry soil = (207 – 120) g = 87 g

Oxidized soil = (167 – 120) g = 47 g

$${}^{\mathrm{b}}\rho_{\mathrm{w}} = \frac{123\ \mathrm{g}}{90\ \mathrm{cm}^3} = 1.367\ \mathrm{g\,cm}^{-3} = 1367\ \mathrm{kg\,m}^{-3}$$

$${}^{\mathrm{b}}\rho_{\mathrm{d}} = \frac{87\ \mathrm{g}}{90\ \mathrm{cm}^3} = 0.967\ \mathrm{g\,cm}^{-3} = 967\ \mathrm{kg\,m}^{-3}$$

$$w = \frac{(123 - 87)\ \mathrm{g}}{87\ \mathrm{g}} = 0.414$$

$$\text{or } w \approx \frac{{}^{\mathrm{b}}\rho_{\mathrm{w}} - {}^{\mathrm{b}}\rho_{\mathrm{d}}}{{}^{\mathrm{b}}\rho_{\mathrm{d}}} = \frac{(1367 - 967)\ \mathrm{kg\,m}^{-3}}{967\ \mathrm{kg\,m}^{-3}} = 0.414$$

$$\theta = \frac{(123 - 87)\ \mathrm{g}/1\ \mathrm{g\,cm}^{-3}}{90\ \mathrm{cm}^3} = 0.400$$

$$\text{or } \theta \approx \frac{{}^{\mathrm{b}}\rho_{\mathrm{w}} - {}^{\mathrm{b}}\rho_{\mathrm{d}}}{\rho_{\mathrm{l}}} = \frac{(1367 - 967)\ \mathrm{kg\,m}^{-3}}{1000\ \mathrm{kg\,m}^{-3}} = 0.400$$

$$\text{or } \theta \approx w\frac{{}^{\mathrm{b}}\rho_{\mathrm{d}}}{\rho_{\mathrm{l}}} = 0.414 \times \frac{967}{1000} = 0.400$$

$$\phi_{\mathrm{min}} = \frac{47\ \mathrm{g}/2.7\ \mathrm{g\,cm}^{-3}}{90\ \mathrm{cm}^3} \approx 0.193$$

$$\phi_{\mathrm{org}} = \frac{(87 - 47)\ \mathrm{g}/1.4\ \mathrm{g\,cm}^{-3}}{90\ \mathrm{cm}^3} \approx 0.317$$

$\phi_{\mathrm{s}} = \phi_{\mathrm{min}} + \phi_{\mathrm{org}} = 0.193 + 0.317 = 0.510$

$\phi = 1 - \phi_{\mathrm{s}} = 0.490$

$\phi_{\mathrm{g}} = \phi - \phi_{\mathrm{l}} = 0.490 - 0.400 = 0.090$

$$\rho_{\mathrm{s}} \approx \frac{{}^{\mathrm{b}}\rho_{\mathrm{d}}}{\phi_{\mathrm{s}}} = \frac{967\ \mathrm{kg\,m}^{-3}}{0.510} = 1896\ \mathrm{kg\,m}^{-3}$$

b. ρ_{s} is the average density of the solid phase. The low value indicates that the soil is chiefly made up of organic matter, with a relatively small amount of mineral matter. This illustrates that one has to be careful with taking $\rho_{\mathrm{s}} = 2650\ \mathrm{kg\,m}^{-3}$. This value is valid only for a mineral soil with a low organic matter content.

Answer 1.24 Moist soil = (123 – 13) g = 110 g

Ovendry soil = (87 – 13) g = 74 g

Oxidized soil = (47 – 13) g = 34 g

$$\text{Volume of stone} = \frac{13\ \text{g}}{2.8\ \text{g cm}^{-3}} = 4.643\ \text{cm}^3$$

$$\text{Volume of soil} = (90 - 4.643)\ \text{cm}^3 = 85.357\ \text{cm}^3$$

$${}^{\text{b}}\rho_{\text{w}} = \frac{110\ \text{g}}{85.357\ \text{cm}^3} = 1.289\ \text{g cm}^{-3} = 1289\ \text{kg m}^{-3}$$

$${}^{\text{b}}\rho_{\text{d}} = \frac{74\ \text{g}}{85.357\ \text{cm}^3} = 0.867\ \text{g cm}^{-3} = 867\ \text{kg m}^{-3}$$

$$w = \frac{(110-74)\ \text{g}}{74\ \text{g}} = 0.487$$

$$\text{or } w \approx \frac{{}^{\text{b}}\rho_{\text{w}} - {}^{\text{b}}\rho_{\text{d}}}{{}^{\text{b}}\rho_{\text{d}}} = \frac{1289-867}{867} = 0.487$$

$$\theta = \frac{(110-74)\ \text{g}/1\ \text{g cm}^{-3}}{85.357\ \text{cm}^3} = 0.422$$

$$\text{or } \theta \approx \frac{{}^{\text{b}}\rho_{\text{w}} - {}^{\text{b}}\rho_{\text{d}}}{\rho_{\text{l}}} = \frac{1289-867}{1000} = 0.422$$

$$\text{or } \theta \approx w\frac{{}^{\text{b}}\rho_{\text{d}}}{\rho_{\text{l}}} = 0.487 \times \frac{867}{1000} = 0.422$$

$$\phi_{\text{min}} = \frac{34\ \text{g}/2.7\ \text{g cm}^{-3}}{85.357\ \text{cm}^3} \approx 0.148$$

$$\phi_{\text{org}} = \frac{(74-34)\ \text{g}/1.4\ \text{g cm}^{-3}}{85.357\ \text{cm}^3} \approx 0.335$$

$$\phi_{\text{s}} = \phi_{\text{min}} + \phi_{\text{org}} = 0.148 + 0.335 = 0.483$$

$$\phi = 1 - \phi_{\text{s}} = 0.517$$

$$\phi_{\text{g}} = \phi - \phi_{\text{l}} = 0.095$$

$$\rho_{\text{s}} \approx \frac{{}^{\text{b}}\rho_{\text{d}}}{\phi_{\text{s}}} = \frac{867\ \text{kg m}^{-3}}{0.482} = 1799\ \text{kg m}^{-3}$$

Answer 1.25 For $l = 2$ cm,

$$s = \frac{6}{2650\ \text{kg m}^{-3} \times 2 \times 10^{-2}\ \text{m}} = 0.113\ \text{m}^2\ \text{kg}^{-1}$$

For $l = 200\ \mu\text{m}$,

$$s = \frac{6}{2650\ \text{kg m}^{-3} \times 2 \times 10^{-4}\ \text{m}} = 11.3\ \text{m}^2\ \text{kg}^{-1}$$

For $l = 2\ \mu\text{m}$,

$$s = \frac{6}{2650\ \text{kg m}^{-3} \times 2 \times 10^{-6}\ \text{m}} = 1132\ \text{m}^2\ \text{kg}^{-1}$$

Answer 1.26 a. The specific surface area is

$$s = \frac{6l^2 + 2l^2(n-1)}{\rho_{\text{s}} l^3} = \frac{4+2n}{\rho_{\text{s}} l} = \frac{4+2n}{\rho_{\text{s}} n\delta}$$

b. The result of Answer 1.26a can be written as

$$s = \frac{4+2n}{\rho_{\text{s}} n\delta} = \frac{2+4/n}{\rho_{\text{s}}\delta}$$

For large n this becomes: $s \approx \frac{2}{\rho_s \delta}$.

Answer 1.27 a. Equations 1.8 can be used for sand and silt and Equations 1.9 for clay. (For clay particles δ is negligible in comparison to a and/or b; Section *1.3.1*).

b. Since sand has sizes that range from 2 mm to 20 μm, its specific surface area ranges between

$$s = \frac{6}{2650 \times 2 \times 10^{-3}} \text{ m}^2 \text{ kg}^{-1} = 1.13 \text{ m}^2 \text{ kg}^{-1} \text{ and}$$

$$s = \frac{6}{2650 \times 2 \times 10^{-5}} \text{ m}^2 \text{ kg}^{-1} = 113 \text{ m}^2 \text{ kg}^{-1}.$$

Hence, the maximum specific surface area for sand is about 113 $\text{m}^2\ \text{kg}^{-1}$.

Silt particles range from 20 μm to 2 μm. Thus, similarly, the maximum specific surface area for silt is about 1130 $\text{m}^2\ \text{kg}^{-1}$.

c. Since clay particles are smaller than those of silt, the specific surface area of clay is even larger than that of silt.

Answer 1.28 The general characteristic is: the smaller the particles, the larger the specific surface area.

Answer 1.29 a. According to Equation 1.9b:

$$s = \frac{2}{\rho_s}\left(\frac{1}{\delta} + \frac{1}{a} + \frac{1}{b}\right) = \frac{2}{2750}\left(\frac{1}{10^{-9}} + \frac{1}{20 \times 10^{-9}} + \frac{1}{20 \times 10^{-9}}\right) \text{ m}^2 \text{ kg}^{-1}$$

$$= 8.0 \times 10^5 \text{ m}^2 \text{ kg}^{-1}.$$

b. 8.0×10^5 $\text{m}^2\ \text{kg}^{-1}$ equals 0.8 $\text{km}^2\ \text{kg}^{-1}$

8.0×10^5 $\text{m}^2\ \text{kg}^{-1}$ equals 80 ha kg^{-1}.

c. The volume of one clay particle is

$10^{-9} \times 20 \times 10^{-9} \times 20 \times 10^{-9} \text{ m}^3 = 4 \times 10^{-25} \text{ m}^3$.

The mass of a single clay particle is then

$4 \times 10^{-25} \text{ m}^3 \times 2750 \text{ kg m}^{-3} = 1.1 \times 10^{-21} \text{ kg}$.

Thus, the number of particles that compose 1 kg of this clay is about 10^{21}.

Answer 1.30 a. $\frac{\delta}{a} = \frac{1 \text{ nm}}{20 \text{ nm}} = 0.05$

b. In Answer 1.29a, Equation 1.9b yielded $s = 8 \times 10^5$ $\text{m}^2\ \text{kg}^{-1}$. This equation involves no approximation.

Equation 1.9c gives:

$$s \approx \frac{2}{\rho_s \delta} = \frac{2}{2750 \times 10^{-9}} \text{ m}^2 \text{ kg}^{-1} \approx 7.3 \times 10^5 \text{ m}^2 \text{ kg}^{-1}$$

Thus, for these plate-shaped particles, Equation 1.9c introduces about 9% error.

Answer 1.31 According to Equation 1.9a, $s_1 = \frac{2}{\rho_s \delta}\left(1 + \frac{2\delta}{a}\right)$.

An approximation of s, according to Equation 1.9c, is $s_2 = \frac{2}{\rho_s \delta}$.

Thus, the percent error in s_2 is $\dfrac{s_1 - s_2}{s_1} \times 100\% = \dfrac{2\delta/a}{1 + 2\delta/a} \times 100\%$.

δ/a	0.05	0.02	0.01	0.005	0.002	0.001	0.0005
percent error	9.1	3.8	2.0	1.0	0.4	0.2	0.1

Answer 1.32 According to Equation 1.9a,

$$s = \frac{2 \times 2.05}{2750 \times 10^{-9}}\,\text{m}^2\,\text{kg}^{-1} = 1.49 \times 10^6\,\text{m}^2\,\text{kg}^{-1}.$$

Equation 1.9c gives $s = \dfrac{2}{2750 \times 10^{-9}}\,\text{m}^2\,\text{kg}^{-1} = 0.73 \times 10^6\,\text{m}^2\,\text{kg}^{-1}$.

Equation 1.9d gives $s = \dfrac{4}{2750 \times 10^{-9}}\,\text{m}^2\,\text{kg}^{-1} = 1.46 \times 10^6\,\text{m}^2\,\text{kg}^{-1}$.

Equation 1.9c introduces a significant error of about 50%, but Equation 1.9d, which was derived specifically for rod-shaped particles, introduces only a 2% error. The large deviation introduced by Equation 1.9c is due to the fact that it was derived for plate-shaped particles ($\delta \ll a$, $\delta \ll b$). For these rod-shaped particles ($\delta = b$), Equation 1.9c cannot be used.

Answer 1.33 If clay particles have circular bases, then

$$s = \frac{2\pi r^2 + 2\pi r\delta}{\rho_s \pi r^2 \delta} = \frac{2}{\rho_s \delta} + \frac{2}{\rho_s r} = \frac{2}{\rho_s}\left(\frac{1}{\delta} + \frac{1}{r}\right)$$

Equation 1.9c can be derived from this equation by neglecting $\dfrac{2}{\rho_s r}$.

This implies that the contribution of the side area, $2\pi r\delta$, to s is neglected in Equation 1.9c.

Answer 1.34 For the sand and silt fraction, we can use Equation 1.8a.

Sand: $s \approx \dfrac{6}{\rho_s d} = \dfrac{6}{2660 \times 10^{-4}}\,\text{m}^2\,\text{kg}^{-1} \approx 22.6\,\text{m}^2\,\text{kg}^{-1}$

Silt: $s \approx \dfrac{6}{2660 \times 10^{-5}}\,\text{m}^2\,\text{kg}^{-1} \approx 226\,\text{m}^2\,\text{kg}^{-1}$

For the clay fraction, we can use Equation 1.9a.

$$s \approx \frac{2}{\rho_s \delta}\left(1 + 2\frac{\delta}{a}\right) = \frac{2(1 + 2 \times 0.025)}{2750 \times 5 \times 10^{-9}}\,\text{m}^2\,\text{kg}^{-1} \approx 152700\,\text{m}^2\,\text{kg}^{-1}$$

The specific surface area of the soil is then

$$s = \sum_i f_i s_i = (0.8 \times 22.6 + 0.1 \times 226 + 0.1 \times 152700)\,\text{m}^2\,\text{kg}^{-1}$$

$$= (18.1 + 22.6 + 15270)\,\text{m}^2\,\text{kg}^{-1} \approx 15310\,\text{m}^2\,\text{kg}^{-1}.$$

Answer 1.35 a. The sand contributes $\dfrac{18.1}{15310} \approx 0.12\%$

the silt contributes $\dfrac{22.6}{15310} \approx 0.15\%$

the clay contributes $\dfrac{15270}{15310} \approx 99.7\%$

b. The soil is classified as loamy sand. This can be found in the triangle with textural class names (Figure 1.2).

c. Since the 10% illite clay contributes 99.7% of the specific surface area of the soil, the chemical and physical properties of the soil are attributable to the clay.

Answer 1.36 The symbols are a and b for the planar extensions, and δ for the thickness.

Answer 1.37 Equation 1.9c shows that the greater δ, the smaller s. This indicates that stacking of clay mineral layers within crystals, thereby forming thicker particles, is the most likely reason for different specific surface areas.

Answer 1.38 a. In Answer 1.29a, $s = 8 \times 10^5$ m^2 kg^{-1} = 800 m^2 g^{-1}. This is a very high value typical of montmorillonite.

b. With $a = b = 20$ nm and $\rho_s = 2750$ kg m^{-3}, Equation 1.9a gives:

δ (nm)	1	2	3	4	5	10
s (10^6 m^2 kg^{-1})	0.80	0.44	0.32	0.26	0.22	0.15

Answer 1.39 The main binding force is an electrostatic force. The cations are electrically attracted but are not chemically bound.

Answer 1.40 Since anions have a negative charge, they are repelled by the negative charge of a clay platelet.

Answer 1.41 Ca^{2+}-ions are attracted twice as strongly by a clay surface as Na^+-ions, because their electrical charge is twice as large.

Answer 1.42 The electrostatic attractive force decreases with distance from a clay platelet (in a rather complicated way).

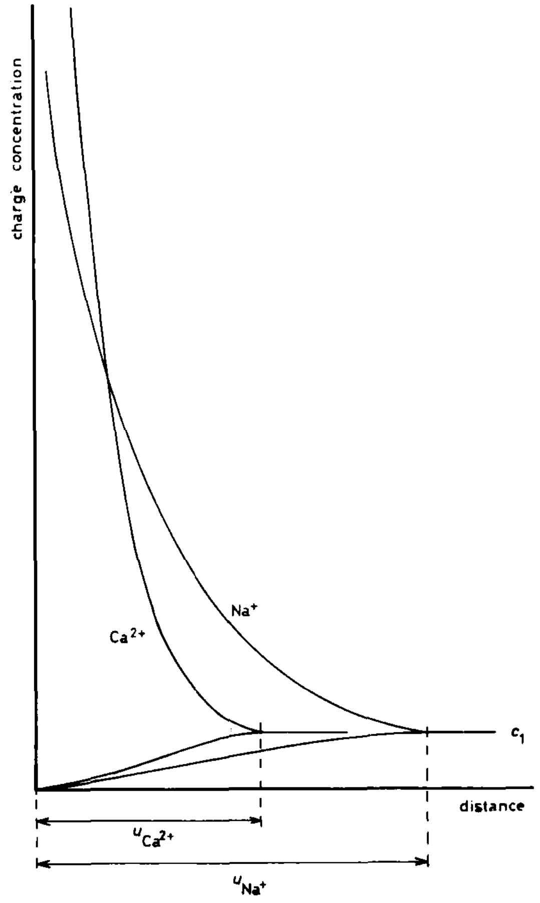
Answer 1.43 All counterions belonging to a clay platelet dried out from a soil are adsorbed on its surface. Thus, the unit is electrically neutral.

Answer 1.44 The diffuse part of the double layer is a result of the electrostatic attraction of the clay platelet on the counterions and the tendency of the counterions to diffuse away from the surface of the platelet, where the concentration is relatively high.

Answer 1.45 Anions are repelled electrostatically by a negatively charged clay particle. The repulsive force decreases with distance. Hence, the anion concentration will increase with distance from a clay particle up to the extent of its double layer.

Answer 1.46 An increase in the salt concentration of the bulk solution suppresses the tendency of counterions to diffuse away from a clay platelet. The extent of its double layer will therefore decrease (Figure 1.5).

Answer 1.47 Because Ca^{2+}-ions are attracted twice as strongly as Na^+-ions, they will on the average be closer to the clay particle than the Na^+-ions. Hence, the charge concentration of Ca^{2+}-ions is higher near the surface of the clay particle than the original charge concentration of Na^+-ions. The area between the cation and anion concentration curves represents the total electrical charge per surface area of the excess cations in the diffuse layer (see Figure below). Since this is equal to the surface charge density of the clay particle, which is constant, the area between the cation and anion curves must be the same for the Na^+-ions and the Ca^{2+}-ions. Since near the clay surface the charge concentration of Ca^{2+}-ions is higher than that of Na^+-ions, it must be lower farther away. The distance over which the concentration of the bulk solution is attained is, therefore, also shorter for Ca^{2+}-ions. Thus, the extent of the double layer decreases when Na^+-ions are replaced by Ca^{2+}-ions. In general, the thickness of the diffuse double layer will decrease as the average valency of the counterions is increased.



Answer 1.48 a. Large

b. Large

c. Small

d. High

e. High

f. High

g. Low

Answer 1.49 a. The contribution of the sand and silt fractions to s is negligible (Answer 1.34). Thus, the specific surface area is:

$$s \approx \frac{33.3}{100} \times 9 \times 10^4 \text{ m}^2 \text{ kg}^{-1} = 3 \times 10^4 \text{ m}^2 \text{ kg}^{-1}$$

b. $\theta = \phi_l \approx \frac{{}^b\rho_w - {}^b\rho_d}{\rho_l} = \frac{1700 - 1300}{1000} = 0.4$

c. A volume fraction of water of 0.4 amounts to $\frac{0.4}{1300 \text{ kg m}^{-3}} = 3.08 \times 10^{-4} \text{ m}^3$ water per kg soil. Since per kg soil the surface area is $3 \times 10^4 \text{ m}^2$ and the water content $3.08 \times 10^{-4} \text{ m}^3$, the average thickness of the water films is

$$\frac{3.08 \times 10^{-4} \text{ m}^3 \text{ kg}^{-1}}{3 \times 10^4 \text{ m}^2 \text{ kg}^{-1}} \approx 10 \text{ nm}.$$

d. Since the thickness of the water films is less than the potential extent of the double layer ($u = 20$ nm), the soil will swell when it starts raining.

Answer 1.50 There are twice as many Na^+-ions than Ca^{2+}-ions needed to compensate the surface charge of the clay platelet. So in a truncated double layer of the same volume, the osmotic pressure in the double layer is about twice as high for a Na-clay as for a Ca-clay, because the osmotic pressure is proportional to the number of particles in solution (Law of Van 't Hoff).

Answer 1.51 The high salt concentration reduces the swelling pressure and thus increases the stability of plate condensation.

Answer 1.52 Irrigation water containing salts, fertilizer application, rain with dissolved salts, weathering of soil minerals, decaying organic matter, etc. are some of the sources of salts in soil.

Answer 1.53 When the double layers of two clay particles in suspension begin to mesh, they start repulsing each other, because their surface charges are no longer completely shielded from each other. If their kinetic energy of approach can overcome this repulsive force to the point where the London-Van der Waals forces between clay faces take effect, plate condensation will take place. If it overcomes the repulsive force during an approach of a positive edge and a negative face, then 'card-house' type of flocculation will occur.

Answer 1.54 a. Deflocculation is enhanced by saturating the clay with Na^+-ions, by lowering the salt concentration, by increasing the pH, by shaking and stirring, etc.

b. This treatment is required in particle-size analysis.

Answer 1.55 H_2O_2 is used for the destruction of humus, while HCl helps the dissolution of lime and sesquioxides.

Answer 1.56 Movement and storage of soil water and gas take place in the soil pores. Hence, the porosity and the size of the pores determine the supply of water, dissolved nutrients and oxygen to plant roots.

Answer 1.57 Soil structure of a heterogeneous soil is characterized, in addition to porosity and pore-size distribution, by the spatial distribution of these factors. The shapes of the pores and their distribution and arrangement in the soil matrix determine the transport possibilities of liquid and gaseous components. For instance, a thin dense surface layer overlying an otherwise homogeneous porous soil profile can block virtually all vertical transport of water and gases.

Answer 1.58 a. Surface soil of ricefields is puddled before sowing or transplanting.
b. It is done to restrict the downward percolation of water in the flooded field.

2 Equilibrium in force fields and theory of potentials

In Chapter 1 it was mentioned that several forces act on soil water due to the nature of the solid particles. Because these forces operate simultaneously and have different magnitudes and directions, it is necessary to know how to add and multiply vectors in order to describe and predict water transport in soils. As in mechanics, the forces acting in soils obey the laws of vector operation. In this chapter we shall first discuss the basic vector operations and the associated conditions for equilibrium. In the second part we shall introduce the concept of potential and define various forms and components.

2.1 EQUILIBRIUM CONDITIONS IN FORCE FIELDS

2.1.1 Forces and equilibria

Intuitively, a force can be defined as an influence that tends to produce a change in the motion of objects. However, a force can also be balanced by another force acting on the object, without change in motion. Fundamentally, a force is defined by Newton's second law of motion

$$\boldsymbol{F} = m\boldsymbol{a} \tag{2.1a}$$

where $\boldsymbol{F}$ is force in N (kg m s^{-2}), m is mass in kg and $\boldsymbol{a}$ is acceleration in m s^{-2}. Force and acceleration are vectors (printed in bold face) which have the same direction, whereas mass is scalar. Vectors are quantities that require specification of direction as well as magnitude for complete characterization. Scalars are quantities that are completely characterized by their magnitude only.

When more than one force is acting on an object at the same time, the forces may be added in the proper way. Scalars can be added just by adding their magnitudes; vectors must be added according to direction and magnitude. This can be done by placing them tail to head in any order. The vector sum or resultant vector is then equal to the single vector leading from the tail of the first vector to the head of the last vector. The component of the resultant force in a certain direction, for instance the vertical direction, is equal to the sum of the components of the individual vectors in that direction.

Question 2.1 Determine the resultant vector, $\boldsymbol{R}$, of the vectors with their tail in the origin and their heads at the x, y-coordinates $\boldsymbol{A}(2,2)$, $\boldsymbol{B}(-3,1)$ and $\boldsymbol{C}(3,-4)$, respectively.

The sum of all forces acting on a body simultaneously, is the vector sum or resultant force, $\Sigma \boldsymbol{F}$. Equation 2.1a is also true for the resultant force. That is:

$$\Sigma \boldsymbol{F} = m\boldsymbol{a} \tag{2.1b}$$

Therefore, if the vector sum of all forces equals zero, the acceleration is zero, and the body is said to be in mechanical equilibrium:

$$\Sigma \boldsymbol{F} = 0 \rightleftarrows \text{mechanical equilibrium} \tag{2.2}$$

The double arrow indicates that Equation 2.2 can be read from left to right and from right to left. When Equation 2.2 is valid, there is no acceleration, but the body can still be in uniform motion.

Question 2.2 Describe Newton's second law of motion in words.
Question 2.3 Define mechanical equilibrium in words.
Question 2.4 A body is placed on a table, where it stays indefinitely. What is its equilibrium status?

The forces in Equation 2.2 can be divided into two types: static forces, $\boldsymbol{F}^{\mathrm{s}}$, and dynamic or drag forces, $\boldsymbol{F}^{\mathrm{d}}$, with $\Sigma \boldsymbol{F} = \Sigma \boldsymbol{F}^{\mathrm{s}} + \Sigma \boldsymbol{F}^{\mathrm{d}}$. Static forces are present irrespective of movement. Drag forces may arise whenever static forces cause a body to move. Thus, they are reaction forces, associated with movement. When mechanical equilibrium and motion occur simultaneously, Equation 2.2 becomes $\Sigma \boldsymbol{F} = \Sigma \boldsymbol{F}^{\mathrm{s}} + \Sigma \boldsymbol{F}^{\mathrm{d}} = 0$. This implies $\Sigma \boldsymbol{F}^{\mathrm{s}} = -\Sigma \boldsymbol{F}^{\mathrm{d}}$. In the absence of motion there are no drag forces, i.e. $\Sigma \boldsymbol{F}^{\mathrm{d}} = 0$. In such a situation $\Sigma \boldsymbol{F}$ equals $\Sigma \boldsymbol{F}^{\mathrm{s}}$, and Equation 2.2 is reduced to $\Sigma \boldsymbol{F} = \Sigma \boldsymbol{F}^{\mathrm{s}} = 0$. This situation, called static equilibrium, is described by

$$\Sigma \boldsymbol{F} = \Sigma \boldsymbol{F}^{\mathrm{s}} = \Sigma \boldsymbol{F}^{\mathrm{d}} = 0 \rightleftarrows \text{static equilibrium} \tag{2.3}$$

Thus, static equilibrium is a special case of mechanical equilibrium.

Question 2.5 Can you imagine a situation (not on earth) to which Equation 2.3 would not apply if read from left to right?
Question 2.6 In Answer 2.4 the sum of the forces is zero ($\Sigma \boldsymbol{F} = 0$)
a. What are the forces to be summed?
b. Is there a drag force?
c. How would you redefine its equilibrium status?

Static forces acting on objects are either body forces, $\boldsymbol{F}^{\mathrm{s}}_{\mathrm{b}}$, or contact forces, $\boldsymbol{F}^{\mathrm{s}}_{\mathrm{c}}$. Contact forces are local reaction forces resulting from contact with other objects. An example of a body force is the gravitational force. Body forces are associated with force fields, which are discussed in the next section. Static forces are the sum of body and contact forces, i.e.

$$\Sigma \boldsymbol{F}^{\mathrm{s}} = \Sigma \boldsymbol{F}^{\mathrm{s}}_{\mathrm{b}} + \Sigma \boldsymbol{F}^{\mathrm{s}}_{\mathrm{c}} \tag{2.4}$$

At static equilibrium, $\Sigma \boldsymbol{F}^{\mathrm{s}} = 0$. Then the sum of the contact forces is equal, but oppositely directed, to the sum of the body forces, i.e.

$$\Sigma \boldsymbol{F}_{\mathrm{c}}^{\mathrm{s}} = -\Sigma \boldsymbol{F}_{\mathrm{b}}^{\mathrm{s}} \tag{2.5}$$

2.1.2 Force fields

A force field is a concept which facilitates the computation of forces, especially when they differ with position. The concept cannot be defined and described here exhaustively. It will only be illustrated by a few examples. The first is the earth's gravitational attraction. The gravitational force field is described by

$$\boldsymbol{F} = m\boldsymbol{g} \tag{2.6}$$

$\boldsymbol{F}$ is the gravitational force, or weight (N), of a body with mass m (kg), and $\boldsymbol{g}$ is the gravitational field strength, which is the gravitational force divided by mass (N kg^{-1}). The gravitational force acts on a body, irrespective whether the body is static or in motion and whether it is in contact with other bodies or not. It is, therefore, also called a body force.

An object falling freely (in a vacuum) in the earth's gravitational field obtains an acceleration described by Equation 2.1a. Experimentally, one can determine that the magnitude of $\boldsymbol{a}$ is 9.81 m s^{-2}. One can show that then also in Equation 2.6, the magnitude of $\boldsymbol{g}$ is 9.81 N kg^{-1}. In this book the magnitude of $\boldsymbol{g}$ will, for convenience, be rounded off to $g = 10.0$ N kg^{-1}.

Question 2.7 a. Define weight in words.
b. What is the weight of a body with a mass of 5 kg?

Question 2.8 Newton's law of gravitation can be written as

$$F = \frac{G m_1 m_2}{r^2} \tag{2.7}$$

where m_1 and m_2 are the masses of two bodies, r is the distance between their centres, G is the universal gravitational constant and F is the magnitude of the attractive force between the masses. By equating Newton's law of gravitation to Equation 2.6, derive an expression for the magnitude of $\boldsymbol{g}$ (g). Take $m = m_1$, as the mass of a body and m_2 as the mass of the earth.

A distinction can be made between homogeneous and nonhomogeneous force fields. A homogeneous force field is a force field in which the magnitude and direction of the force are independent of position, whereas the magnitude and direction of a nonhomogeneous force field vary with position.

Question 2.9 a. Is the gravitational field strength homogeneous? Explain.
b. Is the centrifugal field in a centrifuge homogeneous? Explain.

The field strength of electrostatical force fields can be defined as the force divided by electrical charge. This definition is similar to Equation 2.6, except that mass must be replaced by electrical charge, Q. Equation 2.6 becomes

$$\boldsymbol{F} = Q\boldsymbol{c} \tag{2.8}$$

where $\boldsymbol{c}$ is the electrostatic field strength. This field strength can be translated into a force divided by mass if the specific charge (i.e. charge divided by mass) of the material is known. This subject matter is pertinent to the charges and forces within diffuse double layers of clay particles (Section *1.3.3*).

Question 2.10 A body has a mass of 0.1 g and an electrical charge of 10^{-7} C (coulomb). It experiences a force due to the presence of other charges. The electric field strength at the location of the body is 10^3 N C^{-1}.

a. Express its acceleration as a function of mass, charge and field strength.
b. Compute the acceleration if the body is free to move.
c. Express the force and field strength analogously to Answer 2.8.

2.1.3 Forces and pressures in static equilibrium

Contact forces act not only between different bodies, but also within a body, where they cause material stresses. This can be explained by considering a concrete pillar at static equilibrium in the gravitational force field (Figure 2.1). Each mass m in this pillar is subject to a body force $m\boldsymbol{g}$. But, being at static equilibrium, the body force acting on each mass must be counterbalanced by a net contact force, $-m\boldsymbol{g}$. (The minus sign indicates that the contact force is directed oppositely to the field strength $\boldsymbol{g}$.) This can be shown by dividing the pillar into a number of horizontal, thin slices, each with a mass m. The body force of the top slice exerts a contact force $m\boldsymbol{g}$ on the second slice. Since the pillar, and thus the uppermost slice, is at static equilibrium, the second slice reacts with a contact force $-m\boldsymbol{g}$ on the top slice. The second slice exerts a contact force $2m\boldsymbol{g}$ on the third slice, because the total mass resting on the third slice is that of the upper two slices, i.e. $2m$. But then the third slice reacts with a contact force on the second slice equal to $-2m\boldsymbol{g}$. Thus the net contact force on the second slice is $m\boldsymbol{g} + (-2m\boldsymbol{g}) = -m\boldsymbol{g}$, i.e. $m\boldsymbol{g}$ from above and $-2m\boldsymbol{g}$ from below (Figure 2.1b). Similarly, the third slice is subject to a contact force of $-3m\boldsymbol{g}$ from the fourth slice and the net contact force on the third slice is $2m\boldsymbol{g} + (-3m\boldsymbol{g}) = -m\boldsymbol{g}$. And so forth.

The last paragraph indicates that the contact force in any horizontal plane is the product of $-m\boldsymbol{g}$ and the number of overlying slices, n. Thus, $\boldsymbol{F}_c^s = -nm\boldsymbol{g}$, where nm is the total mass of all the overlying material above the horizontal plane. When this contact force is divided by the area of the horizontal plane, it is called normal material stress, and is expressed in Pa. By 'normal' is meant perpendicular to the horizontal plane. This distinction is necessary, since solid bodies may also exhibit frictional tan-

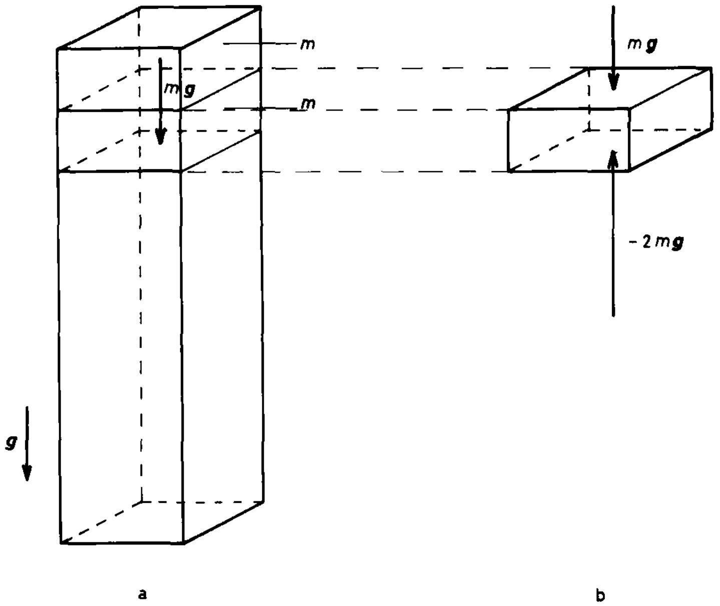


Figure 2.1 a. Concrete pillar divided in horizontal slices, each of mass *m*.
b. Contact forces on the second slice.

gential stresses in other directions, which will not be discussed here. We conclude that, in bodies supported from below, the material stress increases in absolute value in the direction of the gravitational field.

In fluids – substances without rigidity, such as water and air – the material stress acts with equal magnitude in all directions. In that case, the stress can be measured as a pressure, p (Pa), and is, unlike force and acceleration, a scalar quantity.

Question 2.11 Why is p scalar?

Consider now a thin layer in a column of water at rest (Figure 2.2), with horizontal upper and lower square planes with edges l and vertical planes with height Δz. As usual, a coordinate system is chosen with x and y in horizontal directions and z, positive upwards, in the vertical direction. Suppose the average pressure on the vertical plane at the left-hand side is p_1. Then the magnitude of the horizontal force on this plane is $F_1 = l \times \Delta z \times p_1$. Similarly, the magnitude of the horizontal force on the plane at the right is $F_2 = l \times \Delta z \times p_2$. Since the layer is at rest, and $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are acting in exactly opposite directions, the magnitudes of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ must be the same. Thus, $p_1 = p_2$. By letting Δz approach zero, it follows that the pressure at every point in a continuous horizontal layer of water at rest is everywhere the same. From this it follows easily that the horizontal forces on any body completely surrounded by a liquid at rest, always counterbalance each other. Therefore, there is no net horizontal force

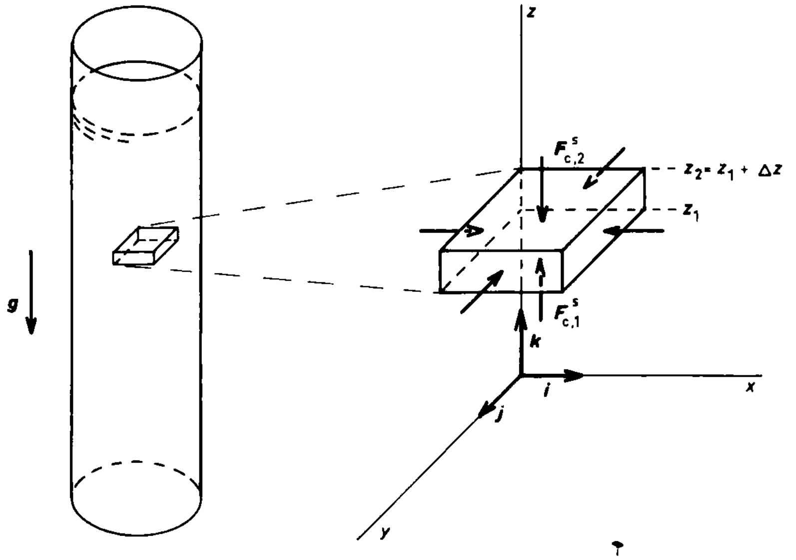


Figure 2.2 Contact forces on a thin layer of water in a column at static equilibrium.

acting on the vertical planes of the water layer in Figure 2.2. Only vertical forces need to be taken into account.

The vertical upward contact force on the lower plane of the layer in Figure 2.2 is

$$\boldsymbol{F}^{s}_{c,1} = p_1 l^2 \boldsymbol{k} = p_1 \frac{V}{\Delta z} \boldsymbol{k} \tag{2.9}$$

where p_1 is the pressure at that plane, $\boldsymbol{k}$ is the unit vector in the z-direction and V is the volume of the layer.

The contact force on the upper layer, directed downward, is similarly

$$\boldsymbol{F}^{s}_{c,2} = -p_2 l^2 \boldsymbol{k} = -p_2 \frac{V}{\Delta z} \boldsymbol{k} \tag{2.10}$$

The net vertical contact force on the layer, $\Sigma \boldsymbol{F}^{s}_{c}$, is then

$$\Sigma \boldsymbol{F}^{s}_{c} = -\frac{\Delta p}{\Delta z} V \boldsymbol{k} \tag{2.11}$$

where $\Delta p = p_2 - p_1$.

As Δz approaches zero, the net vertical contact force divided by volume becomes

$$\frac{\Sigma \boldsymbol{F}^{s}_{c}}{V} = -\boldsymbol{k} \lim_{\Delta z \to 0} \frac{\Delta p}{\Delta z} = -\frac{\mathrm{d}p}{\mathrm{d}z} \boldsymbol{k} \tag{2.12}$$

Now, using $\rho_l = \frac{m}{V}$ for the density of the water and Equations 2.5 and 2.6, we obtain

$$-\frac{1}{\rho_l}\frac{\mathrm{d}p}{\mathrm{d}z}\boldsymbol{k} = \frac{\Sigma \boldsymbol{F}_c^s}{m} = -\frac{\boldsymbol{F}_b^s}{m} = -\boldsymbol{g} \tag{2.13}$$

Since $\boldsymbol{g} = -g\boldsymbol{k}$, Equation 2.13 can be reduced to

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\rho_l g \tag{2.14}$$

Assuming ρ_l is not dependent on p, i.e. water is incompressible, integration of Equation 2.14 yields

$$p = -\rho_l g z + C \tag{2.15}$$

where C is an integration constant. If $z = 0$ is taken at the water surface, where the pressure is the atmospheric pressure, p_{atm}, then $C = p_{\mathrm{atm}}$ and thus:

$$p = -\rho_l g z + p_{\mathrm{atm}} \tag{2.16}$$

Thus, in a liquid at rest, the pressure increases linearly with depth. The difference in pressure at two heights is equal to the magnitude of the weight of the liquid column stretching between those two heights, divided by the cross-sectional area of the column.

Question 2.12 A cube is submerged in a container of water, with two faces parallel to the water surface.

a. Derive general equations for the vertical net contact force, the body force and the total static force on the cube.
b. Calculate $\Sigma \boldsymbol{F}^s$ if the cube has a mass of 30 g and a volume of 27 cm^3.
c. What is the effect of $\Sigma \boldsymbol{F}^s$? (Discuss also $\boldsymbol{F}^d$.)

The result obtained in Answer 2.12a for a cube, is valid for a body of any irregular shape. This is known as Archimedes law: A fluid acts on a body immersed in it with a net contact force that is directed vertically upward and equal in magnitude to the weight of the fluid displaced by the body. The upward force is called buoyant force. This principle is used in soil physics for the determination of the bulk density of irregularly shaped soil samples.

Question 2.13 A clod with a mass of 8.99 g, perfectly sealed with paraffin wax, is lowered into a container of water on a string attached to a torsion balance. At equilibrium, the balance reads 2.79 g. Assume that the masses of the wax coating and the string are negligible.

a. Calculate the weight of the clod.
b. Calculate the body force, the net vertical static force and the net contact force on the clod by the water.
c. What is the bulk density of the clod?

d. Why was it necessary to seal the clod perfectly with paraffin wax? How can you check that it was perfectly sealed?

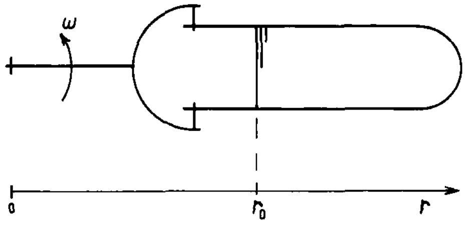
In soils, water is not only subject to the gravitational field, but also to forces of adsorption by the solid phase (Sections 1.3 and *3.1.1*). There, the pressure in the water increases in the direction of the adsorbing surface, similar to the pressure in the gravitational field.

Another force field sometimes used in soil physics, is the centrifugal force field in a centrifuge.

Question 2.14 A liquid in a centrifuge tube rotates horizontally with a constant angular frequency ω about a vertical axis (centrifuge centre, Figure 2.3). Its centripetal acceleration is $\boldsymbol{a} = -\omega^2 r \boldsymbol{i}$ and r_0 is the distance from the vertical axis to the nearest liquid surface in the tube. ($\boldsymbol{i}$ is the unit vector in the direction of r.)

a. If $p = p_{\mathrm{atm}} (\approx 10^5\ \mathrm{Pa})$ at $r = r_0$, derive p at any distance r. (Neglect gravity.)

b. If the centrifuge is turning at 6000 revolutions per minute, $\rho_l = 1000\ \mathrm{kg\ m^{-3}}$ and $r_0 = 0.2\ \mathrm{m}$, calculate the pressure at $r = 0.3\ \mathrm{m}$.



Top view

Figure 2.3 Liquid in centrifuge tube rotating at angular frequency ω.

2.1.4 Sedimentation velocity as mechanical equilibrium

The concept of mechanical equilibrium in the gravitational force field is used in particle-size analysis by sedimentation (Section *1.1.2*). When a soil particle is suspended in water, it first accelerates due to the influence of gravity. As the velocity increases, the drag force exerted by the water on the particle due to the viscosity of the water also increases. The net static force (weight minus buoyant force), however, remains constant. When the drag force has become equal to the net static force, the resultant force is zero, and thus also the acceleration. Then the particle continues to move at constant velocity (Figure 2.4).

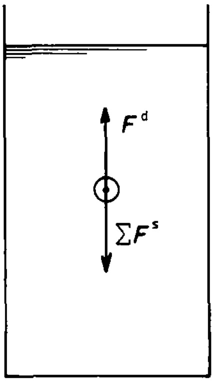


Figure 2.4 Net static force, ΣF^s, and drag force, F^d, on a small solid particle moving through a viscous fluid under the influence of gravity.

When a solid, spherical particle of radius r and density ρ_s moves through a viscous liquid, the net static force acting downward is (Answer 2.12)

$$\Sigma F^s = \frac{4}{3}\pi r^3 (\rho_s - \rho_l) g \tag{2.17}$$

where ρ_l is the density of the liquid. The drag force on a spherical particle under laminar conditions was shown by Stokes to be

$$F^d = -6\pi\eta r v \tag{2.18}$$

where η is the viscosity of the liquid, v is the terminal velocity of the particle, and the minus sign indicates that F^d is directed oppositely to v (and of course also to F^s). Laminar conditions will persist as long as the Reynolds number

$$Re = \frac{2rv\rho_l}{\eta} \tag{2.19}$$

remains smaller than about 0.1, which is true for particles < 50 μm. The sum of forces is zero when the sedimentation velocity has reached the value for which

$$\Sigma F^s + F^d = \frac{4}{3}\pi r^3 g(\rho_s - \rho_l) - 6\pi\eta r v = 0 \tag{2.20}$$

The terminal velocity, also known as settling or sedimentation velocity, is then

$$v = \frac{2gr^2(\rho_s - \rho_l)}{9\eta} \tag{2.21}$$

Since the particles attain a terminal velocity almost instantly, the radius of particles that travel a distance s during a time interval t is then:

$$r = \left(\frac{9\eta s}{2g(\rho_s - \rho_l)t}\right)^{1/2} \tag{2.22}$$

This sedimentation equation is based on a number of simplifying assumptions (see Answer 2.15e). When the equation is used for particle-size analysis, the obtained radius r is not the actual radius but only an equivalent radius of the particle. For nonspherical particles, such as clays, r is the radius of a hypothetical spherical particle with the same theoretical settling velocity.

Question 2.15 A soil sample was taken from a field for particle-size analysis by sedimentation. It was dried and by various means reduced to its primary soil particles. The largest grain sizes were 3 mm and the total mass of solid phase was 65 g. A 1250-cm^3 sedimentation cylinder filled with water at 20°C ($\eta = 10^{-3}$ Pa s), and a 20-cm^3 pipette were used for estimation of grain-size distribution.

a. Outline briefly the steps for estimating the textural components of the soil sample.

b. Which sieve sizes are required?

c. The mass of grains larger than 50 μm and 2 mm were 15 g and 10 g, respectively. The rest of the grains are dispersed in the cylinder. Give the time of sampling after the commencement of sedimentation for a few grain sizes, if samples are taken at 15 cm depth.

d. What is the textural classification of the samples taken at $t \approx 6.82$ min and $t \approx 682$ min?

e. Is the silt content of the sample taken at $t \approx 6.82$ min the same as at $t = 0$?

f. If the mass of solid taken at times about 6.82 min and 682 min are, respectively, 0.3 and 0.1 g in 20 cm^3, find the mass fraction of the different textural fractions in the original soil sample.

g. What is the textural classification of the original soil sample?

h. Justify the use of Stokes' law in the sedimentation method.

2.2 POTENTIALS IN STATIC EQUILIBRIUM

In Section 2.1, we have described equilibrium conditions, their underlying principles and their significance. For a few cases we calculated the resultant force by summing all the component forces according to the rules of vector addition. Unfortunately, the magnitudes and directions of forces are often not easily identified, making vector addition difficult. Therefore, the criterion for static equilibrium stated in Section *2.1.1* ($\Sigma \boldsymbol{F} = 0$ and $\Sigma \boldsymbol{F}^s = 0$), correct as it may be, is not very practical. Much easier to handle is the concept of potential. Since potential is a scalar quantity, it involves only algebraic addition of its components. The potential of soil water which we shall discuss is the energy associated with position. The kinetic energy of soil water, which is the energy associated with its motion, can be neglected, since soil water always moves very slowly.

2.2.1 Potential theory

Consider a mass m subjected to a uniform force field (Figure 2.5). The distance

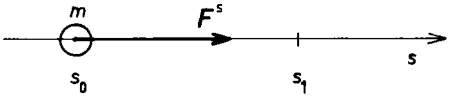


Figure 2.5 A mass *m* moving in the direction of a force field.

s increases (is positive) in the direction of $\boldsymbol{F}^s$, thus $s_1 > s_0$ and $\Delta s = s_1 - s_0 > 0$. When the mass m moves from s_0 to s_1 under the influence of force $\boldsymbol{F}^s$, the system expends an amount of useful work or energy equal to $F^s\Delta s$. To move the mass m back from s_1 to s_0, an external force, acting in the opposite direction must be applied over the distance s_1 to s_0. If the translocation is to occur isothermally and reversibly, this force must be only an infinitesimally small amount larger than $\boldsymbol{F}^s$. In this way the energy of the system is restored to its original value. The energy lost or gained in these transitions between s_0 and s_1 is the difference in potential energy of the mass m in the force field between the two positions. When this potential energy difference is divided by the mass m, one obtains, per definition, the potential difference, $\Delta\psi$:

$$\Delta\psi = -\frac{F^s}{m}\Delta s \tag{2.23}$$

Potential is potential energy divided by mass and is expressed in J kg^{-1}. The minus sign in Equation 2.23 is required, since the potential decreases as the mass moves in the direction of the force. Thus, the force is in the direction of decreasing potential.

Equation 2.23 is valid when F^s is constant over the distance Δs. This is generally not true for soil water. If Δs is confined to a very small (infinitesimal) distance ds, over which F^s can be assumed constant, then Equation 2.23 can be written as:

$$\frac{F^s}{m} = -\lim_{\Delta s\to 0}\frac{\Delta\psi}{\Delta s} = -\frac{d\psi}{ds} \tag{2.24}$$

Or, again in the form of Equation 2.23:

$$d\psi = -\frac{F^s}{m}ds \tag{2.25}$$

By choosing a reference position s_0 where $\psi = 0$, the potential for any point s' can be obtained by integrating Equation 2.25:

$$\psi = -\int_{s_0}^{s'} \frac{F^{\mathrm{s}}}{m} \mathrm{d}s \tag{2.26}$$

In words, the potential at a point in a force field is the amount of useful work or energy required to transfer a body from a reference point s_0 to the point under consideration s', divided by that body's mass. It is not possible to assign an absolute value to the potential at point s' without defining the potential at the reference point s_0. Fortunately, in soil physics, we are never concerned with absolute potentials, but only with differences in potential between points. Thus we can assign any arbitrary value (often zero) to the potential at s_0.

Question 2.16 What influence has ψ_0 at s_0 on the numerical value of ψ at s?

In Figure 2.5, $\boldsymbol{F}^{\mathrm{s}}$ and s have the same direction, but this is not generally true. If the two directions make an angle φ, $\boldsymbol{F}^{\mathrm{s}}$ can be regarded as the sum of two components: one in the direction of s with magnitude $F^{\mathrm{s}} \cos \varphi$ and one perpendicular to s with magnitude $F^{\mathrm{s}} \sin \varphi$ (Figure 2.6). The amount of useful work expended by the component in the direction of s is $F^{\mathrm{s}}\mathrm{d}s \cos \varphi$, while that of the component of $\boldsymbol{F}^{\mathrm{s}}$ perpendicular to s is zero, because this component cannot bring about a translocation in the direction of s. Thus, the total amount of useful work expended by $\boldsymbol{F}^{\mathrm{s}}$ equals $F^{\mathrm{s}}\mathrm{d}s \cos \varphi$.

The product $F^{\mathrm{s}}\mathrm{d}s \cos \varphi$, which is a scalar quantity, is known as the scalar or dot product of the vectors $\boldsymbol{F}^{\mathrm{s}}$ and $\mathrm{d}\boldsymbol{s}$:

$$\boldsymbol{F}^{\mathrm{s}} \cdot \mathrm{d}\boldsymbol{s} = F^{\mathrm{s}}\mathrm{d}s \cos \varphi \tag{2.27}$$

Thus, for any combination of directions of $\boldsymbol{F}^{\mathrm{s}}$ and $\mathrm{d}\boldsymbol{s}$, Equation 2.26 becomes:

$$\psi = -\int_{s_0}^{s'} \frac{\boldsymbol{F}^{\mathrm{s}}}{m} \cdot \mathrm{d}\boldsymbol{s} = -\int_{s_0}^{s'} \frac{F^{\mathrm{s}}}{m} \cos \varphi \, \mathrm{d}s \tag{2.28}$$

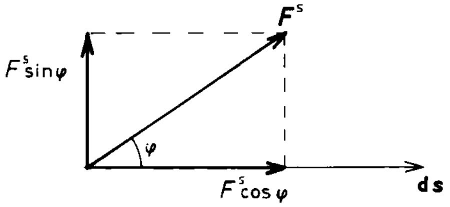


Figure 2.6 Components of a force, F^{s}: $F^{\mathrm{s}} \cos \varphi$ in the direction of movement and $F^{\mathrm{s}} \sin \varphi$ perpendicular to that direction.

If more force fields are present in the same system, each force field contributes its own potential, i.e. the partial potential ψ_i due to the i^{th} force field. The total potential ψ_t is then the algebraic sum of the partial potentials of the different force fields:

$$\psi_t = \sum_i \psi_i = \sum_i -\int \frac{\boldsymbol{F}_i^s}{m} \cdot \mathrm{d}\boldsymbol{s} = -\int \frac{\Sigma_i \boldsymbol{F}_i^s}{m} \cdot \mathrm{d}\boldsymbol{s} \qquad (2.29)$$

At static equilibrium, where $\Sigma \boldsymbol{F}_i^s = 0$, one finds:

$$\psi_t = -\int \frac{\Sigma \boldsymbol{F}_i^s}{m} \cdot \mathrm{d}\boldsymbol{s} = -\int 0 \mathrm{d}\boldsymbol{s} = C \qquad (2.30)$$

where C is a constant of integration.

In a system at static equilibrium, the total potential is constant, i.e. invariant with position in the system. This conclusion is true for all components in the soil.

2.2.2 Hydrostatic equilibrium in the gravitational field. Hydraulic potential

'Hydrostatics' is the study that deals with potentials, pressures, etc. of liquids at hydrostatic equilibrium.

Figure 2.7 shows a cylinder filled with water. The water is subject to the gravitational field and is in hydrostatic equilibrium. Therefore, according to Equation 2.30, ψ_t is constant and depends only on the choice of the reference points for the various partial potentials. In the gravitational field, the water has a gravitational potential, ψ_g. Intro-

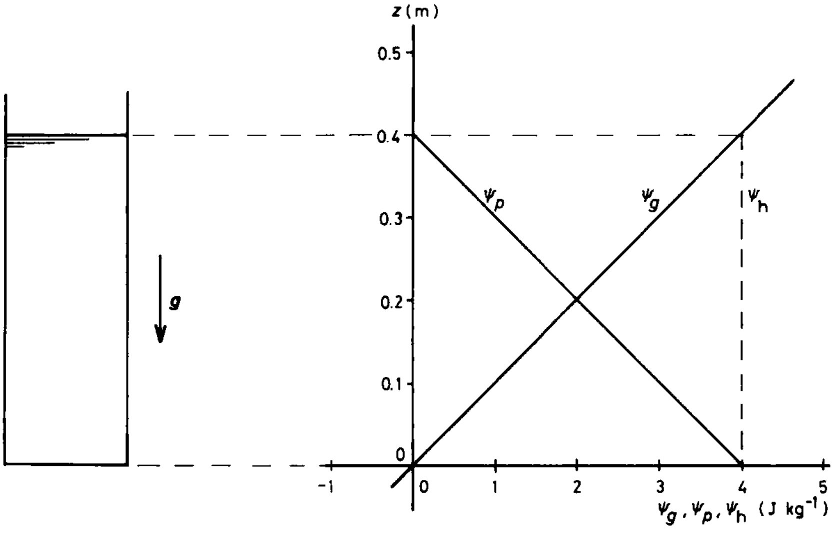


Figure 2.7 Potential diagram of a static column of water in the gravitational field.

duction of ψ_g and $\boldsymbol{F}^{\mathrm{s}}_{\mathrm{b}}$ for ψ and $\boldsymbol{F}^{\mathrm{s}}$ in Equation 2.28, respectively, yields:

$$\psi_g = -\int \frac{\boldsymbol{F}^{\mathrm{s}}_{\mathrm{b}}}{m} \cdot \mathrm{d}\boldsymbol{s} = -\int \boldsymbol{g} \cdot \mathrm{d}\boldsymbol{s} = -\int g \cos\varphi \, \mathrm{d}z = gz + C \tag{2.31}$$

where z refers to the height of the point under consideration and φ is the angle between the direction of z and $\boldsymbol{g}$. Since z increases in the direction opposite to the gravitational field, $\cos\varphi = -1$. Usually, the reference for the gravitational potential is chosen such that $\psi_g = 0$ at $z = 0$. Then the integration constant in Equation 2.31 vanishes, and

$$\psi_g = gz \tag{2.32}$$

Equation 2.32 shows that ψ_g increases linearly with height z. Accordingly, Figure 2.7 shows a linearly increasing ψ_g with height, with $z = 0$ chosen at the bottom of the cylinder.

Question 2.17 a. Compute ψ_g at $z = 0$, 0.1, 0.4, 0.5 and -0.1 m, when $z = 0$ at the bottom of the cylinder.
b. Interpret the meaning of the positive and negative values of ψ_g.

At static equilibrium, the gravitational potential, ψ_g, is compensated by the pressure potential, ψ_p, in the same way that a body force is compensated by a contact force. The compensation of one by the other keeps their sum constant.

The general equation for ψ_p can be obtained from Equation 2.28 by replacing ψ by ψ_p and $\boldsymbol{F}^{\mathrm{s}}$ by $\Sigma \boldsymbol{F}^{\mathrm{s}}_{\mathrm{c}}$. Thus,

$$\psi_p = -\int \frac{\Sigma \boldsymbol{F}^{\mathrm{s}}_{\mathrm{c}}}{m} \cdot \mathrm{d}\boldsymbol{s} = -\int \frac{\Sigma \boldsymbol{F}^{\mathrm{s}}_{\mathrm{c}}}{m} \cdot \mathrm{d}\boldsymbol{z} \tag{2.33}$$

By substituting $-\dfrac{1}{\rho_{\mathrm{l}}}\dfrac{\mathrm{d}p}{\mathrm{d}z}\boldsymbol{k}$ for $\dfrac{\Sigma \boldsymbol{F}^{\mathrm{s}}_{\mathrm{c}}}{m}$ (Equation 2.13) and $\boldsymbol{k}\mathrm{d}z$ for $\mathrm{d}\boldsymbol{z}$ and using $\boldsymbol{k} \cdot \boldsymbol{k} = 1$, we obtain:

$$\psi_p = -\int -\frac{1}{\rho_{\mathrm{l}}}\frac{\mathrm{d}p}{\mathrm{d}z}\mathrm{d}z = \int \frac{1}{\rho_{\mathrm{l}}}\mathrm{d}p \tag{2.34}$$

If ρ_{l} is independent of p, i.e. ρ_{l} is constant, ψ_p becomes:

$$\psi_p = \frac{1}{\rho_{\mathrm{l}}}p + C \tag{2.35}$$

where C is a constant.

Equation 2.35 is valid for water, because water is only slightly compressible and thus ρ_{l} can be regarded as independent of p.

The reference for ψ_p is, by convention, selected at atmospheric pressure. Thus, $\psi_p = 0$ if $p = p_{\mathrm{atm}}$. For instance, in Figure 2.7 the reference for ψ_p is at the water surface, since there the pressure is atmospheric. Now, after substituting p_{atm} for p and zero for ψ_p into Equation 2.35, the integration constant becomes $-\frac{1}{\rho_l} p_{atm}$. This yields

$$\psi_p = \frac{1}{\rho_l} p - \frac{1}{\rho_l} p_{atm} = \frac{1}{\rho_l} (p - p_{atm}) = \frac{1}{\rho_l} p' \tag{2.36}$$

$$\text{where } p' = p - p_{atm} \tag{2.37}$$

In hydrostatics, as well as in hydraulic flow of the soil solution, the gravitational potential and the pressure potential, generally, are the only two components of the total potential that need to be taken into account. The sum of the pressure potential and the gravitational potential is called the hydraulic potential, ψ_h.

$$\psi_h = \psi_p + \psi_g \tag{2.38}$$

In Figure 2.7, both ψ_p and p' are zero at the water surface, where p equals p_{atm}. At the same height, $z = 0.4$ m, the gravitational potential ψ_g is
$\psi_g = gz = 10\ \mathrm{N\ kg^{-1}} \times 0.4\ \mathrm{m} = 4\ \mathrm{J\ kg^{-1}}$
Therefore, at $z = 0.4$ m the hydraulic potential, ψ_h, according to Equation 2.38, is
$\psi_h = \psi_p + \psi_g = 0 + 4\ \mathrm{J\ kg^{-1}} = 4\ \mathrm{J\ kg^{-1}}$
According to Equations 2.30 and 2.38, ψ_h is uniform at static equilibrium, i.e. invariant with position. Hence, $\psi_h = 4\ \mathrm{J\ kg^{-1}}$ throughout the cylinder. Now, ψ_p can be calculated with $\psi_p = \psi_h - \psi_g$. The result of this calculation is shown in Figure 2.7. This illustrates that at hydrostatic equilibrium, the ψ_p and ψ_g-lines have equal but opposite slopes.

Question 2.18 Explain why ψ_p has to be positive below the water surface.
Question 2.19 If the reference point for ψ_g were at half the height of the water column in Figure 2.7:
a. What would be ψ_h?
b. Would there be a change in the plot of ψ_p?

2.2.3 *Potential on volume basis. Pressure equivalent*

It is sometimes easier to work with potentials on volume basis ($\mathrm{J\ m^{-3}}$) than on mass basis ($\mathrm{J\ kg^{-1}}$). Potential on volume basis is obtained from potential on mass basis by multiplying by density ρ (i.e. it becomes $\rho\psi$). It has the dimension (units) of pressure: $\mathrm{kg\ m^{-3}} \times \mathrm{J\ kg^{-1}} = \mathrm{J\ m^{-3}} = \mathrm{N\ m^{-2}} = \mathrm{Pa}$. Particularly for water, the density of which is almost constant, it is convenient to express potentials as pressure equivalents. For convenience, we will assume a constant density of the soil solution of 1000 $\mathrm{kg\ m^{-3}}$.

On the basis of volume, the gravitational potential becomes

$$\rho_l \psi_g = \rho_l g z \tag{2.39}$$

and the pressure potential becomes

$$\rho_l \psi_p = \rho_l \frac{1}{\rho_l} p' = p' \tag{2.40}$$

Hence, the pressure equivalent of the hydraulic potential, p_h, is

$$p_h = \rho_l \psi_h = \rho_l g z + p' \tag{2.41}$$

Pressure equivalent should be expressed in Pa, but, traditionally, the units bar (= 10^5 Pa) and mbar are frequently used in soil physics and related disciplines.

Question 2.20 Convert the potential scale in Figure 2.7 to a Pa scale and a mbar scale.

Question 2.21 What is the water pressure at the bottom of the cylinder in Figure 2.7?

Question 2.22 The barometric pressure at two subsequent days differs by 1 cm mercury. How does this influence the hydraulic potential of the water?

2.2.4 *Potential on weight basis. Head equivalent*

Just as the potential on mass basis was converted to a potential on volume basis (Section *2.2.3*), it can also be converted to a potential related to the weight of the soil solution. Since weight = mass × g (Equation 2.6), to go from J $\mathrm{kg^{-1}}$ to J $\mathrm{N^{-1}}$ requires dividing by g. The convenience of this form of the potential is immediately apparent from the gravitational potential

$$\frac{\psi_g}{g} = \frac{gz}{g} = z \tag{2.42}$$

Thus, the gravitational potential on weight basis has the dimension of height and is called gravitational head. Similarly, the head equivalent of the pressure potential is the pressure head, h,

$$\frac{\psi_p}{g} = h \tag{2.43}$$

The head equivalent of the hydraulic potential is then the hydraulic head, H,

$$\frac{\psi_h}{g} = H = h + z = \frac{\psi_p}{g} + \frac{\psi_g}{g} \tag{2.44}$$

The convenience of using head equivalents will become obvious in Chapters 3 and 5.

Question 2.23 Convert the potential scale in Figure 2.7 to a head scale.

2.2.5 Equilibrium of a compressible medium in the gravitational field

In Answer 2.20 the water pressure increased proportionally with depth, because ρ_l was uniform (incompressible medium). If density is dependent on pressure, the relationship between pressure and height is more complex. Such is the case, for instance, in the atmosphere, because air is compressible. If the air in the atmosphere behaves as an ideal gas and temperature is constant with height, the relationship between air pressure and height can be calculated for the equilibrium state. Then the total potential, $\psi_t = \psi_g + \psi_p$, is constant (Equations 2.30 and 2.38).
Hence (Equations 2.32 and 2.34),

$$\psi_t = gz + \int \frac{1}{\rho_a} \mathrm{d}p = \text{constant} \tag{2.45}$$

where ρ_a is the density of the air.

The relationship between ρ_a and p can be derived from the ideal gas law

$$pV = nRT \tag{2.46}$$

where p is pressure, V is volume, n is amount of gas (mol), R is molar gas constant and T is temperature (K).

$$\text{Also } \rho_a = \frac{nM}{V} = \frac{pM}{RT} \tag{2.47}$$

where M is molar mass of air.
Substitution of ρ_a in Equation 2.45 gives:

$$\psi_t = gz + \int \frac{RT}{pM} \mathrm{d}p = C_1 \tag{2.48}$$

or

$$gz + \frac{RT}{M} \ln p = C_2 \tag{2.49}$$

or

$$\ln p = -\frac{M}{RT} gz + C_3 \tag{2.50}$$

or

$$p = C \exp\left(-\frac{M}{RT} gz\right) \tag{2.51}$$

where C_1, C_2, C_3 and C are constants.

Taking $p = p_0$ at $z = 0$ (sea level) leads to

$$p = p_0 \exp\left(-\frac{M}{RT} gz\right) \tag{2.52}$$

With M (air) ≈ 28.8 g mol^{-1}, $g \approx 10$ N kg^{-1}, $R \approx 8.3$ J mol^{-1} K^{-1} and $T \approx 290$ K, Equation 2.52 reduces to

$$p = p_0 \exp(-1.2 \times 10^{-4}\,\mathrm{m}^{-1} \times z) \tag{2.53}$$

At a height of 1 km, for instance, this gives

$$p = p_0 \exp(-0.12) \approx 0.89\, p_0$$

Question 2.24 If the pressure at sea level is 1000 mbar,

a. what is the pressure at 1, 2 and 3 km height?
b. Plot pressure against height above sea level.
c. What do you conclude?

Density generally increases with pressure. Therefore, pressure increases more than proportionally in the direction of the force field. Although this is to some extent true for water, in soil physics water is always assumed to be incompressible and thus uniform in density.

2.3 SUGGESTED LITERATURE

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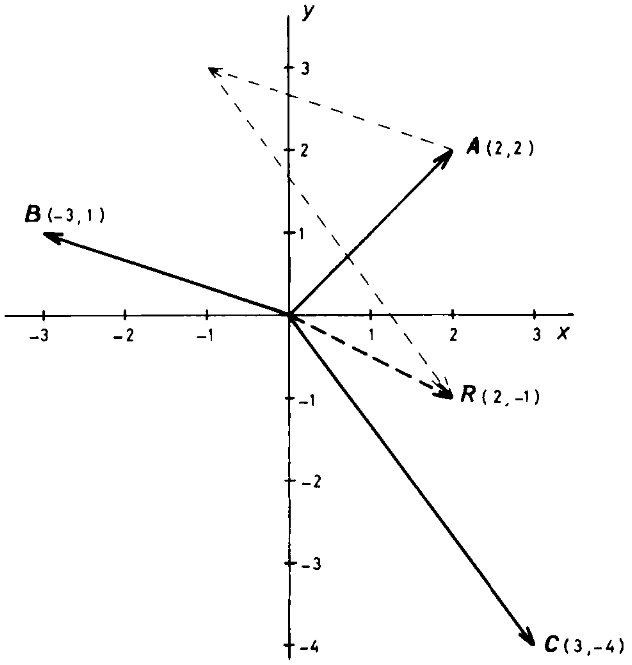
2.4 ANSWERS CHAPTER 2

Answer 2.1 Sum of x-components: $2 - 3 + 3 = 2$

Sum of y-components: $2 + 1 - 4 = -1$

Thus, the x,y-components of the resultant vector, ***R***, are $(2,-1)$. The diagram below shows that the same result is obtained by placing the vectors tail to head in the order ***A***, ***B*** and ***C***. Any other order gives the same result. (Check this.)



Answer 2.2 A resultant external force acting on a body causes it to accelerate. The magnitude of the acceleration is equal to the force divided by the mass of the body. The direction of the acceleration is the same as that of the resultant force.

Answer 2.3 Mechanical equilibrium is the state of a body in which, if at rest, it remains at rest or, if in motion, it continues in uniform motion. No resultant force acts on that body.

Answer 2.4 The body is at rest; there is no acceleration and no resultant force. Thus, the body is (at least) in mechanical equilibrium.

Answer 2.5 Equation 2.3 does not apply to an object moving with constant velocity in outer space. Since it is moving in a virtual vacuum, the resistance to its motion is negligible, or $\Sigma \boldsymbol{F}^{\mathrm{d}} \approx 0$. Since it is moving with constant velocity, there can be no net static force acting on it, i.e. $\Sigma \boldsymbol{F}^{\mathrm{s}} = 0$. However, because it is moving, it is not at static equilibrium. Thus Equation 2.3 would not apply to this situation.

Answer 2.6 a. The forces to be summed are those acting on the body. There are two: the one by gravity (weight) and the one exerted by the table (reaction to the weight).

b. Since the body is at rest, there are no drag forces acting on it: $\Sigma \boldsymbol{F}^{\mathrm{d}} = 0$.

c. Since $\Sigma \boldsymbol{F} = \Sigma \boldsymbol{F}^{\mathrm{s}} + \Sigma \boldsymbol{F}^{\mathrm{d}} = 0$, it follows from Answer 2.6b that $\Sigma \boldsymbol{F}^{\mathrm{s}} = 0$. Thus, the body is, according to Equation 2.3, also at static equilibrium.

Answer 2.7 a. Weight is the gravitational force exerted by the earth on a body.

b. The magnitude of the weight is $F = mg = 5\ \mathrm{kg} \times 10\ \mathrm{N\ kg^{-1}} = 50\ \mathrm{N}$. Its weight is 50 N.

Answer 2.8 By equating the two equations, we find for the magnitude of $\boldsymbol{g}$:

$$g = \frac{Gm_2}{r^2}$$

Answer 2.9 a. Answer 2.8 shows that g depends only on r, because G and m_2 are constants. Thus g decreases with height above the earth surface. Because the radius of the earth is not constant and for several other reasons, even the gravitational field strength is nonhomogeneous if viewed on a large scale. However, on a small scale (e.g. the depth of a soil profile) it is, for all practical purposes, homogeneous.

b. The magnitude of the centrifugal force exerted on a body in a centrifuge is $F = m\omega^2 r$. $\boldsymbol{F}$ is at all times directed away from the centre of rotation, ω is the angular frequency, r is the distance from the centre of rotation and m is the mass of the body. Thus, the centrifugal force increases with distance from the centre of rotation and changes direction constantly. Therefore, the centrifugal field is nonhomogeneous.

Answer 2.10 a. Since $\boldsymbol{F} = m\boldsymbol{a} = Q\boldsymbol{c}$, $\boldsymbol{a} = \frac{Q}{m}\boldsymbol{c}$

b. Since $Q = 10^{-7}\ \mathrm{C}$, $m = 0.1\ \mathrm{g}$ and $c = 10^3\ \mathrm{N\ C^{-1}}$,

$$a = \frac{10^{-7}\ \mathrm{C}}{10^{-4}\ \mathrm{kg}} \times \frac{10^3\ \mathrm{N}}{\mathrm{C}} = 1\ \mathrm{N\ kg^{-1}} = 1\ \mathrm{m\ s^{-2}}$$

c. Analogously to Newton's law of gravitation,

$$F = \frac{kQ_1Q_2}{r^2}$$

where Q_1 and Q_2 are two charges, F is the mutual force of attraction between them, k is constant and r is the distance separating the charges. This equation is known as Coulomb's law. The magnitude of the field strength $\boldsymbol{c}$ is now: $c = \frac{kQ}{r^2}$ where Q is the charge of the body due to which the electric field is present.

Answer 2.11 The pressure in fluids is scalar, because it has only a magnitude and not a particular direction.

Answer 2.12 a. Combination of Equations 2.12 and 2.14, using $g\boldsymbol{k} = -\boldsymbol{g}$, gives:

$$\Sigma \boldsymbol{F}_{\mathrm{c}}^{\mathrm{s}} = \rho_1 g V \boldsymbol{k} = -\rho_1 V \boldsymbol{g}$$

The body force is $\boldsymbol{F}_\mathrm{b}^\mathrm{s} = m\boldsymbol{g} = \rho_\mathrm{c} V\boldsymbol{g}$
where ρ_c is the density of the cube.
Thus, $\Sigma \boldsymbol{F}^\mathrm{s} = (\rho_\mathrm{c} V - \rho_\mathrm{l} V)\boldsymbol{g} = (\rho_\mathrm{c} - \rho_\mathrm{l}) V\boldsymbol{g}$

b. $\Sigma \boldsymbol{F}^\mathrm{s} = (30\,\mathrm{g} - 27\,\mathrm{cm}^3 \times 1\,\mathrm{g\,cm}^{-3}) \times (-10\,\mathrm{N\,kg}^{-1}) = -0.03\,\mathrm{N}$ (downward).

c. Since $\Sigma \boldsymbol{F}^\mathrm{s}$ is negative, the cube will accelerate downwards. $\boldsymbol{F}^\mathrm{d}$ will increase with speed (magnitude of velocity) until $\boldsymbol{F}^\mathrm{d} = -\Sigma \boldsymbol{F}^\mathrm{s}$. From that moment on, the cube will sink at constant velocity until it collides with the bottom of the container (See Section *2.1.4*).

Answer 2.13 a. The weight of the clod is $\boldsymbol{F} = m\boldsymbol{g} = 8.99\,\mathrm{g} \times (-10^{-2})\,\mathrm{N\,g}^{-1} = -8.99 \times 10^{-2}\,\mathrm{N}$.

b. $\boldsymbol{F}_\mathrm{b}^\mathrm{s} = m\boldsymbol{g} = -8.99 \times 10^{-2}$ N. The clod is in static equilibrium, thus $\Sigma \boldsymbol{F}^\mathrm{s} = 0$ and $\Sigma \boldsymbol{F}_\mathrm{c}^\mathrm{s} = 8.99 \times 10^{-2}$ N. Part of this force, $2.79\,\mathrm{g} \times 10^{-2}\,\mathrm{N\,g}^{-1} = 2.79 \times 10^{-2}$ N, is provided by the balance. Thus, the net contact force by the water (upthrust) equals $8.99 \times 10^{-2}\,\mathrm{N} - 2.79 \times 10^{-2}\,\mathrm{N} = 6.2 \times 10^{-2}\,\mathrm{N}$.

c. The net contact force by the water is 6.2×10^{-2} N (Answer 2.13b) and this force equals $-\rho_\mathrm{l} V\boldsymbol{g}$ (Answer 2.12a).

Therefore, $V = \dfrac{6.2 \times 10^{-2}\,\mathrm{N}}{-\rho_\mathrm{l} g} = \dfrac{6.2 \times 10^{-2}\,\mathrm{N}}{-1\,\mathrm{g\,cm}^{-3} \times (-10^{-2})\,\mathrm{N\,g}^{-1}} = 6.2\ \mathrm{cm}^3$. The mass of the clod is 8.99 g. Thus, its bulk density is $\dfrac{8.99\,\mathrm{g}}{6.2\,\mathrm{cm}^3} = 1.45\ \mathrm{g\ cm}^{-3}$ or $1450\,\mathrm{kg\,m}^{-3}$.

d. The clod needed to be perfectly sealed to prevent water from entering. If water enters the clod, the balance reads higher and one would calculate a higher density than the actual density of the clod. Perfect sealing can be checked by reweighing the clod after it is taken out of the water and comparing the weight with that before it was immersed in water.

Answer 2.14 a. For an observer who is part of the rotating system, a body of liquid is in static equilibrium, because it does not move in the direction of $\boldsymbol{r}$.

The centrifugal force field exerts a body force $\dfrac{\boldsymbol{F}_\mathrm{b}^\mathrm{s}}{m} = \omega^2 r\boldsymbol{i}$, directed away from the axis of rotation. This body force is compensated by an equal, but oppositely directed net contact force exerted by the surrounding liquid.

Thus, analogously to Equation 2.13:

$$\frac{\Sigma \boldsymbol{F}_\mathrm{c}^\mathrm{s}}{m} = -\frac{1}{\rho_\mathrm{l}}\frac{\mathrm{d}p}{\mathrm{d}r}\boldsymbol{i} = -\omega^2 r\boldsymbol{i}$$

For an observer outside the system, there is no static equilibrium. The velocity of the liquid changes in direction as a result of the centripetal acceleration $-\omega^2 r\boldsymbol{i}$ (towards the axis of rotation). This acceleration results from the net contact force $\dfrac{\Sigma \boldsymbol{F}_\mathrm{c}^\mathrm{s}}{m}$, which leads to the same conclusion as above.

Therefore, $dp = \rho_l \omega^2 r \, dr$ or, after integration, $p = \frac{1}{2} \rho_l \omega^2 r^2 + C$, where C is an integration constant.

Because $p = p_{atm}$ at $r = r_0$, $C = p_{atm} - \frac{1}{2} \rho_l \omega^2 r_0^2$

Thus, $p = \frac{1}{2} \rho_l \omega^2 (r^2 - r_0^2) + p_{atm}$

b. Since 1 revolution equals 2π rad, $\omega = \frac{6000 \times 2\pi}{60 \text{ s}} = 200\pi \text{ s}^{-1}$ (radians are dimensionless).

$$\text{Therefore, } p = \frac{1}{2} \rho_l \omega^2 (r^2 - r_0^2) + p_{atm}$$

$$= \frac{1}{2} \times 1000 \text{ kg m}^{-3} \times (200\pi)^2 \text{ s}^{-2} \times (0.09 - 0.04) \text{ m}^2 + p_{atm}$$

$$\approx (10^7 + 10^5) \text{ Pa} = 1.01 \times 10^7 \text{ Pa.}$$

Answer 2.15 a. The procedure is as follows:

1. Sieve the grains.
2. Disperse the finer grains in water in such a way that a homogeneous suspension is formed.
3. Sample the suspension at a given depth at various times.
4. Dry the samples and determine the mass fraction of each textural fraction by weighing.

b. 2 mm and 50 μm sieves are required. A 2 mm sieve is needed to separate grains larger than 2 mm from the smaller ones. Grains between 2 and 3 mm are classified as gravel. These are not included in the textural analysis of soils or fine earth (Section 1.1.2). Grains larger than 50 μm can easily be separated by sieves.

c. Equation 2.22 can be written as

$$t = \frac{9\eta s}{2gr^2(\rho_s - \rho_l)}$$

To find the sand content according to the ISSS classification, sample for $r = 10$ μm at 15 cm depth at time:

$$t = \frac{9 \times 10^{-3} \text{ Pa s} \times 0.15 \text{ m}}{2 \times 10 \text{ N kg}^{-1} \times (10 \text{ μm})^2 \times 1.65 \times 10^3 \text{ kg m}^{-3}} \approx 409 \text{ s} \approx 6.82 \text{ min}$$

To find the silt content, sample for $r = 1$ μm at:

$$t = \frac{9 \times 10^{-3} \text{ Pa s} \times 0.15 \text{ m}}{2 \times 10 \text{ N kg}^{-1} \times (1 \text{ μm})^2 \times 1.65 \times 10^3 \text{ kg m}^{-3}} \approx 40909 \text{ s} \approx 682 \text{ min} =$$

11 h 22 min

d. At $t \approx 6.82$ min, all particles larger than 20 μm are beyond 15 cm depth; therefore, the sample contains only clay and silt. At $t \approx 682$ min, all silt particles are also beyond 15 cm depth, thus only clay remains in the sample.

e. Yes. At $t = 0$, all particle sizes are assumed to be uniformly distributed throughout the cylinder. It is also assumed that all particles attain their terminal velocity instantly or within a negligible distance and that they all settle independently of each other. Then at any depth, the concentration of particles of a particular size remains at the original value, until it drops instantly to zero when the particles of that size that started settling at the water surface, pass that depth. Thus a sample taken at $t \approx 6.82$ min from a layer of infinitesimal thickness at 15 cm depth contains all the particles smaller than $r = 10$ μm in their original concentration, and none of the particles larger than $r = 10$ μm.

f. The clay content in the sedimentation cylinder, as well as in the original soil sample, is: $\frac{0.1\,\text{g}}{20\,\text{cm}^3} \times 1250\,\text{cm}^3 = 6.25$ g.

Clay and silt content in the original soil sample is:

$\frac{0.3\,\text{g}}{20\,\text{cm}^3} \times 1250\,\text{cm}^3 = 18.75$ g.

Thus, the silt content is: 18.75 g − 6.25 g = 12.50 g.

The mass of grains less than 2 mm is 65 g − 10 g = 55 g (Question 2.15c).

So the sand content is 55 g − 18.75 g = 36.25 g.

Of this, (15 − 10) g = 5 g is between 2 mm and 50 μm (Question 2.15c) and (36.25 − 5) g = 31.25 g is between 50 and 20 μm.

The mass fractions are then:

sand $\frac{36.25\,\text{g}}{55\,\text{g}} \approx 66\%$

silt $\frac{12.50\,\text{g}}{55\,\text{g}} \approx 23\%$

clay $\frac{6.25\,\text{g}}{55\,\text{g}} \approx 11\%$

g. The textural classification of the soil sample is sandy loam (Figure 1.2).

Note (erratum): In order to use Figure 1.2, one should recalculate the fractions according to a boundary between sand and silt at 50 μm, rather than 20 μm. The result is: sand: 5.00 g or 9.1 %; silt: 43.75 g or 79.5 %; clay: 6.25 g or 11.4 %. The textural classification of the soil sample is silt loam.

h. Stokes' law is valid for $Re < 0.1$.

Combination of $Re = \frac{2rv\rho_l}{\eta}$ and $v = \frac{2gr^2(\rho_s - \rho_l)}{9\eta}$ gives

$$r^3 = \frac{9\,Re\,\eta^2}{4g\rho_l(\rho_s - \rho_l)}$$

$$\text{At } 20\,^\circ\text{C, } r^3 = \frac{9 \times 0.1 \times 10^{-6}\,(\text{Pa s})^2}{4 \times 10\,\text{N kg}^{-1} \times 10^3\,\text{kg m}^{-3} \times 1.65 \times 10^3\,\text{kg m}^{-3}}$$

$$\approx 1.36 \times 10^{-14}\,\text{m}^3$$

$$r \approx 2.39 \times 10^{-5}\,\text{m} \approx 24\,\mu\text{m}$$

Thus the maximum particle diameter for which Stokes' law can be used is about 50 μm. This is the same as the maximum grain size used in the sedimentation method. Therefore, it is justified to use Stokes' law.

nswer 2.16 Carrying out the integration of Equation 2.25, assuming $\frac{F^s}{m}$ independent of s, yields

$$\psi = -\frac{F^s}{m}s + C$$

where C is a constant of integration.

Substitution of ψ_0 for ψ at $s = s_0$ yields

$C = \psi_0 + \frac{F^s}{m}s_0$, and thus

$$\psi = -\frac{F^s}{m}(s - s_0) + \psi_0$$

This result shows that the numerical value of ψ at s depends on the value of ψ_0 at s_0. The difference in ψ between two points is not affected by the choice of ψ_0 at s_0.

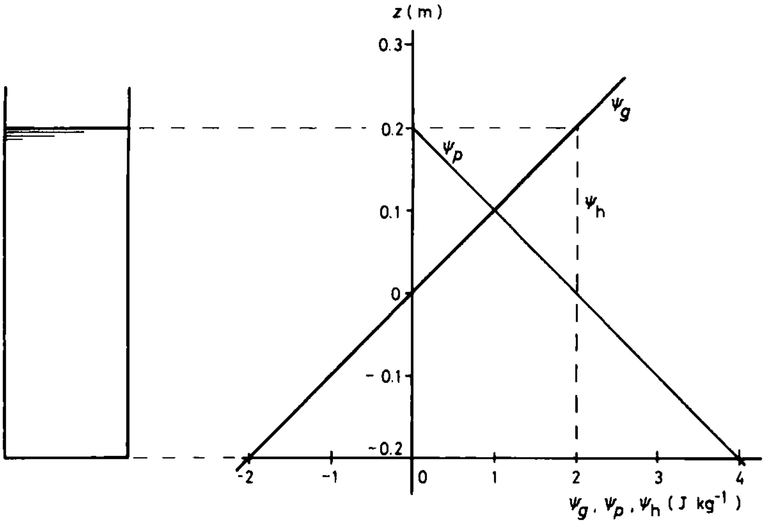
Answer 2.17 a. From Equation 2.32, $\psi_g = gz = 10\ \text{N kg}^{-1} \times z$. For $z = 0$, 0.1, 0.4, 0.5 and -0.1 m, $\psi_g = 0$, 1, 4, 5 and $-1\ \text{J kg}^{-1}$, respectively.

b. A positive value of ψ_g means that the point is located above the selected reference level for ψ_g, here the bottom of the cylinder. A negative value means that the point is located below the reference level.

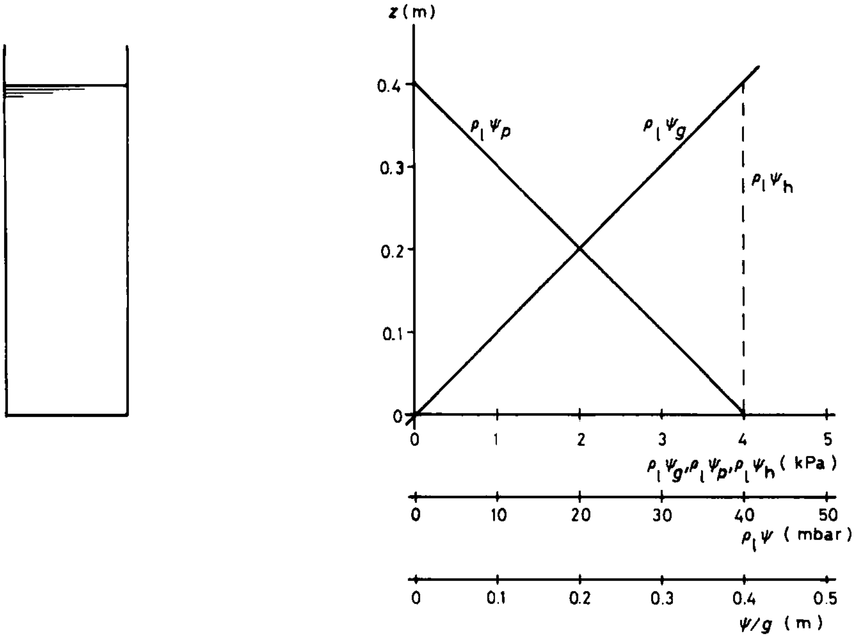
Answer 2.18 According to Equation 2.36, $\psi_p = \frac{1}{\rho_l}(p - p_{atm})$. Below the water surface p is greater than p_{atm} because of the overlying water (remember F^s_b). Therefore, ψ_p is positive.

Answer 2.19 a. Now, at the surface, $z = 0.2$ m and $\psi_p = 0$, so $\psi_g = gz = 10\ \text{N kg}^{-1} \times 0.2\ \text{m} = 2\ \text{J kg}^{-1}$. Therefore, $\psi_h = \psi_p + \psi_g = 2\ \text{J kg}^{-1}$.

b. The plot of ψ_p does not change because the reference point for ψ_p remains at the water surface.



Answer 2.20 This amounts to changing potential on mass basis (J kg^{-1}) into potential on volume basis (J m^{-3}). If the density of water is assumed constant at 1000 kg m^{-3}, then the pressure equivalent of $\psi = 1$ J kg^{-1} is $\rho_l\psi = 10^3$ kg $m^{-3} \times 1$ J $kg^{-1} = 10^3$ J $m^{-3} = 1$ kPa $= 10$ mbar. Thus the potential scale can be changed into a pressure equivalent scale by substituting 1 kPa or 10 mbar for every J kg^{-1}.



Answer 2.21 Since at the bottom $\psi_p = 4$ J kg^{-1}, $\rho_l\psi_p = 4$ kPa or 40 mbar. Thus at hydrostatic equilibrium, the water pressure in mbar equals the depth in centimeter below the water surface, again assuming that $\rho_l = 1000$ kg m^{-3} (and $g = 10$ N kg^{-1}).

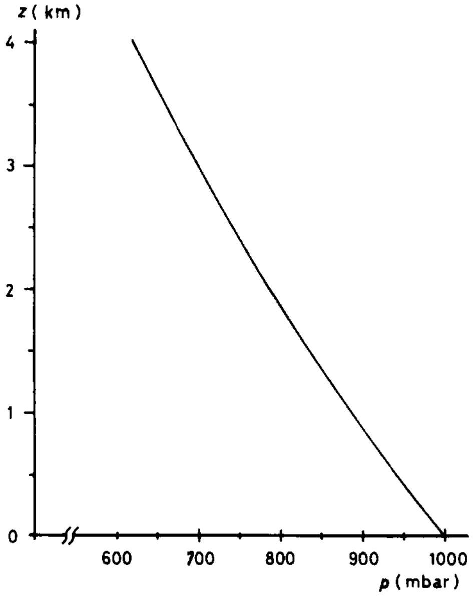
Answer 2.22 A change in barometric pressure does, by convention, not influence the hydraulic potential of the water, because ψ_p is expressed relative to atmospheric pressure. At the water surface, ψ_p remains zero, irrespective of the barometric pressure.

Answer 2.23 Converting a potential scale into a head scale amounts to changing potential on mass basis (J kg^{-1}) into potential on weight basis (J N^{-1}). Therefore, the potentials on mass basis have to be divided by g. If $\psi = 1$ J kg^{-1}, then $\psi/g = 0.1$ J $N^{-1} = 0.1$ m. So every J kg^{-1} on the potential scale is substituted by 0.1 m. This scale is also indicated in the figure of Answer 2.20.

Answer 2.24 a. $p = p_0 \exp -(1.2 \times 10^{-4}\ \text{m}^{-1} \times z) = 1000\ \text{mbar} \times \exp -(1.2 \times 10^{-4}\ \text{m}^{-1} \times z)$

At $z = 1$ km, $p = 887$ mbar; at $z = 2$ km, $p = 787$ mbar, and at $z = 3$ km, $p = 698$ mbar.

b.



c. In an isothermal atmosphere, the pressure decreases exponentially with height.

3 Static equilibria in soils

3.1 HYDROSTATIC EQUILIBRIUM

3.1.1 Binding of water by soils

As described in Chapter 1, soils consist of a solid skeleton (matrix) with pores in between. The pores have different sizes, shapes and spatial distributions and provide the space for storage and transport of soil water and gas.

Storage or retention of water by soils is a result of attractive forces between the solid and liquid phases. These 'matric' forces enable the soil to hold water against forces or processes such as gravity, evaporation, uptake by plant roots, etc. There are three mechanisms for binding of water to the solid matrix:

- direct adhesion of water molecules to solid surfaces by London-van der Waals forces
- capillary binding of water
- osmotic binding of water in double layers.

Adhesion is the attraction of dissimilar substances for each other; cohesion is the mutual attraction of particles of the same substance.

Water molecules are attracted to solid surfaces by various types of London-van der Waals forces. These are strong, but very short-range forces; they diminish with about the sixth power of the distance. Thus only a very thin water layer is adsorbed in this way around soil particles. Since these adhesive forces are so strong that the water cannot be extracted by plant roots, this form of water retention in itself is insignificant for storage, transport and plant growth. However, the adhesive forces, together with the cohesive forces between water molecules, form the basis for capillary binding of soil water. This is the most important mechanism of binding of water in coarse soils. In clay soils the osmotic binding of water in diffuse electric double layers may exceed the capillary binding.

The capillary and osmotic binding of water in soils, the components of the soil water potential associated with them, and standard equipment for measuring some of these components are discussed in detail in the remainder of this chapter. This discussion is started with an analysis of the binding of water in a glass capillary to provide a sound physical basis for understanding capillary binding of water in soils.

3.1.2 Binding of water in a capillary

The adhesive and cohesive forces together are responsible for the binding of water in capillaries.

The adhesion or repulsion of water by solid surfaces can be characterized with the potential energy of water molecules near the solid surface, divided by area of solid-water interface, γ (J m^{-2}). Taking as reference, as usual, zero potential for bulk water, a positive value of γ means that the energy of water molecules near solid surfaces is higher than the energy of molecules in bulk water. Water will not adhere to such surfaces. They are called hydrophobic. Surfaces with negative γ values will spontaneously attract water, if available, because near such surfaces water molecules have a lower energy than in bulk water. Such surfaces are called hydrophilic. The water is said to 'wet' such surfaces. This is the normal situation for soil particles, glass, etc.

A similar characterization can be made for the cohesive forces between water molecules. Inside bulk water a water molecule experiences the same cohesive attraction in all directions. Thus, the net force on that molecule is zero. In contrast, water molecules at an air-water interface are attracted only towards the interior, because there are no water molecules on the air side of the interface. Thus the energy content of water molecules at the interface is higher than that of molecules inside bulk water. This is characterized by the surface tension, σ, which is the interface potential energy divided by interface area (J m^{-2}). Again, taking the potential of water molecules in bulk as zero, σ is positive. The result is that water in contact with air will always attempt to reduce its surface area. The smallest surface area for a given mass of water is that of a sphere. This explains the formation of spherical droplets of rain, sprays, etc. Any other shape would represent a larger surface area and thus a higher energy content. For the same reason the meniscus of water in a capillary assumes the spherical shape which is compatible with the angle of contact, φ (Figure 3.1a).

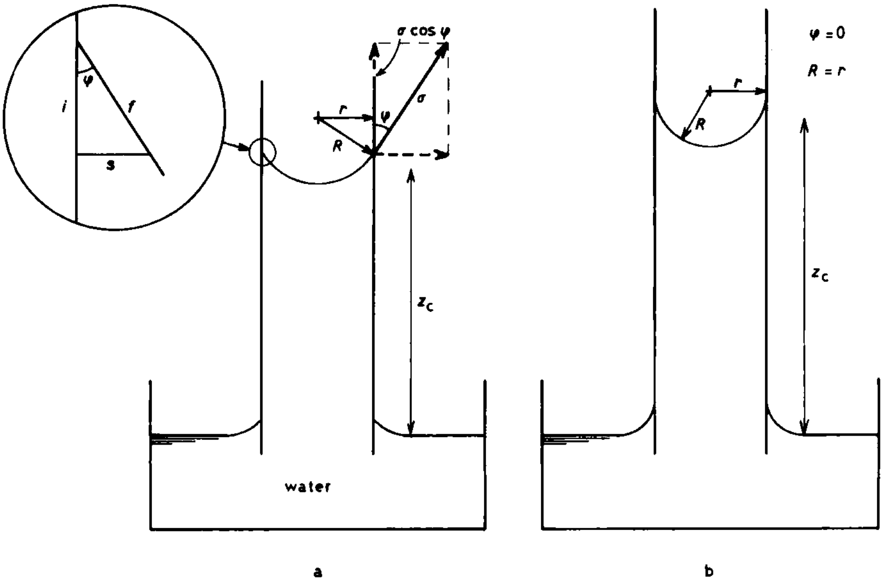


Figure 3.1 a. Capillary rise in a capillary with wetting angle φ.
b. Same, with wetting angle $\varphi = 0$ (complete wetting).

When a solid, water and air phase come together at one point, such as in a capillary, the shape of the interface, as characterized by the angle of contact, φ, depends on whether γ is positive, negative or zero, and on its magnitude with respect to σ. This can be shown using the enlargement of the situation around the point of contact shown in Figure 3.1a. The angle between the meniscus and the solid surface at that point is φ. Consider the water in the triangular volume of unit width perpendicular to the paper, for which the maximum distance to the solid surface is s. The distance s is arbitrary, except that it should be large compared to the distance over which the adhesive and cohesive forces are active, and small compared to the radius of curvature of the meniscus. The area of solid-water interface associated with the triangular volume is i, that of the water-air interface is f. The total energy associated with both types of interfaces is then:

$$E = \sigma f + \gamma i = \sigma s/\sin \varphi + \gamma s/\tan \varphi \tag{3.1}$$

The angle φ will adjust itself, until E is at a minimum. For this situation

$$\frac{\mathrm{d}E}{\mathrm{d}\varphi} = -\frac{\sigma s \cos \varphi}{\sin^2 \varphi} - \frac{\gamma s}{\sin^2 \varphi} = 0 \tag{3.2}$$

$$\text{or} \cos \varphi = -\frac{\gamma}{\sigma} \tag{3.3}$$

$$\begin{aligned} \text{Thus, } \varphi &= 180°, \text{ when } \gamma = \sigma \\ \varphi &= 90°, \text{ when } \gamma = 0 \\ \varphi &= 0°, \text{ when } \gamma = -\sigma \end{aligned}$$

Since $\varphi = 180°$ and $\varphi = 0$ are the physical limits, it should be clear that $\varphi = 180°$ for $\gamma \geqslant \sigma$ and $\varphi = 0$ for $\gamma \leqslant -\sigma$.

The latter case is normal for soil-water and glass-water systems. It is called complete wetting. An example of nonwetting is mercury on clean glass, for which $\varphi = 140°$. Clean steel has no attraction, nor repulsion for water, and thus $\gamma = 0$ and $\varphi = 90°$.

To illustrate the principle of capillary binding of water in soils, consider a clean glass capillary being inserted in a container with water, such as in Figure 3.1b. Since $\gamma < -\sigma$, the angle of contact φ is zero. Since $\gamma < 0$, the inside and outside surfaces of the capillary attract a water film. The large air-water interfacial area of these films represent a relatively high energy ($\sigma > 0$) and thus the system would move spontaneously to a state of lower potential energy if the air-water interfacial area could be reduced. On the outside surface of the capillary there is no inherent physical possibility to reduce the air-water interfacial area. But inside the capillary the interfacial area would be reduced if the capillary would fill with water. The reduction in air-water interfacial energy can be used to draw water up into the capillary against gravity. Above and below the water level in the capillary, the walls of the capillary are covered with a water film. Thus, after the initial wetting of the capillary walls, the energy of the solid-water interfaces remains the same independent of the height of capillary rise inside the capillary. Thus, water will continue to enter the capillary until the reduction in the air-water interfacial energy brought about by an additional incremental volume of water has become equal to the increase in gravitational energy of that incremental volume in the capillary. Since $\varphi = 0$, the meniscus will assume a spherical shape with radius of curvature, R, equal to the radius of the capillary, r.

Now suppose the water in the capillary has reached its final height of capillary rise, z_c (Figure 3.1b). A further incremental rise of Δz would decrease the air-water interfacial area by $2\pi r\Delta z$ (m^2) and thus the interfacial energy by $2\pi r\Delta z\sigma$ (J). The water that would fill the thin layer Δz would have to come from the container. This would mean an increase in potential energy in the gravitational force field of $\pi r^2\Delta z\rho_l g z_c$ (J). At the point of maximum capillary rise these two amounts of energy are the same, thus:

$$2\pi r\sigma\Delta z = \pi r^2 \rho_l g z_c \Delta z \tag{3.4}$$

$$\text{or } z_c = \frac{2\sigma}{\rho_l g r} \tag{3.5}$$

As the synonym 'surface tension' suggests, the air-water interfacial energy divided by area, σ, manifests itself as a force divided by length ($J\,m^{-2} = N\,m^{-1}$). By considering σ in this way, the influence of the angle of contact, φ, on height of capillary rise is easily derived. In Figure 3.1a the surface tension force acts over the entire circumference, $2\pi r$, and is everywhere directed tangentially to the meniscus and thus at an angle φ with the wall. The vertical component of the resultant force is then $2\pi r\sigma\cos\varphi$, directed upward. The horizontal component of the resultant force is zero, because the horizontal components, $\sigma\sin\varphi$, along the circumference cancel each other. The resultant upward force is balanced by the downward force on the liquid in the capillary tube which equals $\pi r^2 z_c \rho_l g$. Thus

$$2\pi r\sigma\cos\varphi = \pi r^2 z_c \rho_l g \tag{3.6}$$

from which:

$$z_c = \frac{2\sigma\cos\varphi}{\rho_l g r} \tag{3.7}$$

From Figure 3.1a it is clear that the radius of curvature, R, of the meniscus in a capillary with radius r and contact angle φ is:

$$R = r/\cos\varphi \tag{3.8}$$

Question 3.1 Calculate the height of capillary rise of water in a clean glass tube of 70 μm inner diameter. The surface tension of water $\sigma = 0.07\ N\,m^{-1}$.

The water in the capillary tube of Figure 3.1 is at static equilibrium. Thus its potential components can be calculated in the same way as was done in connection with

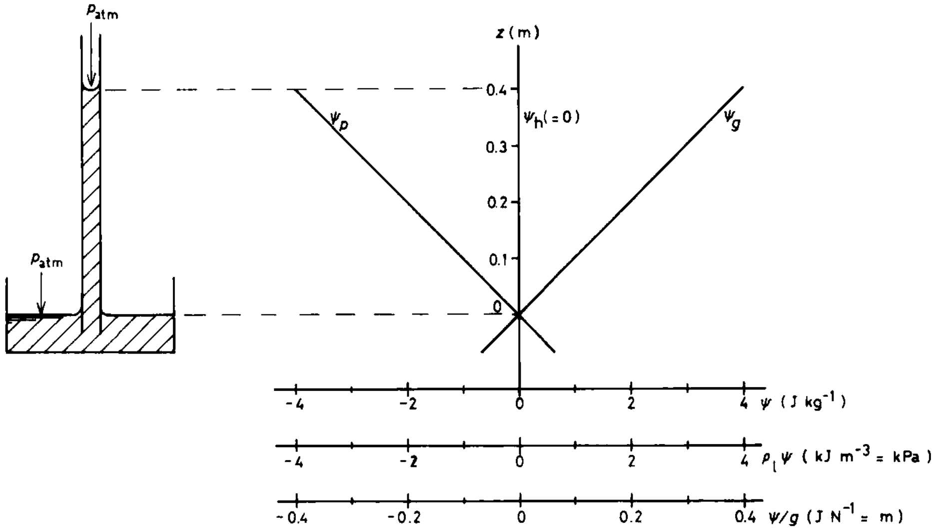


Figure 3.2 Potential diagram of water in a capillary.

Figure 2.7. Figure 3.2 presents a diagram of the potentials on mass basis, as well as on volume and weight basis. As in Figure 2.7, the pressure potential in the big container increases with depth from zero at the water surface to positive values below the surface and compensates the decreasing gravitational potential. Likewise, the pressure potential of the water in the capillary tube decreases with height to compensate the increasing gravitational potential. Thus, the pressure of the water in the capillary tube is lower than atmospheric pressure, creating a pressure difference between both sides of the curved meniscus.

Question 3.2 What is in Figure 3.2 the pressure potential of the water at the air-water interface in the capillary tube? Give your answer in J kg^{-1}, Pa and m.

Question 3.3 Find an expression for the pressure potential on volume basis (Pa) of the water at the meniscus, based on Equation 3.7.

The pressure potential in the air above the curved meniscus in the capillary equals zero. Therefore, Answer 3.3 gives also the pressure potential difference across a curved interface. Since, according to Equation 3.8, $r = R \cos \varphi$, a general equation for the pressure difference across a curved interface with radius of curvature R is

$$\Delta p = 2\sigma/R \qquad (3.9)$$

where the highest pressure is at the concave side. Thus the pressure difference across a curved interface is inversely proportional to the radius of curvature. This is why reference points for pressure potentials are not chosen at curved menisci but at flat water surfaces under atmospheric pressure, free from any influence of solid surfaces, etc.

Question 3.4 Derive Equation 3.9 for a free spherical droplet of water.

Consider now the capillary tube of Figure 3.2 first being immersed in a water container and then raised slowly (Figure 3.3). Except for the gradual decrease of the radius of curvature of the meniscus, the capillary will remain filled with water until its top reaches a height above the water level in the container which is equal to the maximum height of capillary rise, z_c, in this case, $z_c = 0.4$ m. If the capillary is raised still further, air will enter from above, such that the water column remains 0.4 m high. This is why the pressure equivalent, $-\rho_l g z_c$, of the maximum height of capillary rise in a capillary is called air-entry value. In the case of positive gas pressure, it is also called bubbling pressure. Since $-z_c$, and its pressure equivalent $-\rho_l g z_c$, is also the pressure head of the water at the curved air-water interface, the concept can be extended to capillaries, pores, etc., in general. Thus, if the pressure potential of the water in a capillary, pore, etc. tends to become lower than the air-entry value, because of gravity or any other reason, the adhesive and cohesive forces are no longer able to hold onto the water and the capillary will empty to the extent necessary to keep the pressure potential of the water at the air-water interface at the air-entry value.

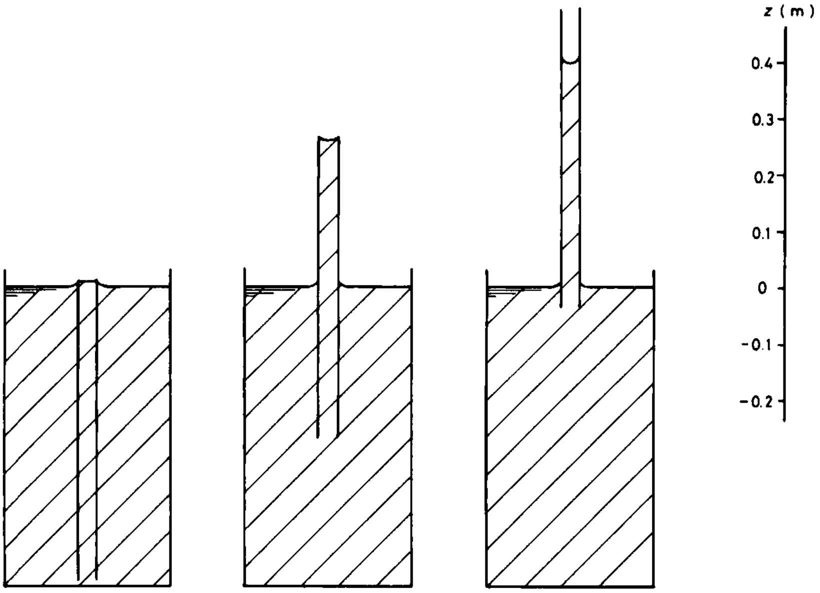


Figure 3.3 Illustration of air-entry value.

Question 3.5 A cylindrical capillary has a diameter of 20 μm. What is the air-entry value of this capillary in case of complete wetting?

Capillary rise of water takes place also in noncylindrical capillaries. The equivalent diameter of such a capillary is defined as the diameter of the cylindrical capillary that has the same height of capillary rise (or the same air-entry value).

Question 3.6 A noncylindrical capillary with complete wetting has an air-entry value of −14 kPa. What is its height of capillary rise and the equivalent diameter of the capillary?

A glass capillary is a first, crude approximation or model of a soil pore. With respect to the capillary binding of water, soils can be considered to consist of an assembly of irregularly shaped capillaries. Therefore, the results obtained in this section can, qualitatively, be applied to the capillary binding of water in soils. This will be done in the following sections.

3.1.3 Hydraulic potential of soil water at static equilibrium

A cylinder with an overflow at 0.4 m above the bottom is equipped with a noncapillary glass gauge (Figure 3.4). The cylinder and the gauge are filled with water up to

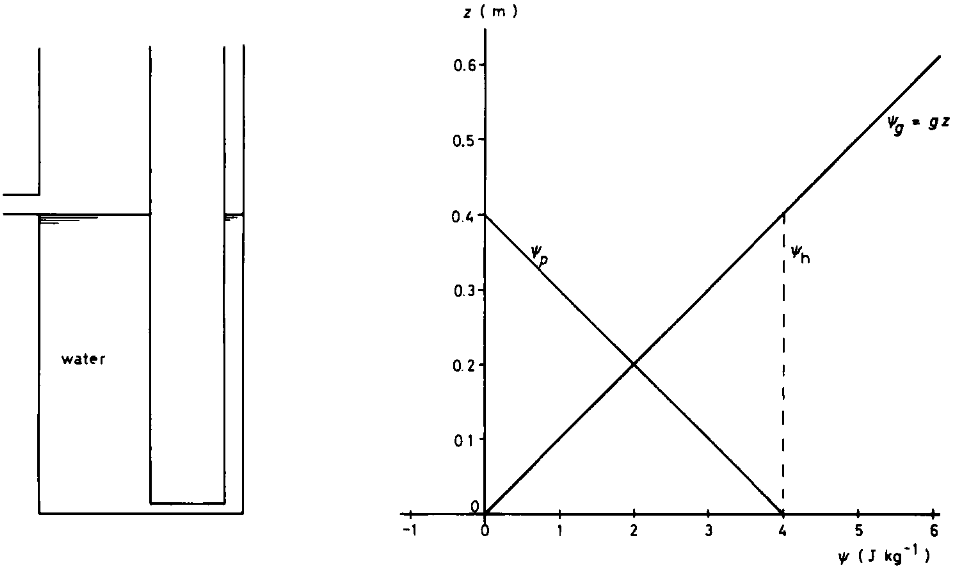


Figure 3.4 Potential diagram of water in container with noncapillary glass gauge and overflow.

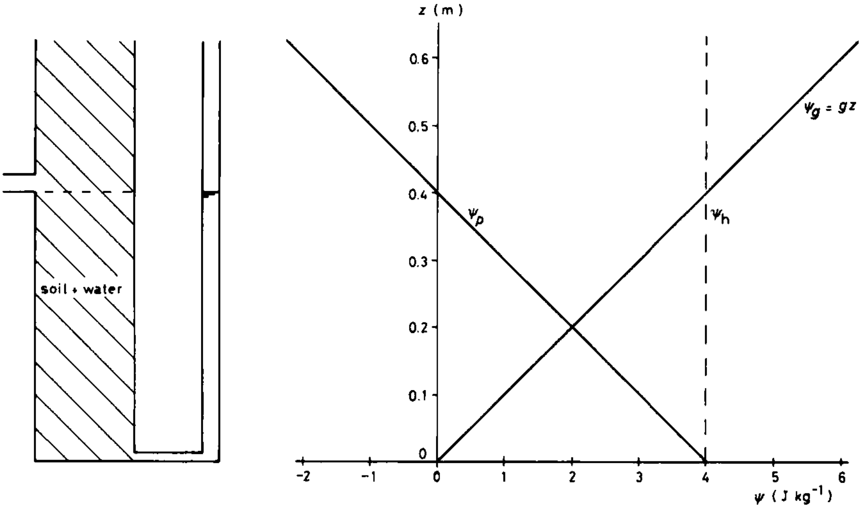


Figure 3.5 Potential diagram of water in container filled with soil, with noncapillary glass gauge and overflow.

the level of the overflow. If the reference point for the gravitational potential is chosen at the bottom of the cylinder, the potential diagram is exactly the same as that in Figure 2.7.

The empty cylinder is filled to the rim with dry soil and water is added slowly through the gauge until it starts discharging from the overflow. The water surface in the gauge will then again establish itself at the level of the overflow (Figure 3.5). This means that below $z = 0.4$ m the potential diagram is the same as in Figure 3.4. The presence of soil decreases the volume fraction of water but leaves the pressure potential unchanged.

Question 3.7 Why is the pressure potential below $z = 0.4$ m still the same as in Figure 3.4?

The soil above the overflow will attract water, due to capillary rise in the soil pores, osmotic binding, etc., until static equilibrium is attained.

Question 3.8 What is the condition for static equilibrium of the water above the overflow?

Since the hydraulic potential is everywhere the same and the gravitational potential increases linearly with height, the pressure potential must decrease linearly with height. Above the overflow, then, ψ_p is negative, because at the overflow, where the water surface is flat under atmospheric pressure, ψ_p is zero. This negative pressure potential is caused by the capillary action of the soil pores (similar to Figure 3.2) and other matric forces (Section *3.1.1*). The level where the pressure potential in a soil column or profile is zero, is called phreatic level. In field situations it is also called the groundwater table.

Question 3.9 A hole is made in the cylinder of Figure 3.5 at 0.1 m above the bottom.

a. What happens to the phreatic level?

b. What is the new hydraulic potential, ψ_h?

Draw the potential diagram for the new equilibrium situation that will be established after some time. (N.B.: do not change the reference level of the gravitational potential!)

Question 3.10 Two cylinders, filled with soil and placed at different levels, are connected with a tube. The tube connection is closed by means of a stopcock. The phreatic level in each cylinder is at the level of its overflow (Figure 3.6).

a. Plot in one diagram the water potentials in the cylinders against height. Choose the reference level of the gravitational potential at the phreatic level of the lowest cylinder.

b. Describe what happens when the stopcock is opened until a new static equilibrium is attained.

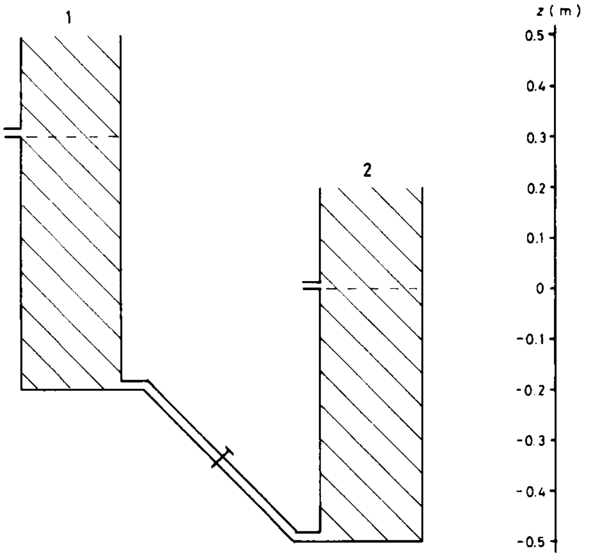


Figure 3.6 Two cylinders filled with soil and water, connected by a tube with stopcock, at different phreatic levels.

Below the phreatic level or groundwater table, water has a hydrostatic pressure which is larger than atmospheric pressure. Therefore, it will displace any air present at atmospheric pressure. However, some 'blocked' air may remain present at higher than atmospheric pressure, because it could not escape during wetting of the soil. The volume of this blocked air is usually neglected and the soil is still called saturated. However, the blocked air may amount to 10% of the pore volume, especially when the soil is saturated quickly from above.

The soil will also be saturated above a groundwater table in the zone where the pressure potential is higher than the air-entry value of the largest capillaries. This saturated zone above the groundwater table is called the full-capillary zone. Above the full-capillary zone, progressively smaller pores with lower air-entry values are empty. Thus, in a homogeneous soil, the water content decreases with height above the full-capillary zone.

Question 3.11 Which are the extreme values of the volume fraction of liquid, θ, above the groundwater table?

The presence of a full-capillary zone makes that the groundwater table is not the 'visible' boundary between saturated and unsaturated soil zones. A groundwater table can be made visible only by augering a vertical hole in which a free water surface can be formed.

Question 3.12 So far in this section only potentials on mass basis were used. Make a potential diagram for the soil column of Figure 3.5 using potentials on weight basis ('heads').

3.1.4 The piezometer and tensiometer

A piezometer is a tube of a few cm inner diameter, open at both ends, which is installed in a soil profile. If the lower end is below the groundwater table, a piezometer is partially filled with water. By determining the height of the water level in a piezometer it is possible to calculate the (positive) pressure potential of the soil water at the lower end of the tube. The diameter of piezometers is chosen large enough that capillary rise and resistance to water flow are negligible. As a result, any variation in hydraulic potential that may arise inside the piezometer, is instantaneously equalized. Thus, even if the pressure potential at the lower end is changing rapidly, the water inside a piezometer goes through a series of static equilibria and at any moment it can be assumed that the hydraulic head is uniform and equal to the hydraulic head of the soil water at the open lower end. There exchange of water takes place such that the pressure is always locally uniform. The static hydraulic head in piezometers can be determined by measuring the depth of the water level, since at the flat air-water interface the pressure potential is zero.

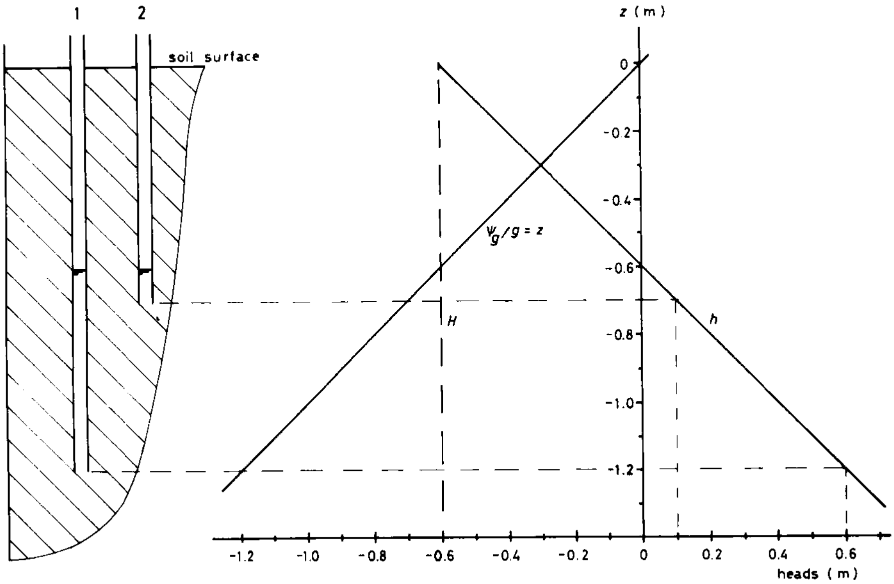


Figure 3.7 Two piezometers in a soil profile at static hydraulic equilibrium.

Figure 3.7 shows two piezometers in a soil profile in which the water is at static equilibrium. In the potential diagram the reference point $z = 0$ is taken at the soil surface. In both piezometers, at the water level, $H = h + z = 0 - 0.6$ m $= -0.6$ m. Thus at both lower ends H is also -0.6 m, which indicates that indeed the soil water is at static equilibrium.

From the water level in piezometer 1 at $z = -0.6$ m, it follows that at its lower end, where $z = -1.2$ m, $h = H - z = (-0.6 + 1.2)$ m $= 0.6$ m. Similarly, for piezometer 2, the pressure head at its lower end, where $z = -0.7$ m, is $h = 0.1$ m. With these results the potential diagram can be drawn.

Question 3.13 At what level is the groundwater table in Figure 3.7?

Since the water in the soil in Figure 3.7 is at static equilibrium, the potential diagram for the soil water is also valid for the water in each of the piezometers. In general, at static equilibrium of the soil water, the water level in piezometers is at the level of the groundwater table, independent of the depths of the lower ends. These results are somewhat trivial for this case of static equilibrium of the soil water, but this is no longer true when the water in the soil is flowing. Although water transport in soils is discussed extensively in Chapter 5, the functioning of piezometers in such a case is illustrated in the next question.

Question 3.14 Suppose that in Figure 3.7 the liquid level is in piezometer 1 at $z = -0.65$ m, and in piezometer 2 at $z = -0.55$ m.

a. Draw a potential diagram, assuming that the hydraulic head is a linear function of height.
b. At what height is the groundwater table?
c. Is this an equilibrium situation?
d. In which direction will the water flow?

Piezometers cannot be used to measure negative pressure heads because any water in the tubes will be absorbed by the soil. Negative pressure heads can be measured with so-called tensiometers. A tensiometer consists of a liquid-filled unglazed porous ceramic cup connected to a pressure measuring device, such as a vacuum gauge, via a liquid-filled tube (Figure 3.8). If the ceramic cup is embedded in soil, the soil solution can flow into or out of the tensiometer through the very small pores in the ceramic cup. Analogously to the situation discussed for piezometers, this flow continues until the pressure potential of the liquid in the cup has become equal to the pressure potential of the soil water around the cup. Thus the pressure potential of soil water can be measured with a tensiometer. It is, therefore, also called tensiometer pressure potential, and the pressure potential on volume basis is also called tensiometer pressure, p_t.

The vacuum gauge does not indicate the pressure in the cup when there is a difference in height between the two, such as in Figure 3.8. The liquid in the tube between the

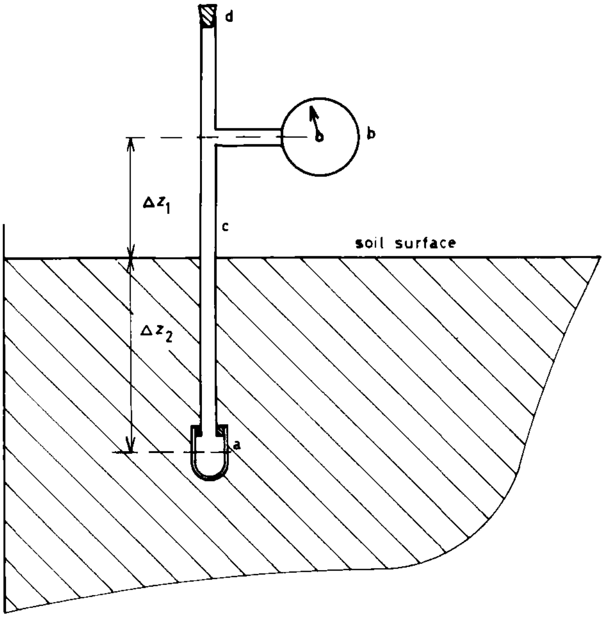


Figure 3.8 Tensiometer with vacuum gauge. a. porous cup; b. vacuum gauge; c. connecting tube; d. removable rubber stopper for deaeration.

cup and the vacuum gauge is at static equilibrium and thus the pressure in this liquid column increases linearly with depth. Therefore, in general, the tensiometer pressure in the cup is:

$$p_t = p_{\text{gauge}} + \rho_l g(\Delta z_1 + \Delta z_2) \tag{3.10}$$

where p_{gauge} is the pressure relative to atmospheric pressure of the liquid in the vacuum gauge (which is usually, but not necessarily the gauge reading), Δz_1 is the height of the gauge above the soil surface (which can also be negative!), Δz_2 is the depth of the tensiometer cup below the soil surface and ρ_l is the density of the liquid in the connecting tube. The pressure head of the liquid in the tensiometer cup is then:

$$h = h_{\text{gauge}} + \Delta z_1 + \Delta z_2 \tag{3.11}$$

where h_{gauge} is the pressure head of the liquid in the gauge.

As in piezometers, the liquid inside the tensiometer is always at static equilibrium. Thus, with $z = 0$ at the soil surface and using Equation 3.11:

$$H_{\text{tens}} = h_{\text{gauge}} + \Delta z_1 = h - \Delta z_2 = \text{uniform} \tag{3.12}$$

where H_{tens} is the hydraulic head of the liquid in the tensiometer.

While there is no resistance to flow in piezometers, so that they are always instantaneously at equilibrium with the soil water at the lower, open end, this is not necessarily true for tensiometers. The porous cup usually presents considerable resistance to flow and the water pressure inside may adjust only slowly to changes in the potential of the soil water at the cup. If the two are at equilibrium with each other, then the hydraulic head of the soil water at the cup is

$$H = h_{\text{gauge}} + \Delta z_1 \tag{3.13}$$

From this it follows immediately that for two tensiometers at equilibrium with the soil water, which have the gauge at the same height above the soil surface (Δz_1), the difference in H of the soil water at the cups is equal to the difference in gauge readings, independent of the depths of the cups.

If the hydraulic potential in the soil is not uniform, the tensiometer can only indicate an average hydraulic potential of the soil water around the cup. If the hydraulic potential varies mostly with height (Answer 3.14), which is usually the case, the vertical dimension of the cup should be kept small and Δz_2 should be related to the average depth of the permeable part of the cup.

Question 3.15 The cups of tensiometers 1 and 2 are at a depth of 0.6 m and 0.8 m, respectively. The gauges are 0.2 m above the soil surface. The gauge of tensiometer 1 indicates $h_{\text{gauge}} = -0.9$ m.

a. Draw the potential diagram, assuming that the water in the soil is at static equilibrium.

b. Calculate the gauge reading of tensiometer 2 and comment on the result.

Question 3.16 At another moment the tensiometers of Question 3.15 indicate $h_{\text{gauge}} = -0.9$ m for tensiometer 1 and $h_{\text{gauge}} = -1.0$ m for tensiometer 2.

a. Draw the potential diagram for this new situation, assuming H is linear with z.

b. What is the heigth of the groundwater table?

c. How can one easily find the difference in hydraulic potential of the soil water at the two cups?

Instead of vacuum gauges, mercury manometers can be used to measure the pressure head of the liquid in the tensiometer (Figure 3.9). The pressure in the cup relative to atmospheric pressure (= tensiometer pressure, p_t) can be calculated by starting at the flat air-mercury interface in the reservoir where the pressure potential is zero, and moving through the manometer to the cup while applying two rules for the pressure in a uniform liquid at rest:

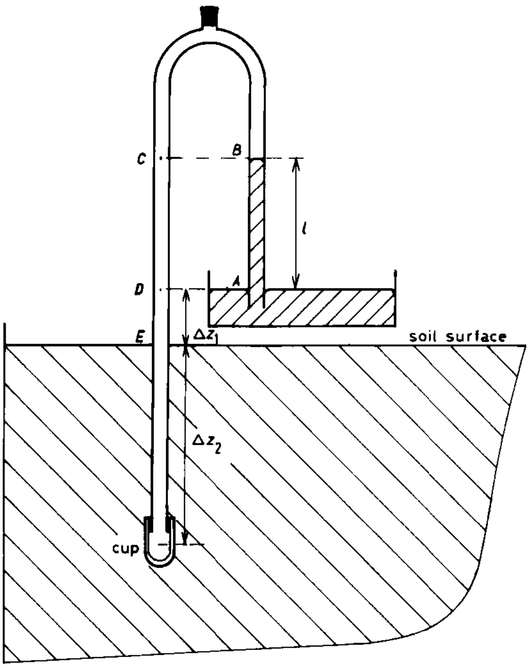


Figure 3.9 Tensiometer with mercury manometer.

a. the pressure in a horizontal plane is uniform

b. the pressure changes with height according to $\Delta p = -\rho_l g \Delta z$.

Thus,

$$p_1 = \underset{A}{0} - \underset{B,C}{\rho_{Hg} g l} + \underset{D}{\rho_l g l} + \underset{E}{\rho_l g \Delta z_1} + \underset{\text{cup}}{\rho_l g \Delta z_2} \tag{3.14}$$

The letters under the terms in Equation 3.14 refer to the positions indicated in Figure 3.9, l is the length of the mercury column above the reservoir, Δz_1 the height of the mercury level in the reservoir above the soil surface and Δz_2 the depth of the tensiometer cup below the soil surface. Dividing Equation 3.14 by $\rho_l g$ gives the pressure head in the cup, and thus of the soil water in contact with the cup:

$$h = -\frac{\rho_{Hg} - \rho_l}{\rho_l} l + \Delta z_1 + \Delta z_2 \tag{3.15}$$

Using $\rho_{Hg} = 13600$ kg m^{-3} and $\rho_l = 1000$ kg m^{-3} for the density of mercury and liquid in the cup, respectively, and assuming that the variation in the mercury level in the reservoir is negligible, Equation 3.15 reduces to

$$h = -12.6 l + \Delta z_1 + \Delta z_2 \tag{3.16}$$

What was said in connection with Equations 3.12 and 3.13 for tensiometers with pressure gauges, is also true for tensiometers with mercury manometers if h_{gauge} is replaced by $-12.6\ l$. Therefore, subject to the same restrictions ($z = 0$ at the soil surface and equilibrium between the water inside and outside the cup),

$$H = -12.6 l + \Delta z_1 \tag{3.17}$$

Thus, for manometers in the same reservoir, the difference in lengths of mercury columns, multiplied by −12.6, is equal to the difference in hydraulic head of the soil water at the cups, again irrespective of the depths of the cups.

Question 3.17 Two tensiometers are installed in a soil profile and connected to the same mercury reservoir. The cups of tensiometers 1 and 2 are at depths 0.4 and 0.8 m, respectively. The mercury level in the reservoir is at 0.1 m above the soil surface. The length of the mercury column in tensiometer 1 is 7.5 cm, that of tensiometer 2 is 9.0 cm.

a. Draw the potential diagram assuming H is linear with z.
b. Calculate the height of the groundwater table.
c. What do you expect about the mercury levels in the manometer tubes when the water in the soil is at static equilibrium?

Nowadays so-called pressure transducers are frequently used instead of pressure gauges or mercury manometers. A pressure transducer is a device that converts pressures to voltages linearly. With these devices tensiometer pressures can be measured very accurately and they can also be registered automatically. Pressure transducers require very small displacements of their sensing element to register changing pressures. This displacement must be effected by minute volumes of liquid flowing into or out of the tensiometer cup. In contrast, mercury manometers need exchange of considerable volumes of water between the soil and the tensiometer. Thus it takes considerable time for a tensiometer with a mercury manometer to equilibrate with the pressure potential of the soil water. Tensiometers with a pressure transducer often equilibrate almost instantaneously. A disadvantage of the use of pressure transducers with tensiometers is that the readings are usually strongly influenced by changing temperatures, mainly due to thermal expansion and contraction of the liquid.

The lowest soil water pressure potential that, theoretically, can be measured with tensiometers is limited by the lowest possible absolute pressure in the instrument, which is the vapour pressure of water (e.g., 2.3 kPa at 20 °C). In practice, however, already at absolute pressures below $\approx$ 10 kPa (which is -90 kPa or -9.0 m relative to an atmospheric pressure of 100 kPa or 10.0 m) problems arise from dissolution of gases from the water.

The measuring range of tensiometers is further limited by the size of the largest pores in the cup. If the pressure in the cup falls below the air-entry value of the largest pores, air will enter the tensiometer and all the water may be absorbed by the soil.

Question 3.18 What is the equivalent diameter of the largest pore in a tensiometer cup that can be used to measure pressure heads of -9.0 m (at complete wetting)?

Since air is compressible and has a large thermal expansion coefficient, isolated air bubbles inside tensiometers make them very sluggish in following changes in soil water potential and make them sensitive to temperature changes. If air is abundant in the system, it can make accurate measurements altogether impossible.

The problems mentioned above can be nearly eliminated by filling tensiometers with deaerated water and selecting ceramic cups with pores smaller than 2 µm. However, air can still diffuse through the walls of the tubing connecting the cup with the pressure measuring device (often nylon tubing) and through the water in the pores of the cup walls. The former can be eliminated by using impermeable tubing, such as copper, but the latter can only be reduced by using thick ceramic walls of low porosity. Therefore, tensiometers must be flushed periodically with deaerated water to drive accumulated air out of the system.

3.1.5 Water retention (pF curve)

In dealing with problems of flow of water in unsaturated soil, there is need for a relationship between the volume fraction of water and an appropriate component of the water potential. Recalling the discussion in Section *3.1.3*, the pressure potential would appear to be suitable for that purpose, but there is one problem associated with it. This problem is illustrated in Figure 3.10. A soil column with a tensiometer is placed in a container, in which the air pressure can be made to differ from atmospheric. The vacuum gauge on the tensiometer is located outside the container, 1.0 m above the groundwater table in the column.

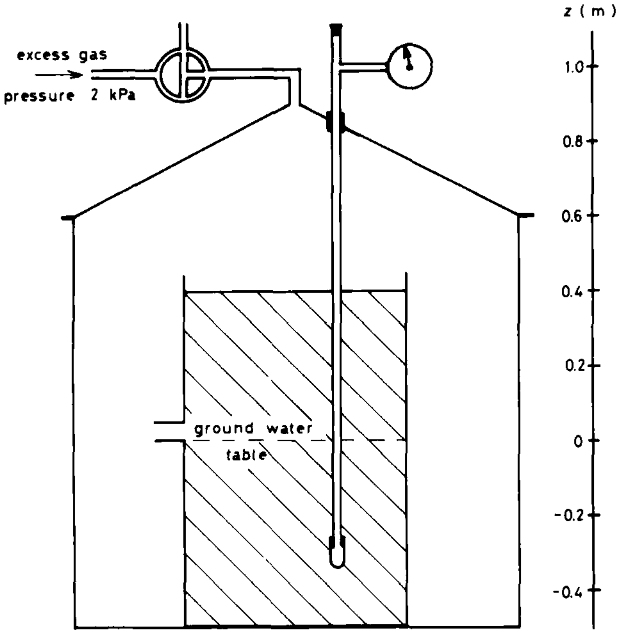


Figure 3.10 Apparatus for illustrating the influence of nonatmospheric pressure of the soil gas phase on tensiometer readings.

Question 3.19 What is the pressure head at the vacuum gauge when the soil column is under atmospheric pressure?

Question 3.20 The gas pressure inside the container is raised to 2 kPa above atmospheric pressure by turning the stopcock.

a. What is the change in the pressure head at the gauge?

b. What do you conclude about the pressure potential of the soil water in the column?

Question 3.21 Has the volume fraction of water changed at any position in the soil profile?

Answers 3.20 and 3.21 illustrate that there is no unique relationship between the volume fraction of water and the tensiometer pressure potential of soil water, if the soil air pressure changes with respect to atmospheric pressure. This occurs occasionally in laboratory and field conditions when the soil air becomes isolated from the atmosphere by a zone with a low permeability for air, such as a water-saturated zone. In such situations, the nonuniqueness of the relationship makes it unsuitable for solving problems of water flow in unsaturated soil. This problem can be circumvented by dividing the tensiometer pressure potential into two components: the matric potential, ψ_m, resulting exclusively from the soil matrix, and the pneumatic potential, ψ_a, due to the soil gas pressure in excess to atmospheric pressure.

Thus $\psi_p = \psi_m + \psi_a$ (3.18)

The pressure equivalents or potentials on volume basis are:

$$p_t = p_m + p_a \tag{3.19}$$

where p_m is the matric pressure and p_a is the excess gas pressure. The head equivalents or potentials on weight basis are:

$$h = \frac{p_m}{\rho_l g} + \frac{p_a}{\rho_l g} = h_m + h_a \tag{3.20}$$

where h_m is the matric head and h_a is the pneumatic head.

These equations show that under normal atmospheric conditions ($p_a = 0$), the matric potential is equal to the tensiometer pressure potential.

The relationship between the volume fraction of water and the matric potential is not dependent on the pressure in the gas phase. It is characteristic for different types of soil and is, therefore, often called soil water characteristic. A more descriptive name for it is soil water retentivity curve or function. Figure 3.11 shows the water retentivity function of a coarse sandy soil with grains of more or less equal size.

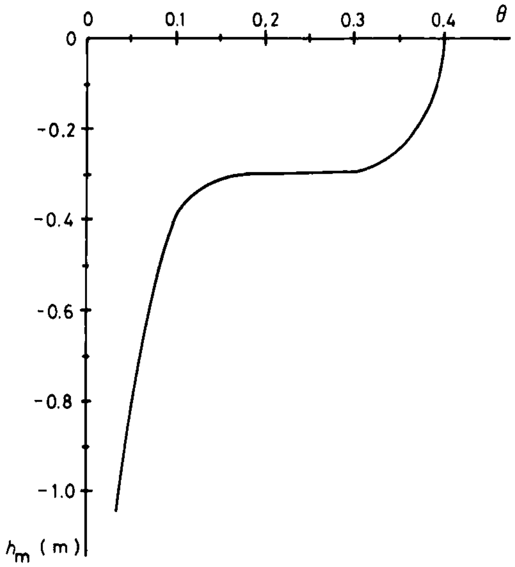


Figure 3.11 Water retentivity curve of a coarse sandy soil.

Question 3.22 What is the porosity of this soil?

If water is extracted from the soil of Figure 3.11, starting from saturation, h_m drops at first quite sharply from zero to about –0.3 m. Thereafter, much water can be extracted without a substantial change of h_m. At these high h_m values the binding of water takes place almost exclusively by capillary action. Therefore, the above soil water characteristic shows that the soil contains a large number of pores with the same equivalent diameter.

Question 3.23 What is this equivalent diameter with complete wetting?

As the water content is decreased further, the capillary binding of the remaining water becomes stronger, because ever smaller capillaries are emptied, leaving only progressively smaller radii of curvature and lower values of the matric head. The last amounts of soil water are bound very strongly. This water occurs only in the smallest parts of the pores around the contact points between grains and as thin films around soil particles bound by adsorptive forces.

In soils that contain a wider range of grain sizes one finds a more gradual release of water when the matric head is decreased from $h_m = 0$ to $h_m \approx -10^4$ m. In such cases it is difficult to plot the whole soil water characteristic in one graph, unless h_m is plotted on a logarithmic scale. The value of ${}^{10}\log(-h_m/\text{cm})$, where $h_m < 0$, is called the *pF* value and the graph showing the relationship between the *pF* value and θ is known as the *pF* curve of a soil.

Question 3.24 a. Give the *pF* values corresponding with the following h_m values: $h_m = -160, -10, -1, -0.1, -0.01$ and 0 m.
b. Do negative *pF* values exist?
c. What is the *pF* value at the phreatic level?
d. What is the *pF* value at 10 cm below the groundwater table (at static equilibrium)?

Question 3.25 Draw the *pF* curve corresponding to Figure 3.11 and compare the two curves.

The shape of *pF* curves is determined by the contributions of different matric forces to the matric potential. Capillary binding of water can take place only at *pF* values of less than about 3 ($h_m > -10$ m). This is due to the fact that negative absolute pressures do not exist.

Question 3.26 Why are negative absolute pressures not possible?

Matric heads smaller than $h_m = -10$ m are caused exclusively by the adhesive binding of water on solid surfaces and by the osmotic binding of water in double layers (Section *3.1.9*). Figure 3.12 presents the *pF* curves of a number of Dutch soils. The curves reflect that sandy soils have mainly capillary binding and that clay soils are

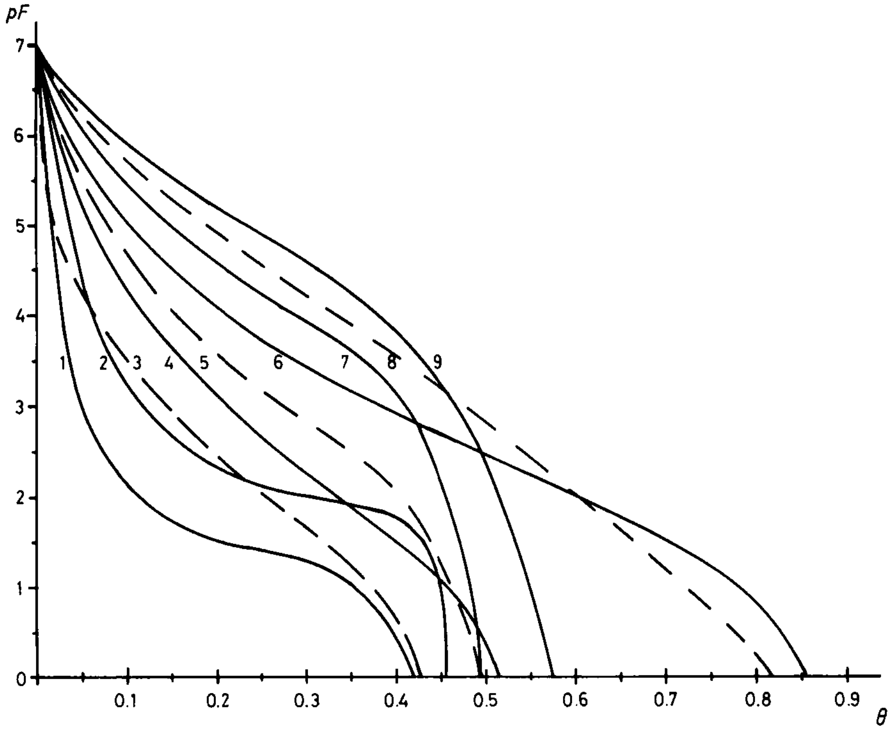


Figure 3.12 *pF* curves of some Dutch soils: 1. Dune sand; 2. loamy sand; 3. calcareous fine sandy loam; 4. calcareous loam; 5. silt loam derived from loess; 6. young oligotrophous peat soil; 7. marine clay; 8. eutrophous peat soil; 9. river basin clay. (Data: ICW, Wageningen.)

dominated by adhesive and osmotic binding.

Question 3.27 How can you see this from the *pF* curves?

Question 3.28 a. What can you say about loamy and peaty soils in this context?

b. What is the most significant difference between these two soil types that can be observed in the *pF* curves?

The soil water characteristic produced by progressive extraction of soil water from saturation (desorption) differs from that obtained by gradual addition of water to air-dry soil (absorption). This phenomenon is called hysteresis. Hysteresis can be caused by a number of factors. It is to a large extent due to the irregular and composite sizes and shapes of soil pores. In addition, the contact angle (Section *3.1.2*) may be lower during wetting than during drying and cause hysteresis. Also during wetting usually some air is entrapped which keeps the water content lower than it could be. Finally, clays can change in water content while remaining saturated. Different arrangements of the clay particles during drying and wetting can cause hysteresis. Question and Answer 3.29 explain the principle of hysteresis as a result of varying pore geometry.

Question 3.29 Figure 3.13a shows a 'pore' that has the shape of a cylindrical bottle with an upper and lower neck. The diameter and height of the open-ended bottle necks are a and $2a$, respectively. The belly has a diameter $2a$ and a height $4a$. The pore is inserted into a cylinder with diameter $2\frac{1}{4}a$ and height $8a$ (Figure 3.13b). The space between the walls of the pore and the cylinder is filled with solid material of zero porosity. This total configuration can be used as a model for an undisturbed soil sample.

a. What is its porosity?

b. The pore is completely filled with water. What is the volume fraction of water of the 'soil sample' at saturation?

c. Water is extracted from the soil sample by applying a gradually decreasing pressure at the lower end of the pore. Draw the water characteristic (desorption curve) of the soil sample if given that $\sigma = 7 \times 10^{-2}\ \mathrm{N\,m^{-1}}$, $a = 14\ \mu\mathrm{m}$ and that there is complete wetting. (The pore is small enough that differences in the gravitational potential can be ignored.)

d. What happens when the pressure is gradually increased again to zero? Draw the absorption curve in the same graph.

e. Is it correct to draw the curves with sharp angles?

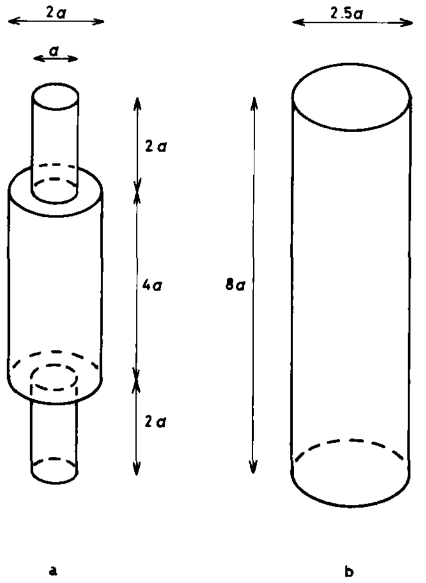


Figure 3.13 Elements of soil pore model used to explain the principle of hysteresis.

Answer 3.29 shows that the water content at a certain matric potential is lower during absorption than during desorption. In general, after any reversal the soil is wetter during drying than during wetting. The same principle applies to real soils, but the curves are smoother, as shown in Figure 3.14.

Question 3.30 Can you explain this?

Figure 3.14 shows so-called primary scanning curves inside the envelope formed by the characteristic, main, or boundary absorption and desorption curves of a soil. The latter are obtained when the soil is dried from saturation to air-dryness, or wetted from air-dryness to saturation, without reversal of the process somewhere in between these two states. Anywhere on a boundary curve one could reverse the process from wetting to drying, or vice versa, and create a primary scanning curve. The relationship between matric potential and volume fraction of water then depends on the value at which the reversal is made. Reversal on a primary scanning curve produces a secondary scanning curve, etc. Thus the recent history of wetting and drying must be known to be able to obtain the water content for a particular matric potential, or vice versa, from a given set of scanning curves of a soil. The *pF* curves in Figure 3.12 are characteristic desorption curves.

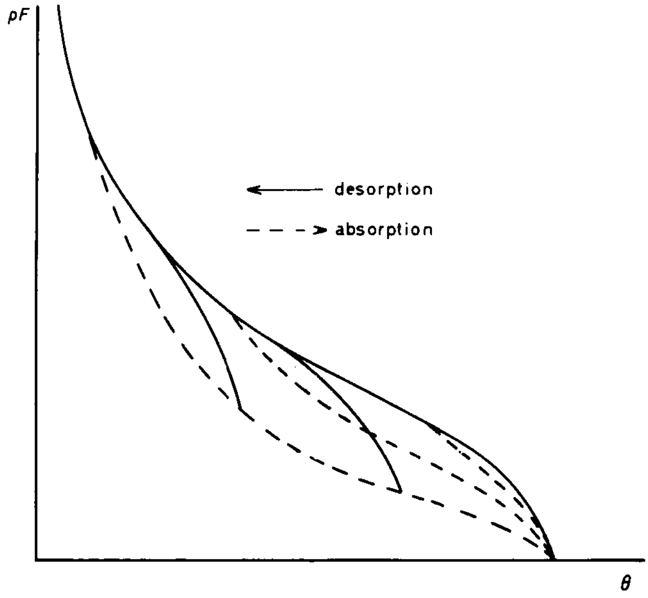


Figure 3.14 Absorption and desorption characteristics with primary scanning curves.

Absorption of water by plant roots becomes more difficult as the matric potential decreases. Yield reductions may result from still relatively high matric heads. This depends much on atmospheric conditions, type of crop, growth stage of crop, etc. However, most plants are unable to take up any more water when the matric head has decreased to $h_m = -160$m or $pF \approx 4.2$. This value is, therefore, called the permanent wilting point of a soil. Water held at very high matric potentials is also not available to plants if it percolates through the root zone to the subsoil under the influence of gravity. The water content in a soil profile when deep percolation has nearly stopped after the soil was thoroughly wetted by heavy rain or irrigation, is called the field capacity. The matric potential at field capacity varies with the circumstances. When the groundwater table is stable at a relatively shallow depth, field capacity corresponds with hydrostatic equilibrium with the groundwater table. Then the water content profile is identical to an inverted soil water retentivity curve, such as in Figure 3.11.

Soils with deep groundwater tables at field capacity generally show matric heads of about -3.0 m. The corresponding pF value is about 2.5.

The difference between the water contents at field capacity and wilting point is called the available soil water for plants. One should use this concept with caution. On the one hand, this amount is often augmented with water supplied by capillary rise from a groundwater table, and precipitation and irrigation water during the growing season. On the other hand, yield reductions result well before soils reach $pF = 4.2$, while plants are seldom able to extract water uniformly throughout the root zone.

Question 3.31 Calculate for soils 1, 3, 6, 7 and 9 in Figure 3.12 the water available for plants between $pF = 2$ and $pF = 4.2$. Give your answer in volume fraction, θ, and in equivalent mm (rainfall) in a root zone of 0.6 m.

Question 3.32 A rain shower of 8 mm falls on soil 4 of Figure 3.12, which is at wilting point.

a. Calculate the penetration depth of the rainwater if the soil is wetted up to field capacity ($pF = 2$).

b. How many cubic metres of water are required per hectare to wet 0.3 m of this soil to field capacity?

Question 3.33 a. Make for soils 1 and 7 in Figure 3.12 a diagram of θ versus depth for static equilibrium with a groundwater table at 1.0 m depth.

b. Do the same for a soil that consists of soil 1 in the upper 0.5 m and soil 7 in the subsoil, with the groundwater table at a depth of 1.0 m.

c. Do the same for a soil that has 0.5 m of soil 7 overlying a subsoil of soil 1, again with the groundwater table at 1.0 m depth.

d. Comment on the results.

3.1.6 Equipment for determining soil water characteristics

Normally, the soil water characteristics determined in the laboratory are main desorption curves. These are obtained by subjecting a wet soil sample to a constant h_m value until hydraulic equilibrium is reached, after which the soil water content is determined.

For relatively high values of h_m a water-saturated soil sample is placed on a fine-porous plate. Thereafter the matric head in the pores of the suction plate is decreased by means of a hanging water column (Figure 3.15). After some time, depending on the head value required and the kind of soil, hydraulic equilibrium is established. Then:

$$H = h + z = \text{uniform} \tag{3.21}$$

At the overflow, $h = 0$ and $H = z_{\text{overflow}}$

$$\text{Thus: } h_{\text{sample}} = H - z_{\text{sample}} = z_{\text{overflow}} - z_{\text{sample}} \tag{3.22}$$

If $z = 0$ is taken at the soil sample as in Figure 3.15, then

$$h_{\text{sample}} = z_{\text{overflow}} \tag{3.23}$$

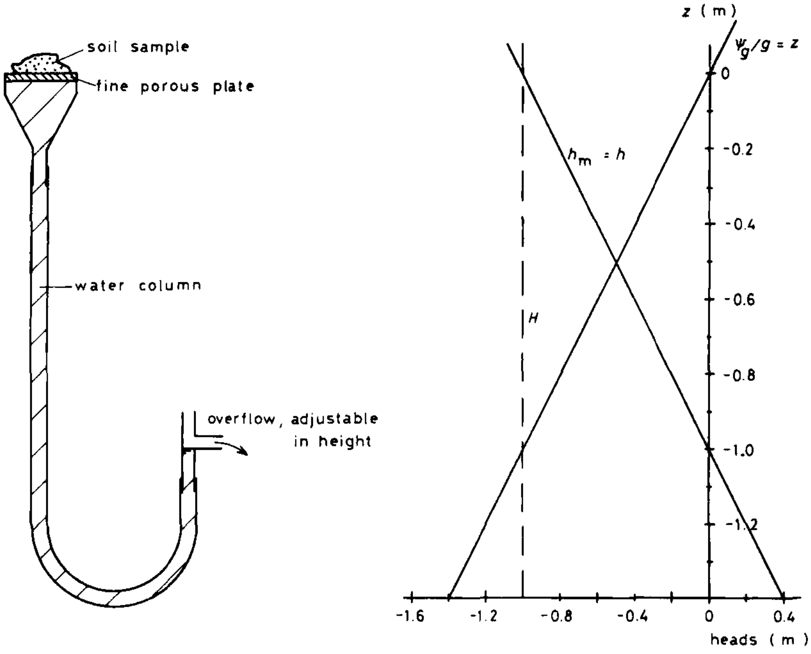


Figure 3.15 Apparatus for determination of soil water characteristics at high matric heads.

Because the system is under atmospheric pressure, $p_a = 0$ and $h_m = h$ (Equations 3.19 and 3.20). Thus the pressure head of the sample is also its matric head.

Question 3.34 a. What is the value of the matric head for the soil sample in Figure 3.15?

b. What must be done to determine the water content at $pF = 1$?

Of course, the pores of the porous plate must be small enough to prevent that the air-entry value is reached. The minimum head for which this apparatus can be used is about -8 m. The lower values between about -5 m and -8 m are usually obtained with a partial vacuum rather than a hanging water column because of problems with dissolution of gas.

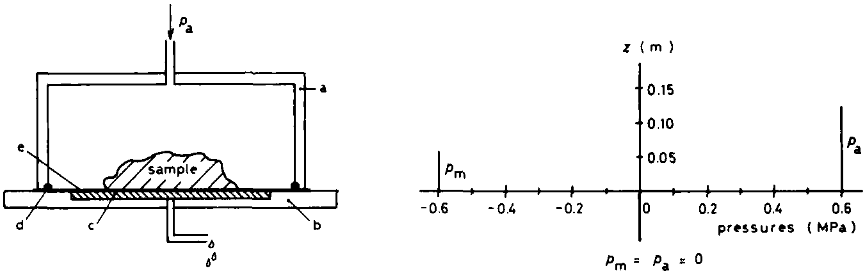


Figure 3.16 Pressure membrane apparatus for measuring desorption characteristic at low matric heads: a. steel cylinder; b. steel base plate; c. porous plate; d. O-ring; e. membrane.

For lower head values so-called pressure plates or pressure membranes are used. In such apparatus moist soil samples are placed on fine-porous plates or membranes and subjected to external gas pressures. Below the plate or membrane the water is discharged at atmospheric pressure (Figure 3.16). Thus, for this 'free' water, $h = 0$ and $H = z_{\text{discharge}}$. At hydraulic equilibrium,

$$H_{\text{sample}} = H_{\text{discharge}} = z_{\text{discharge}} \tag{3.24}$$

Equations 2.44 and 3.20 for the water in the soil sample give:

$$H_{\text{sample}} = h_m + h_a + z_{\text{sample}} \tag{3.25}$$

Disregarding the small difference in height between the soil sample and the discharging water, combination of Equations 3.24 and 3.25 gives immediately:

$$h_m = -h_a \tag{3.26}$$

or when multiplied by $\rho_l g$:

$$p_m = -p_a \tag{3.27}$$

The pressure membrane apparatus is used for head values between $\approx$ -5 and -200 m.

Question 3.35 What is the external gas pressure required for the determination of the water content at the permanent wilting point ($pF = 4.2$)?

3.1.7 Potentials of the pure component water. Vapour equilibrium

In general, dissolved salts can move freely with water through soil. Salts also pass porous materials such as tensiometer cups and porous plates and membranes used for the determination of soil water characteristics. Therefore, osmotic effects did not need to be considered up to this point and only potentials of the soil solution were discussed. There are circumstances, however, under which only the pure component water can move from one place to another. This is the case when there are semi-permeable membranes in the system, such as in plant roots. Another example is a gas barrier, because water can be transmitted via the vapour phase, but solutes remain behind in the liquid phase. Then osmotic effects must be taken into account.

The total potential of the pure component water is not only influenced by gravity, matric forces and excess gas pressure, but also by the presence of dissolved salts. To account for this effect, the so-called osmotic potential, ψ_o, is introduced. Pure water is taken as reference for the osmotic potential, i.e. $\psi_o = 0$ for pure water. The total potential of the pure component water is then:

$$\psi_t = \psi_p + \psi_g + \psi_o \tag{3.28}$$

While the gravitational potential, $\psi_g = gz$, is the same for the pure component water as for the soil solution as a whole, the tensiometer pressure potential of the pure component is slightly different from the tensiometer pressure potential of the soil liquid, $\psi_p = \frac{1}{\rho_l} p_t$. This is due to the small difference between the density of the soil solution, ρ_l, and that of pure water, ρ_w. In practice, however, this difference can be neglected.

The osmotic potential can be illustrated by Figure 3.17. A container, divided in two parts by a semi-permeable membrane, holds a salt solution at one side of the membrane and pure water at the other side. Both sides are at atmospheric pressure. Because the concentration of water molecules is higher in the pure water than in the solution, more water molecules will move in the direction of the solution than in the other direction. As a result, the surface of the soil solution will rise slowly and that of the pure water will drop. One can prevent this flow of water across the membrane and maintain static equilibrium by exerting on the salt solution a gas pressure in excess of atmospheric pressure. At the surface of the solution the total potential of the pure water is then:

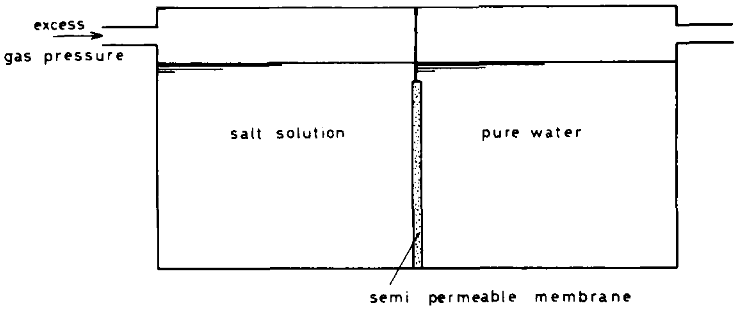


Figure 3.17 Apparatus for illustrating the concept of osmotic pressure.

$$\psi_t = \frac{1}{\rho_w} p_a + \psi_g + \psi_o \tag{3.29}$$

where p_a is the gas pressure in excess of atmospheric pressure. At the surface of the pure water, the total potential is:

$$\psi_t = \psi_p + \psi_g + \psi_o = 0 + \psi_g + 0 = \psi_g \tag{3.30}$$

At static equilibrium, with the liquid surfaces at the same level, ψ_t and ψ_g are equal at both sides. Thus, from Equation 3.29 and 3.30:

$$\frac{1}{\rho_w} p_a + \psi_o = 0 \tag{3.31}$$

The excess gas pressure needed to maintain equilibrium is equal to the so-called osmotic pressure, Π, of the solution. Then the osmotic potential can be written as:

$$\psi_o = -\frac{1}{\rho_w} \Pi \tag{3.32}$$

The total potential of the component water in a soil solution is then, according to Equations 3.28 and 3.32:

$$\psi_t = \frac{1}{\rho_w} p_t + \psi_g - \frac{1}{\rho_w} \Pi = gz + \frac{1}{\rho_w} (p_t - \Pi) \tag{3.33}$$

Water is present in soil not only in the liquid phase, but also in the gas phase, i.e. as water vapour. At static equilibrium the total potentials in both phases are equal. Water vapour consists of pure water, and thus $\psi_o = 0$. Also, it is not influenced by matric forces, and thus $\psi_m = 0$. Therefore, its total potential is determined only by its vapour pressure, p, and by its position in the gravitational field (Section *2.2.5*):

$$\psi_t \text{(vapour)} = \int \frac{1}{\rho_{vap}} \mathrm{d}p + gz \tag{3.34}$$

where ρ_{vap} is the density of the water vapour. Substitution of ρ_{vap} with an equation similar to Equation 2.47 yields for the total potential of the water vapour:

$$\psi_t\,(\text{vapour}) = \int \frac{RT}{pM}\,\mathrm{d}p + gz = \frac{RT}{M}\ln p + gz + C \tag{3.35}$$

where M is the molar mass of water and C is an integration constant.

Per definition, the saturated vapour pressure, p_0, is the vapour pressure at a free surface of pure water under atmospheric pressure. The total potential of the liquid water at this water surface, according to Equation 3.33, is:

$$\psi_t\,(\text{liquid}) = \psi_g = gz \tag{3.36}$$

Question 3.36 Why is this so?

At the gas-liquid interface, gz is the same for the liquid water and the water vapour. Thus:

$$\psi_t\,(\text{liquid}) = gz = \psi_t\,(\text{vapour}) = \frac{RT}{M}\ln p_0 + C + gz \tag{3.37}$$

It follows:

$$C = -\frac{RT}{M}\ln p_0 \tag{3.38}$$

Substitution of Equation 3.38 into Equation 3.35 yields the total potential of water vapour with a vapour pressure p:

$$\psi_t\,(\text{vapour}) = \frac{RT}{M}\ln\frac{p}{p_0} + gz \tag{3.39}$$

At static equilibrium, the total potential of the component water in the soil solution is equal to the total potential of the water vapour in the soil air. Thus, at the gas-liquid interface of the soil water is, according to Equations 3.33 and 3.39:

$$\frac{1}{\rho_w}(p_t - \Pi) = \frac{RT}{M}\ln\frac{p}{p_0} \tag{3.40}$$

With $\rho_w \approx 1000$ kg m^{-3}, $RT \approx 2.4 \times 10^3$ J mol^{-1} at 15°C and $M \approx 18 \times 10^{-3}$ kg mol^{-1}, this reduces to:

$$\ln\frac{p}{p_0} = 7.5 \times 10^{-9}\,\text{Pa}^{-1}(p_t - \Pi) \tag{3.41}$$

This equation indicates that the relative vapour pressure p/p_0 (also called relative humidity) at the gas-liquid interface is determined by the tensiometer pressure and the osmotic pressure of the soil solution. If the soil air is at atmospheric pressure, Equation

3.41 can be written as:

$$\ln \frac{p}{p_0} = 7.5 \times 10^{-9}\,\mathrm{Pa}^{-1}\,(p_m - \Pi) \tag{3.42}$$

or in the head equivalent form, using $\rho_l g \approx 10^4\ \mathrm{N\,m^{-3}}$,

$$\ln \frac{p}{p_0} = 7.5 \times 10^{-5}\,\mathrm{m}^{-1}\,(h_m + h_o) \tag{3.43}$$

in which $h_o = -\dfrac{\Pi}{\rho_l g}$ is the osmotic pressure head of the soil solution.

Question 3.37 A small soil sample and a cup containing a saturated NaCl-solution are placed side by side in a closed, air-tight jar. The osmotic pressure of the NaCl-solution is 300 bar.

a. What is the relative vapour pressure in the jar?

b. What is the *pF* value of the soil sample if the salt concentration of the soil water is negligible?

Question 3.38 The relative humidity in a laboratory is maintained at 50 %.

a. What is the *pF* value of 'air-dry' soil samples for which the osmotic potential can be neglected?

b. Derive a general relationship between *pF* and relative humidity, under the same restriction.

c. What is the head value registered by a tensiometer kept in this laboratory?

Question 3.39 Make a graph showing the relative vapour pressure as a function of *pF*. Plot the *pF* values in the range between 0 and 7 on the horizontal scale of the graph.

Question 3.40 A colomn of fine-sandy soil, with an air-entry value of −1.2 m, has a groundwater table at 1.0 m below the surface. The water in the soil column has a salt concentration of 50 mol NaCl per m^3. The osmotic pressure of the water can be calculated by $\Pi = RTc$, where c is the ionic concentration.

An air-tight container is placed over the soil column. The air is under atmospheric pressure and contains water vapour in equilibrium with the soil-water system.

a. A soil sample A (Figure 3.18) is placed in direct contact with the surface of the soil column. Calculate h_m of the water in sample A at equilibrium.

b. A second sample B is placed on the soil column in a small beaker preventing direct contact. What is the *pF* value in sample B, if the salt concentration in sample B is negligible?

c. Calculate the relative vapour pressure above the soil column.

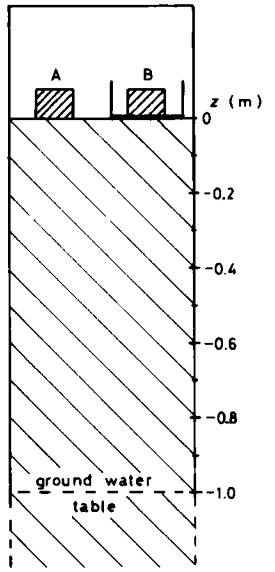


Figure 3.18 Soil column with groundwater table under air-tight container. Soil sample A is in direct contact with soil column, soil sample B is not.

3.1.8 Criteria for equilibrium of the component water

So far, two different cases of equilibrium of soil water have been discussed. These can be described as follows:

- If dissolved salts or solutes can move freely through the soil, which is the case in sandy soils, there will be equilibrium of the soil solution if the hydraulic potential $\psi_h = \psi_p + \psi_g$ is uniform, i.e. invariant with position. If the concentration of solutes varies with position, they will diffuse in the direction of decreasing concentration until the concentration is everywhere the same. Any increase in volume occupied by the solutes associated with this diffusion will be compensated by the movement of an equal volume of the component water in the opposite direction. Thus there will be total equilibrium only if ψ_h is uniform and ψ_o is uniform.
- If the system contains semi-permeable barriers that allow movement of the component water but not of solutes, there is equilibrium only if the total potential $\psi_t = \psi_p + \psi_g + \psi_o$ is uniform. This situation is found where the transport has to take place through the gas phase (Question 3.37), e.g. in dry soils where the water is stored in isolated small pores and/or in double layers.

A third situation can be distinguished in soils with barriers that obstruct the transport of solutes without completely preventing it. This can be illustrated by Figure

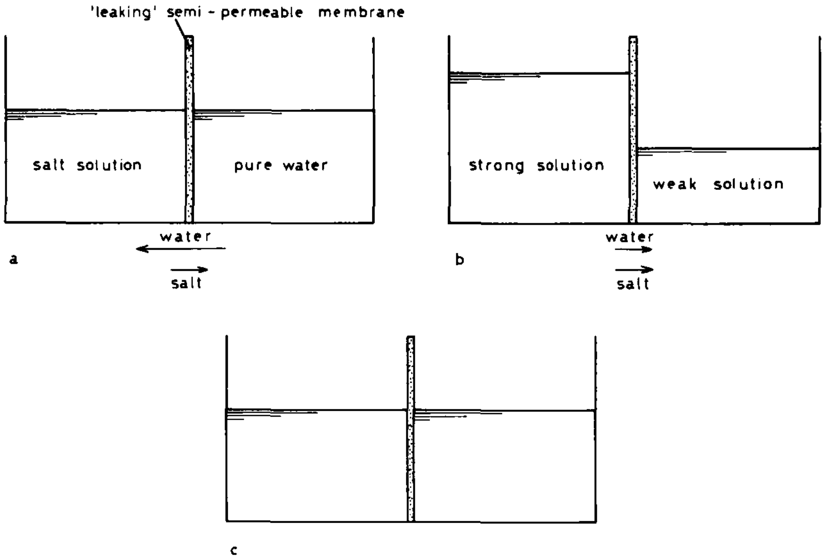


Figure 3.19 Salt solution and pure water separated by a 'leaking' semi-permeable membrane.
a. Shortly after installation.
b. Some time after direction of flow of water reverses.
c. Equilibrium with equal concentrations at both sides.

3.19, where a container is divided in two parts by an imperfectly semi-permeable membrane. Initially, there is a salt solution at one side of the membrane and pure water at the other side, with the liquid surfaces at the same height. This will result in net diffusion of water in the direction of the solution and leakage of very small quantities of solutes through the membrane in the direction of the pure water. The liquid level of the salt solution will rise at first, but after some time a situation will be reached where the total potential of the component water will be equal at both sides. Then ψ_t = uniform, while the difference in hydraulic potential at both sides balances the difference in osmotic potential. But, because the solute concentration is still different at both sides, the leakage of solutes through the membrane will continue. Final equilibrium will be reached when the solute concentration and the liquid levels have both become equal at both sides of the membrane. Thus, total equilibrium will exist only if ψ_h = uniform and ψ_o = uniform.

This situation is found in moist clay soils, where the soil water is present in thin films on the clay particles. Salt transport is then difficult because the anions are repelled from the double layer by the negative electrostatic charge of the clay particles.

Summarizing these three situations one can state that generally there is hydraulic equilibrium (no bulk transport) if ψ_h is uniform. Exceptions to this are found in all nearly dry soils and in moist clay soils. In these situations there will be bulk transport if ψ_o is not uniform, even if ψ_h is uniform.

3.1.9 Osmotic binding of water in the double layer

Water molecules in the double layer are subject to several kinds of forces. Close to the surface of a clay particle water molecules are attracted by London-van der Waals forces. These short range forces decrease very rapidly with distance from the clay surface and are not dependent on the composition and extent of the double layer.

Due to their dipolar nature, water molecules orient themselves in the electrostatic field resulting from the negative charge of clay particles. Because the field strength decreases with distance from the clay particle, the water molecules experience a net static electrical force in the direction of the clay surface. This force decreases gradually with distance from the surface and is zero at the end of the double layer (either truncated or fully extended).

Question 3.41 Why is the electrostatic field strength, resulting from the charge of a clay particle, zero at the end of the double layer?

The effect of the two kinds of forces mentioned above can be represented by an 'adsorption' potential, ψ_α. The reference $\psi_\alpha = 0$ is chosen at a point where these forces are zero, i.e. outside the double layer. Within the double layer, ψ_α decreases towards the surface of the clay particle to a very low (negative) value (Figure 3.20).

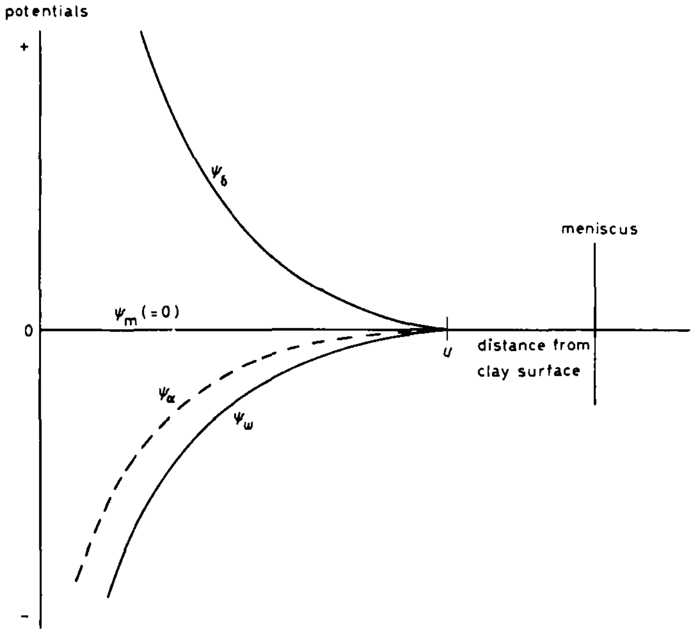


Figure 3.20 Water potentials in a fully extended double layer near a flat gas-liquid interface.

A third kind of force acting on water molecules in the double layer results from the increasing ion concentration towards the solid surface. This results in an osmotic potential in the double layer, $\psi_{o,dl}$, which decreases from the outside soil solution towards the clay particle. Similar to the situation of water in the gravitational field, where the decreasing gravitational potential is compensated by an increasing pressure potential, the decrease of ψ_α and $\psi_{o,dl}$ is compensated by an increase of a pressure potential, ψ_δ. This potential will be taken relative to the pressure of the soil air. The total potential of the component water in the double layer can then be written as:

$$\psi_t = \psi_\alpha + \psi_{o,dl} + \psi_\delta + \psi_a + \psi_g \qquad (3.44)$$

The total potential for water outside the double layer is given by Equation 3.28:

$$\psi_t = \psi_p + \psi_g + \psi_o \qquad (3.28)$$

Combination of Equations 3.18, 3.28 and 3.44 and neglecting very small differences of ψ_g yields:

$$\psi_m + \psi_o = \psi_\alpha + \psi_{o,dl} + \psi_\delta \qquad (3.45)$$

Defining $\psi_\omega = \psi_{o,dl} - \psi_o$ as the osmotic potential due to the ions in the double layer in excess to those in the soil solution, Equation 3.45 can be written as:

$$\psi_m = \psi_\alpha + \psi_\omega + \psi_\delta \qquad (3.46)$$

Figure 3.20 shows the various potentials in a fully extended double layer adjacent to the soil solution with which it is in equilibrium. If the soil solution has a flat meniscus with the gas phase (at the same height of the double layer), then the matric potential of the soil solution is zero. (A tensiometer in situ would read $\psi_m = 0$.) Both ψ_α and ψ_ω are also zero in the solution outside the double layer and thus ψ_δ there is also zero. In fact, ψ_δ is the partial potential of the water associated with the actual pressure in the water relative to the pressure of the gas phase. Near the clay surface ψ_δ, and thus the actual pressure in the water, increases to very high values (probably 10 à 100 bar). If the meniscus is curved, ψ_δ at the meniscus may have either a negative or positive value depending on whether the meniscus is concave or convex. In both cases is at the gas-water interface

$$\psi_m = \psi_\delta \qquad (3.47)$$

Figure 3.21 shows the various potentials when the double layer is truncated (see Figure 1.7). The value of ψ_δ at the gas-liquid interface again depends on the radius of curvature of that interface. Figure 3.21 is drawn for a situation where ψ_δ has a negative value at the interface. ψ_ω now has a finite negative value at the interface, because the ionic concentration is larger than it would be if the double layer were fully extended, in equilibrium with the soil solution. As stated earlier, at the interface ψ_α is zero. Thus, at the gas-water interface of a truncated double layer:

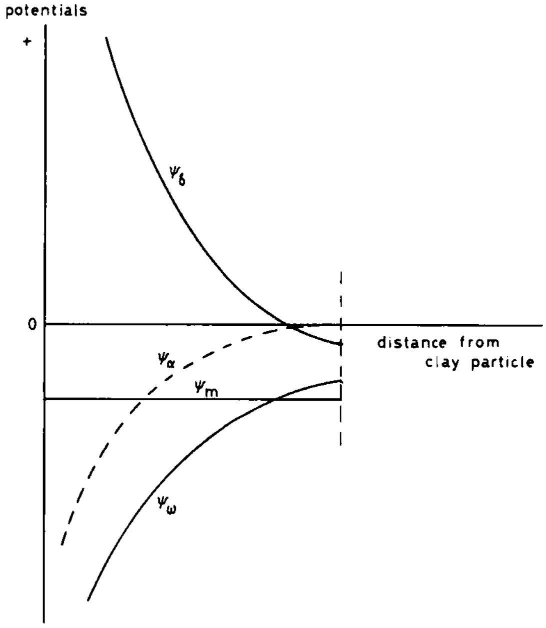


Figure 3.21 Water potentials in a truncated double layer.

$$\psi_m = \psi_\delta + \psi_\omega \tag{3.48}$$

Equations 3.47 and 3.48 describe the behaviour of double layer systems with respect to their environment. They show that the matric potential is equal to the potential due to the actual liquid pressure at the gas-liquid interface relative to the pressure of the gas phase, plus the osmotic potential due to the excess ions at the same interface, if the double layer is truncated. It follows that for soils with truncated double layers – that is, for almost every soil except pure sand – Equation 2.41 must be rewritten as:

$$p_h = p_t + \rho_l g z \tag{3.49}$$

or, if the soil gas phase has atmospheric pressure:

$$p_h = p_m + \rho_l g z \tag{3.50}$$

Equation 3.48 explains why matric pressures of less than –1 bar (matric heads less than –10 m) can occur. The matric pressure is the pressure *equivalent* of the matric potential and is not the actual pressure (relative to the pressure in the gas phase) in the liquid. The actual absolute pressure will always be larger than zero (Question 3.26), or the pressure relative to atmospheric pressure always higher than ≈ –1 bar. Matric pressures less than −1 bar are caused mainly by the osmotic potential, ψ_{ω}, and in very dry soils also by a negative value of ψ_{α}.

3.2 STATIC EQUILIBRIUM OF THE SOIL SYSTEM

3.2.1 Soil pressure

The condition of static equilibrium in the gravitational force field was so far applied to the soil water only. In a similar way, this condition can be applied to the soil system as a whole, if the various (partial) potentials of the soil system are defined properly. By 'soil system' we mean the soil matrix and all substances (mass) in the pore space associated with the soil matrix.

At static equilibrium the total potential of the soil system, ψ_t, has to be uniform, which means that the gradient of the gravitational potential, ψ_g, has to be compensated by an equal but opposite gradient of the pressure potential, ψ_p. Thus, similar to Equation 2.38, but now for the soil system:

$$\psi_t = \psi_g + \psi_p = \text{uniform} \tag{3.51}$$

If the water in the cylinder of Figure 2.7 is replaced by soil, the potential diagram for the soil system is the same as it was for water, if the same reference points are chosen for ψ_g and ψ_p. This is so, because these potentials are defined on mass basis. If, however, potentials on volume basis (pressure equivalents) are used, the diagram for the soil system will be different from that for water. This is due to the difference between the density of water, ρ_l, and the bulk density of the soil, ${}^{b}\rho$. Then, static equilibrium of the soil system is described by:

$${}^{b}\rho\psi_t = {}^{b}\rho\psi_g + {}^{b}\rho\psi_p = \text{uniform} \tag{3.52}$$

In soil mechanics, the pressure potential on volume basis is called soil pressure and is indicated with the symbol σ_s:

$$\sigma_s = {}^{b}\rho\psi_p \tag{3.53}$$

Since soil is usually rigid, σ_s is hardly a real pressure. It is better to define σ_s as the 'normal' component (i.e. the component perpendicular to a certain plane) of the material stress at a certain location. The term soil pressure will here be used for the vertical force exerted by the weight of the overlying material, divided by surface area of a horizontal plane.

Question 3.42 What is the soil pressure at static equilibrium at a depth of 1 m in a dry sandy soil with a dry bulk density of 1500 kg m^{-3}? Draw a diagram of the soil potentials in pressure units.

3.2.2 Intergranular pressure

The intergranular pressure, σ_g, is defined as the total contact force between the grains divided by area. In a dry soil, the vertical intergranular pressure equals the soil pressure, because at any depth the weight of the overlying soil material is borne solely by the contact forces between the grains at that depth. Hence:

$$\sigma_s = \sigma_g \tag{3.54}$$

The intergranular pressure causes friction between the grains, which in turn determines the resistance against motion of grains relative to each other. For instance, the friction determines the resistance of the soil against ploughing, its 'bearing capacity' for buildings and its resistance against compaction by heavy farm machinery.

Question 3.43 What is σ_g at 1 m depth for the soil of Question 3.42?

In a water-saturated soil, the weight of the overlying soil is borne not only by the contact force between the grains, but also by the contact force in the water:

$$\sigma_s = \sigma_g + \sigma_w \tag{3.55}$$

Similar to σ_s, σ_w is called water pressure, and it is defined as $\sigma_w = \rho_l \psi_p$. Then, according to Equation 2.37,

$$\sigma_w = p' \tag{3.56}$$

Thus σ_w is the actual pressure of the water relative to atmospheric pressure.

Question 3.44 a. Make a diagram of σ_s and σ_w versus depth for the soil of Question 3.42, if the groundwater table is at the soil surface and $\phi = 0.40$.
b. What is now the value of σ_g at a depth of 1 m?
c. Is the bearing capacity of the wet soil higher or lower than the bearing capacity of the dry soil?

Question 3.45 Calculate σ_g at a depth of 1 m when the groundwater table is at 0.10 m above the surface of the sandy soil (submerged condition). What is now σ_g at the soil surface?

In a rigid soil not saturated with water, σ_w is negative. This negative pressure is supported by the soil particles, which are pressed together with a force in excess of the weight of the overlying soil. The water pressure is active only in the water-filled pore space, while σ_w, σ_s and σ_g are defined as if they act over the total cross-sectional area. For an unsaturated, rigid soil Equation 3.55 must be changed to:

$$\sigma_s = \sigma_g + \frac{\theta}{\phi}\sigma_w \tag{3.57}$$

The coefficient θ/ϕ in front of σ_w varies from zero at air-dryness to one at saturation. This is as it should be, according to Equations 3.54 and 3.55. In nonrigid or swelling soils the situation is more complex and beyond the scope of this text.

Question 3.46 What is σ_g at the surface of the sandy soil if the groundwater table is at a depth of 0.20 m and the air-entry value of the soil equals −0.30 m?

Answer 3.46 shows that if the groundwater table is lowered from the soil surface, the intergranular pressure at the surface will increase. This continues until the air-entry value is reached. After that σ_g will decrease again, due to the decrease of θ/ϕ, according to:

$$\sigma_{g(\text{surface})} = -\frac{\theta}{\phi}\sigma_w \tag{3.58}$$

This general rule can be confirmed at the beach. Beyond the waterline it is difficult to use a bicycle, because $\sigma_g = 0$ at the surface. The same applies to the dry sand higher up. In between those areas is a zone where the sand is saturated with water at a negative σ_w value. There the intergranular pressure at the surface is positive and cycling or walking is most easy.

3.3 SUGGESTED LITERATURE

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Answer 3.1 The contact angle of water on clean glass is zero, and thus cos $\varphi = 1$. The radius of the capillary tube is 35×10^{-6} m. Using Equation 3.7:

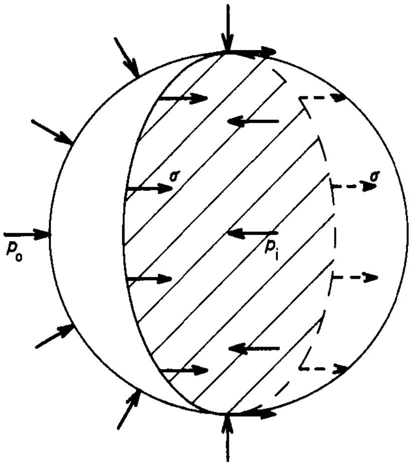
$$z_c = \frac{2 \times 0.07\,\mathrm{N\,m^{-1}} \times 1}{1000\,\mathrm{kg\,m^{-3}} \times 10\,\mathrm{N\,kg^{-1}} \times 35 \times 10^{-6}\,\mathrm{m}} = 0.40\,\mathrm{m}$$

Answer 3.2 The pressure potential of the water at the meniscus can be read from the diagram. It is $-4\,\mathrm{J\,kg^{-1}}$, -4000 Pa or -0.4 m.

Answer 3.3 If the reference point for z is taken at the flat water surface, then everywhere $\psi_h = 0$ and $\psi_p = -\psi_g$. So, the pressure potential on volume basis at the meniscus is:

$$\rho_l \psi_p = -\rho_l \psi_g = -\rho_l g z_c = -\frac{2\sigma \cos \varphi}{r}$$

Answer 3.4 Consider the forces acting on the left-hand hemisphere of the spherical droplet in the figure below. If the pressure inside thesphereis p_i, then the total horizontal force on the vertical cross-section, directed towards the left, is $F_i = \pi r^2 p_i$.



Similarly, the net horizontal force on the hemispherical outside surface as a result of the outside pressure p_o, directed towards the right, is $F_o = \pi r^2 p_o$.

The vertical components of the outside forces cancel each other. The surface tension in the spherical surface acts over the circumference of the cross-sectional area in a horizontal direction. This total force is $F_\sigma = 2\pi r \sigma$, exerted by the right-hand hemisphere on the left-hand one and, therefore, directed towards the right. At static equilibrium, $\Sigma \boldsymbol{F} = 0$. Thus

$$\pi r^2 p_o + 2\pi r \sigma = \pi r^2 p_i$$

From this follows immediately: $p_i - p_o = \dfrac{2\sigma}{r}$

Thus the inside pressure is higher than the outside. By using the sign convention for the radius of concave (positive) and convex (negative) surfaces, the proper sign of the pressure difference is automatically obtained.

Answer 3.5 At complete wetting, the height of capillary rise for a capillary of 20 μm diameter is:

$$z_c = \frac{2\sigma\cos\varphi}{\rho_l g r} = \frac{2 \times 0.07 \times 1}{1000 \times 10 \times 10^{-5}}\ \text{m} = 1.4\ \text{m}$$

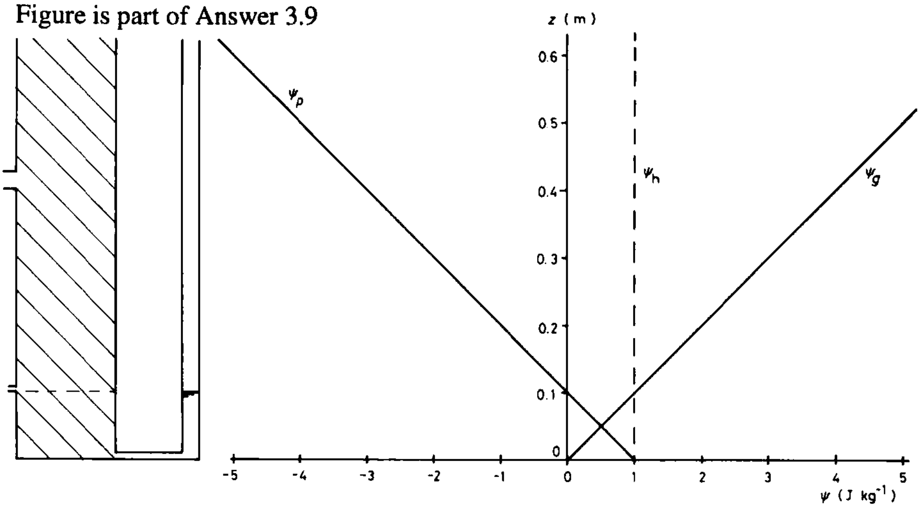
Thus the air-entry value is $-z_c = -1.4$ m or $-\rho_l g z_c = -14$ kPa.

Answer 3.6 The air-entry value of the noncylindrical capillary is the same as that of the cylindrical capillary of Question 3.5. Therefore, its height of capillary rise is 1.4 m and its equivalent diameter is 20 μm.

Answer 3.7 At static equilibrium the gradient of the gravitational potential must be compensated by an equal but opposite gradient of the pressure potential. The reference level of the pressure potential is again at the overflow, since there the water surface is flat and in contact with atmospheric pressure. So, the pressure potential of the water below $z = 0.4$ m remains unchanged.

Answer 3.8 At static equilibrium the hydraulic potential is everywhere the same. Thus the hydraulic potential above the overflow is equal to that below the overflow.

Figure is part of Answer 3.9

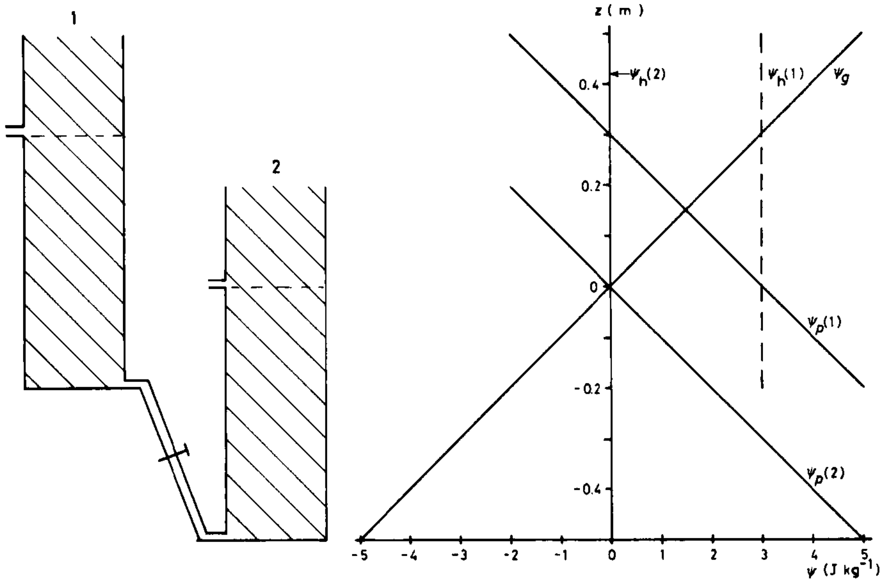


Answer 3.9 a. After the hole is made, the pressure there drops to atmospheric. Thus the phreatic level drops 0.30 m to the level of the hole.

b. The drop in phreatic level causes a decrease of ψ_p, and hence a decrease of ψ_h, of 3 J kg^{-1}. Thus, the new hydraulic potential of the water is everywhere 1 J kg^{-1}.

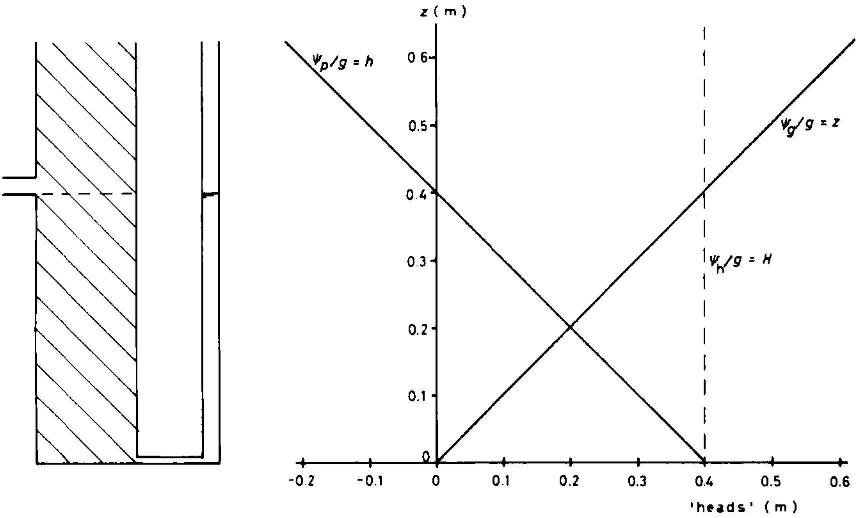
Answer 3.10 a. With the stopcock closed the water in the two cylinders has different hydraulic potentials. The hydraulic potential in cylinder 2, $\psi_h(2)$, is zero, because the gravitational and pressure potential have the same reference level, namely the phreatic level. In cylinder 1, $\psi_h(1) = 3$ J kg^{-1}, because its phreatic level is located at $z = 0.3$ m.

b. When the stopcock is opened water will flow from cylinder 1 to cylinder 2, because the hydraulic potential in the former is higher than in the latter. This water will be discharged through the overflow of cylinder 2. When the phreatic level in cylinder 1 has dropped to the phreatic level in cylinder 2, a new static equilibrium will be attained. In this equilibrium situation the hydraulic potential in both cylinders is the same, and equal to the hydraulic potential in cylinder 2 before the stopcock was opened.



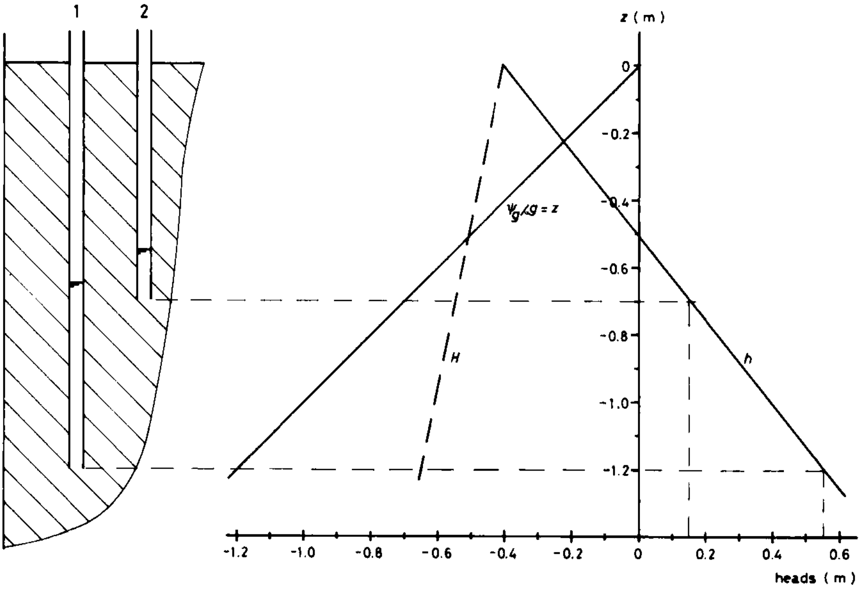
Answer 3.11 $0 \leqslant \theta \leqslant \phi$, which is the range from completely dry to saturation.

Answer 3.12 Conversion of potentials on mass basis to potentials on weight basis requires division by g (Section *2.2.4*). The result is given in the diagram below.



Answer 3.13 The diagram shows that the pressure head is zero at $z = -0.6$ m. So, the groundwater table is at $z = -0.6$ m.

Answer 3.14 a. The hydraulic head of the water in piezometer 1 is constant and equal to that at the air-water interface; thus $H = h + z = 0 - 0.65 \text{ m} = -0.65$ m. This is also the value of H in the soil at the depth of the lower end, thus $H = -0.65$ m at $z = -1.20$ m. Similarly, from piezometer 2 it follows that $H = -0.55$ m at $z = -0.70$ m. Since it is given that H is a linear function of z, H is determined, as indicated in the potential diagram. The h line can be obtained from $h = H - z$.

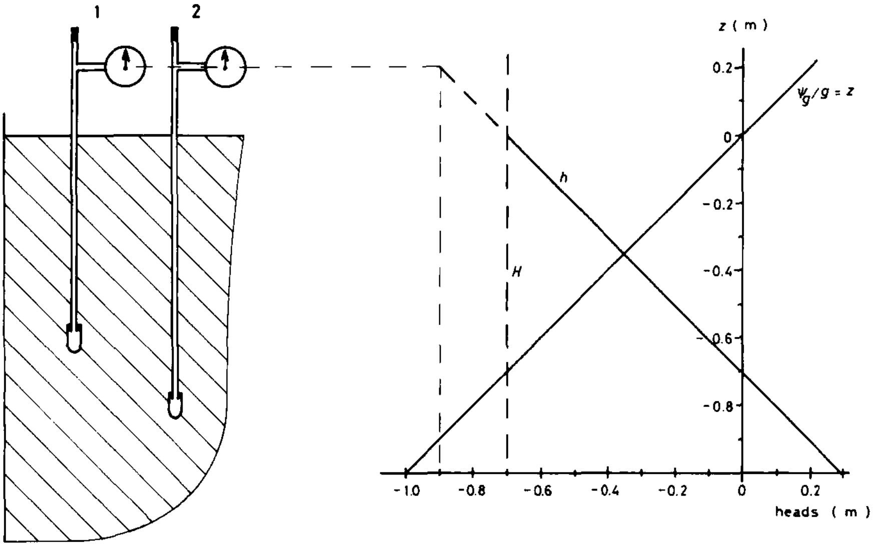


b. The groundwater table is at the depth where $h = 0$. The diagram shows this at $z = -0.51$ m.

c. This is not an equilibrium situation since H is not constant with height.

d. Water flows in the direction of decreasing hydraulic potential. Thus, in this case, the water flows downwards.

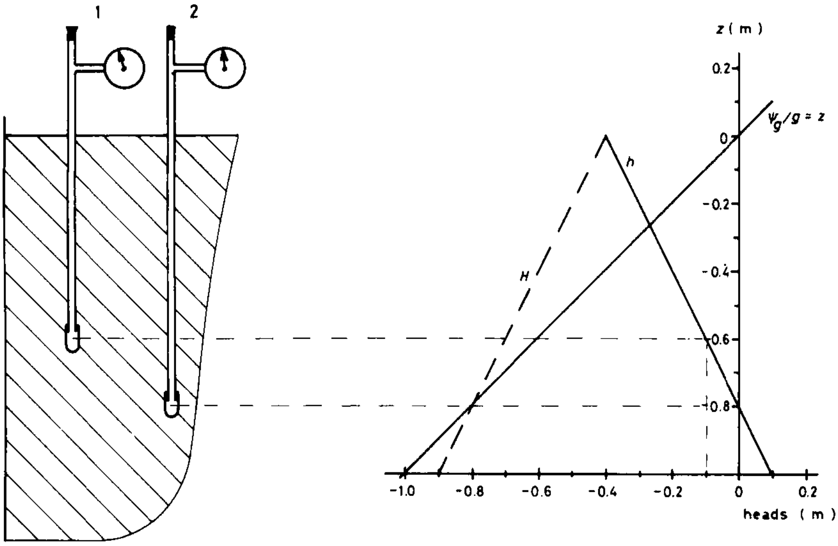
Answer 3.15 a. For the soil water at tensiometer cup 1 is, according to Equation 3.11:
$h = h_{\text{gauge}} + \Delta z_1 + \Delta z_2 = (-0.9 + 0.2 + 0.6)$ m $= -0.1$ m and $H = h + z = (-0.1 - 0.6)$ m $= -0.7$ m. The potential diagram at static equilibrium can now be drawn.



b. For tensiometer cup 2: $h = H - z = -0.7$ m $- (-0.8$ m$) = 0.1$ m. Thus the cup is below the groundwater table. While negative pressure heads cannot be measured with piezometers, it is of course possible to measure positive pressure heads with tensiometers.

For tensiometer 2, $h_{\text{gauge}} = h - \Delta z_1 - \Delta z_2 = (0.1 - 0.8 - 0.2)$ m $= -0.9$ m. This is the same value as for tensiometer 1. Thus, like the piezometers in Figure 3.7, at static equilibrium the tensiometer gauges give the same readings if they are at the same height, irrespective of the depths of the cups. Note that at static equilibrium the pressure heads in the soil and in both tensiometer tubes have the same value at the same height. The hydraulic head, of course, has everywhere the same value.

Answer 3.16 a. The gauge reading of tensiometer 1 is the same as in Answer 3.15 and thus $H = -0.7$ m and $h = -0.1$ m at $z = -0.6$ m. For tensiometer cup 2 at $z = -0.8$ m, $h = (-1.0 + 1.0)$ m $= 0$ and $H = 0 - 0.8$ m $= -0.8$ m. Assuming H is linear with z, the diagram can now be drawn.



b. The groundwater table is at the cup of tensiometer 2, where $h = 0$.

c. If the water in the tensiometers is at static equilibrium with the soil water in contact with the cups, then, according to Equation 3.13, the difference in hydraulic head of the soil water at the cups is equal to the difference in the readings of the gauges (0.1 m), because the gauges are located at the same height above the soil surface.

Answer 3.17 a. Applying Equation 3.16 to tensiometer 1 gives:

$h = (-12.6 \times 0.075 + 0.1 + 0.4)\,\mathrm{m} = -0.445\,\mathrm{m}$.

Similarly for tensiometer 2:

$h = (-12.6 \times 0.09 + 0.1 + 0.8)\,\mathrm{m} = -0.234\,\mathrm{m}$.

Using $H = h + z$, and assuming H is linear with z, the diagram can be drawn.

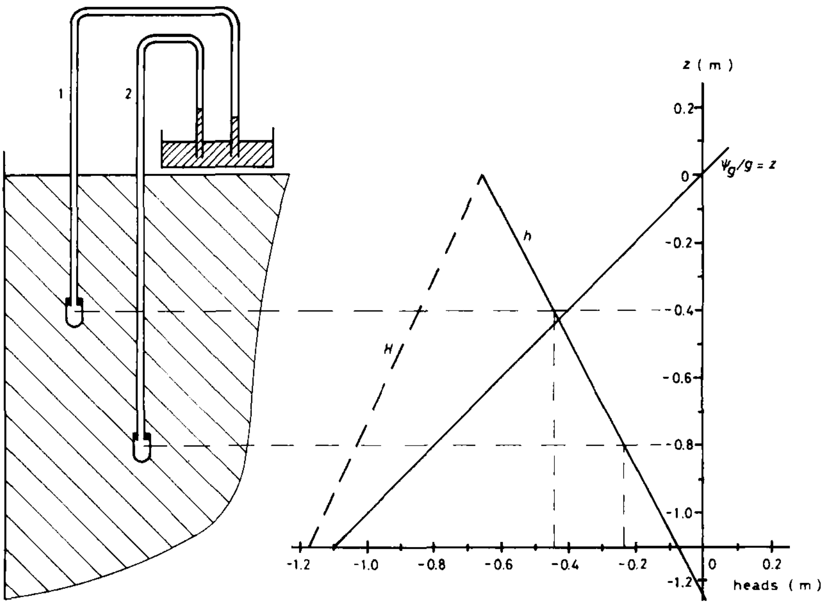
b. The h line can be represented by $h = az + b$

at cup 1: $-0.445 = -0.4a + b$

at cup 2: $-0.234 = -0.8a + b$.

Subtraction of these two equations gives: $-0.211 = 0.4a$ or $a = -0.5275$.

From the first equation: $b = -0.445 - 0.4 \times 0.5275 = -0.656$.



At the groundwater table $h = 0$, and thus $z = -b/a$. Thus the groundwater table is at $z = -1.244$ m.

c. When the water in the soil is at static equilibrium, H is uniform with position. Then, according to Equation 3.17, the mercury levels in the manometers will be at the same height.

Answer 3.18 According to Equations 3.8 and 3.9, the maximum pressure difference that can be maintained across a curved air-water interface in a capillary with radius r is:

$$\Delta p_{\max} = \frac{2\sigma}{r}$$

Thus the maximum equivalent pore diameter that can occur in a tensiometer cup for measuring pressure heads of -9.0 m is:

$$d_{\max} = 2r_{\max} = \frac{4\sigma}{\Delta p_{\max}} = \frac{-4\sigma}{\rho_1 g h_{\min}} = \frac{4 \times 0.07\ \mathrm{N\,m^{-1}}}{1000\ \mathrm{kg\,m^{-3}} \times 10\ \mathrm{N\,kg^{-1}} \times 9.0\ \mathrm{m}} = 3.1\ \mu\mathrm{m}$$

Answer 3.19 The water in the soil and that in the tensiometer are both at static equilibrium and are also in equilibrium with each other. Thus the pressure head in the soil and in the tensiometer tube is zero at the level of the groundwater table. The pressure head at the gauge is then -1.0 m, because the gauge is at 1.0 m above the groundwater table.

Answer 3.20 a. The increase in gas pressure of 2 kPa inside the container increases the pressure on the soil water everywhere with the same amount. Thus the pressure head of the soil water increases with

$$\frac{2\,\text{kPa}}{\rho_l g} = \frac{2000\,\text{Pa}}{10^3\,\text{kg m}^{-3} \times 10\,\text{N kg}^{-1}} = 0.2\,\text{m}$$

and the gauge will now indicate $-1.0\,\text{m} + 0.2\,\text{m} = -0.8\,\text{m}$.

b. Since the pressure head of the soil water is read, per definition, relative to atmospheric pressure, it has changed 0.2 m.

Answer 3.21 According to Equation 3.9, the curvatures of the air-water interfaces in the soil, and with them the volume fraction of water, are determined by the pressure difference across the interface and not by the absolute values. Since the pressure in the water and in the air changed everywhere by the same amount, the soil water content will not change.

Answer 3.22 The porosity of this soil is 40%, because at $h_m = 0$ the soil is saturated with water.

Answer 3.23 Pores that are emptied at $h_m = -0.3$ m have, according to Answers 3.5 and 3.6, an equivalent diameter:

$$2r = \frac{4\sigma}{\rho_l g |h_m|} = \frac{4 \times 7 \times 10^{-2}}{10^3 \times 10 \times 0.3}\,\text{m} \approx 93\,\mu\text{m}$$

Answer 3.24 a. $pF = {}^{10}\log(-h_m/\text{cm}) = {}^{10}\log(160\,\text{m/cm}) = {}^{10}\log 16000 \approx 4.2$

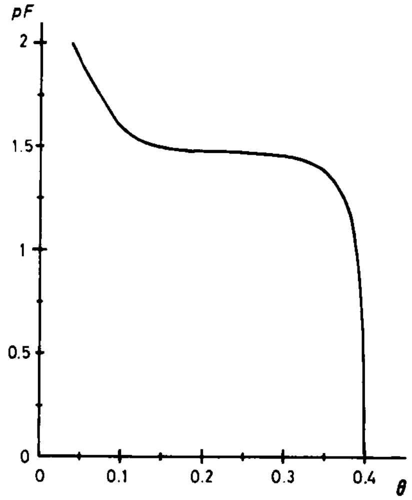
Similarly, for $h_m = -10$ m, $pF = 3$
-1 m, 2
-0.1 m, 1
-0.01 m, 0
0, $-\infty$

b. Negative pF values do exist for $-1.0\,\text{cm} < h_m < 0$.

c. At the phreatic level $pF = \log 0 = -\infty$.

d. At 10 cm below the groundwater table there is no negative pressure head and hence no pF value.

Answer 3.25



Note that the scale on the vertical axis has been compressed in the lower matric potential range and stretched in the higher matric potential range.

Answer 3.26 If the absolute pressure is zero, there is vacuum. So negative absolute pressures do not exist.

Answer 3.27 Sandy soils (curve 1 and 2) release most of their water below $pF = 3$, where mainly capillary binding is active. Clay soils release much of their water at higher pF values.

Answer 3.28 a. Loamy and peaty soils exhibit both capillary and osmotic binding of water.

b. The most significant difference between these two soil types is that the total amount of water that can be stored, and hence the total porosity, is much larger in peat soils than in loam soils.

Answer 3.29 a. Volume of pore $= 2 \times \frac{1}{4}\pi a^2 \times 2a + \pi a^2 \times 4a = \pi a^3 + 4\pi a^3 = 5\pi a^3$

$$\text{Volume of sample} = \frac{25}{16}\pi a^2 \times 8a = 12.5\pi a^3$$

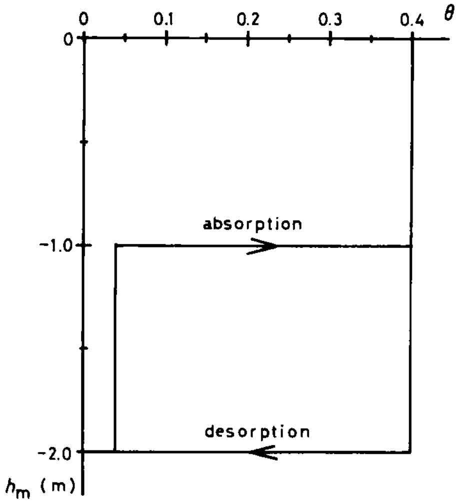
$$\text{Thus, } \phi = \frac{5\pi a^3}{12.5\pi a^3} = 0.4$$

b. $\theta = \phi = 0.4$ at $h_m = 0$.

c. The opening of the upper bottle-neck will be able to 'hold' the meniscus against the applied negative pressure as long as h_m is larger than the air-entry value, or

$$h_m > \frac{-2\sigma}{\rho_l g r} = \frac{-2 \times 7 \times 10^{-2}}{1000 \times 10 \times 7 \times 10^{-6}}\,\text{m} = -2.0\,\text{m}$$

Thus the pore remains full while h_m is being decreased from 0 to -2.0 m, except for a slight decrease of the water content due to a gradual change of the shape of the meniscus from flat to concave hemispherical. At $h_m = -2.0$ m the whole pore will empty at once. The corresponding desorption curve is drawn below.



d. The lower bottle-neck will refill with water as soon as h_m becomes larger than its air-entry value, i.e. at $h_m = -2.0$ m.

The belly will refill at $h_m = \frac{-2\sigma}{\rho_l gr} = \frac{-2 \times 7 \times 10^{-2}}{1000 \times 10 \times 14 \times 10^{-6}}$ m $= -1.0$ m.

As soon as the belly is filled, the upper bottle-neck will also fill immediately, because its air-entry value is lower than -1.0 m. When the whole pore is filled, $\theta = 0.4$. When only the lower bottle-neck is filled,

$$\theta = \frac{\frac{1}{4}\pi a^2 \times 2a}{12.5\pi a^3} = \frac{1}{25} = 0.04$$

The corresponding absorption curve is drawn in the same figure.

e. As indicated under c, the meniscus at the top will change gradually from flat to a hemisphere. At maximum curvature the decrease in water content, associated with this curvature, is

$$\frac{1}{2} \times \frac{4}{3} \times \pi \times (\tfrac{1}{2}a)^3 = \frac{1}{12}\pi a^3$$

This is 1/60-th of the total pore volume of $5\pi a^3$. Thus, the vertical branch of the desorption curve should deflect gradually from $\theta = 0.4$ to $\theta = 0.393$ at $h_m = -2.0$ m. This is barely more than the accuracy with which the figure can be drawn. The same holds true, of course, for the absorption curve.

Answer 3.30 The smoother absorption and desorption curves are due to the fact that real soils have pores of many different sizes and shapes, which change gradually into each other.

Answer 3.31 The volume fraction of water available in soil 1 for plant use is 0.09. Integrated over a root zone of 0.6 m this is equivalent to a rainfall of 0.09 × 600 mm = 54 mm. The equivalents for the various soils are tabulated in the last column.

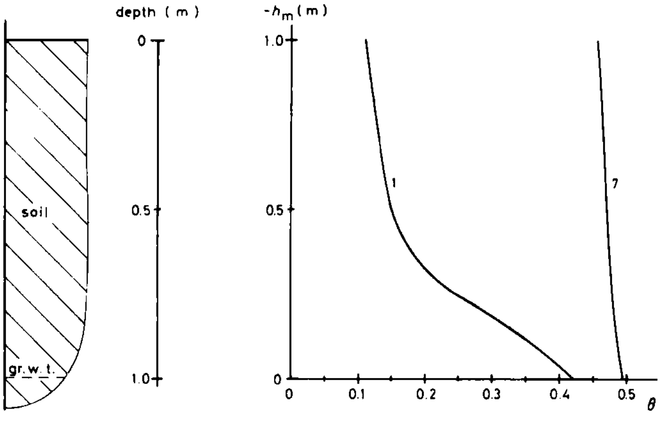
Soil	θ at $pF = 2$	θ at $pF = 4.2$	$\Delta\theta$	equivalent mm rainfall
1	0.11	0.02	0.09	54
3	0.31	0.09	0.22	132
6	0.60	0.18	0.42	252
7	0.47	0.26	0.21	126
9	0.50	0.36	0.14	84

Answer 3.32 a. Figure 3.12 gives for soil 4:
Wilting point, $pF = 4.2$, $\theta \approx 0.10$
Field capacity, $pF = 2.0$, $\theta \approx 0.35$

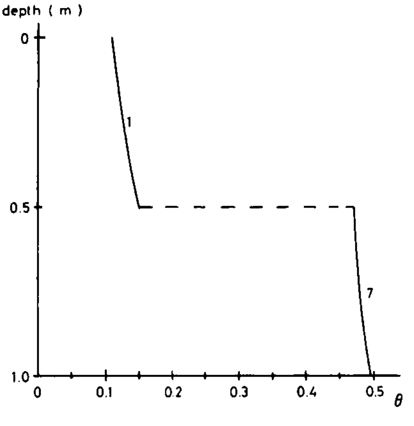
Thus, to wet this soil from wilting point to field capacity requires filling a volume fraction of about $0.35 - 0.10 = 0.25$ with water. The penetration depth of the rainshower is then 8 mm/0.25 = 32 mm.

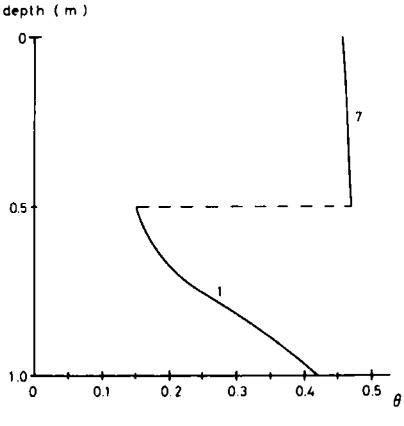
b. 0.3 m topsoil is wetted to field capacity by a rainshower of 0.25×300 mm $= 75$ mm. Per hectare this is a volume of 10^4 m^2 $\times 75 \times 10^{-3}$ m $= 750$ m^3.

Answer 3.33 a. At static equilibrium, $-h_m$ is equal to the height above the groundwater table. From the definition $pF = {}^{10}\log(-h_m/\text{cm})$ it follows:
height above groundwater table $= -h_m = 10^{pF}$ cm.



b/c.





d. Even though there is static equilibrium and soil 7 has lower values of h_m than soil 1, soil 7 is much wetter than soil 1. This result illustrates that in an inhomogeneous soil the water content may be discontinuous. The matric potential, however, is always continuous.

Answer 3.34 a. The matric head of the soil sample varies from -1.00 m at the bottom to -1.05 at the top of the sample. This difference usually causes a negligible difference in water content of the top and bottom. When the reference for z is chosen half way the height of the soil sample, the average water content should give a satisfactory result.

b. $pF = 1.0$ when $h_m = -10$ cm. So the overflow should be kept at 10 cm below the soil sample. With the set-up of Figure 3.15 it is physically impossible to do this for the whole sample. At this high value of h_m the difference between the top and bottom of the soil sample is no longer negligible. The thickness of the soil sample should then be limited to about 1 cm.

Answer 3.35 At $pF = 4.2$, the matric head is $-10^{4 \cdot 2}$ cm ≈ -158.5 m. Using Equations 3.20 and 3.26, one finds for the air pressure required:

$$p_a = -\rho_l g h_m \approx 1.6 \times 10^6 \text{ Pa}.$$

Answer 3.36 In pure water $\Pi = 0$; at the 'free' water surface under atmospheric pressure $p_t = 0$. Thus, according to Equation 3.33, $\psi_t = gz$.

Answer 3.37 a. For the conditions stated, Equation 3.40 is valid, and at 15°C thus also Equations 3.41 to 3.43. At the free surface of the NaCl-solution, $p_m = 0$. Hence, according to Equation 3.42,

$$\frac{p}{p_0} = \exp(7.5 \times 10^{-9}\ \text{Pa}^{-1} \times (-3 \times 10^7\ \text{Pa})) = \exp(-0.225) \approx 0.80 \text{ or } 80\%.$$

b. If the salt concentration of the soil water is negligible, the osmotic pressure head h_o of the soil solution is negligible and the total potential of the soil water is composed only of the matric potential and the gravitational potential.

Thus the matric potential of the soil water is equal to the osmotic potential of the solution, or -300 bar. Therefore:

$$h_m = -\frac{300 \times 10^5\ \text{Pa}}{\rho_l g} = -3000 \text{ m and}$$

$$pF = {}^{10}\log\left(\frac{-h_m}{\text{cm}}\right) = {}^{10}\log\left(\frac{3000\ \text{m}}{\text{cm}}\right) \approx 5.48$$

Answer 3.38 a. When the relative humidity is 50%, then according to Equation 3.43:

$$h_m + h_o = \frac{\ln 0.50}{7.5 \times 10^{-5}\ \text{m}^{-1}} = -9.24 \times 10^3 \text{ m}$$

If the salt in the soil sample is negligible, then $h_o = 0$ and

$$pF = {}^{10}\log\left(\frac{-h_m}{\text{cm}}\right) = {}^{10}\log\left(\frac{9.24 \times 10^3\ \text{m}}{\text{cm}}\right) = 5.97$$

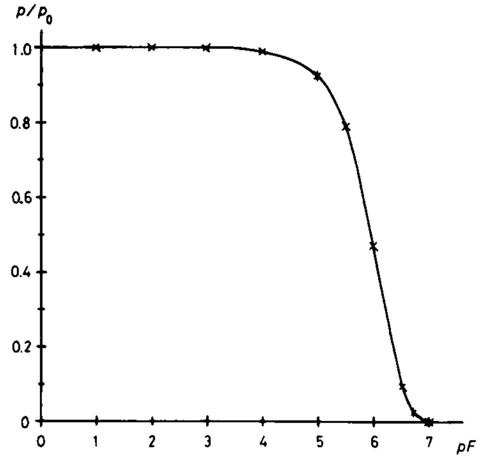
b. $$pF = {}^{10}\log\left(\frac{-h_m}{\text{cm}}\right) = {}^{10}\log\left(\frac{-100\,h_m}{\text{m}}\right) =$$

$${}^{10}\log\left(\frac{-100 \ln(p/p_0)}{7.5 \times 10^{-5}}\right) \approx 6.12 + {}^{10}\log(-\ln(p/p_0))$$

c. A tensiometer can not register pF values above 3. The water in the tensiometer will evaporate from the porous cup until the whole tensiometer is filled with air only.

Answer 3.39 Using Equation 3.43 the following table is easily composed:

pF	h_m (m)	$\ln (p/p_0)$	p/p_0
0	-0.01	-7.5×10^{-7}	1.0000
1	-0.10	-7.5×10^{-6}	1.0000
2	-1.0	-7.5×10^{-5}	0.9999
3	-10	-7.5×10^{-4}	0.9993
4	-10^2	-7.5×10^{-3}	0.993
5	-10^3	-7.5×10^{-2}	0.928
5.5	-3.2×10^3	-0.237	0.789
6	-10^4	-0.750	0.472
6.5	-3.2×10^4	-2.37	0.093
7	-10^5	-7.50	0.0006



Answer 3.40 a. At equilibrium the salt concentration will be the same in sample A and in the soil column, because the salt can diffuse freely.

The condition for equilibrium is, therefore:

ψ_h = uniform, or H = uniform.

Since $p_a = 0$, $h = h_m$ and $H = h_m + z$ = uniform.

Thus at the same height, h_m is the same in the sample as in the soil column. Hence, $h_m = -1.0$ m, since sample A is located 1.0 m above the groundwater table (neglecting the thickness of the sample).

b. In the case of sample B, salt diffusion is not possible but water can diffuse between the sample and the column through the vapour phase. It is through the vapour phase that equilibrium is attained. The condition for equilibrium is, therefore:

ψ_t(sample) = ψ_t(vapour) = ψ_t(column surface)

Neglecting the salt concentration in sample B and the small differences in height:

ψ_m(sample) = $\psi_m + \psi_o$(column surface), or

h_m (sample) = $h_m + h_o$ (column surface)
h_m (column surface) = -1.0 m
The NaCl is fully dissociated, and thus
$c = 2 \times 50 \text{ mol m}^{-3} = 100 \text{ mol m}^{-3}$.

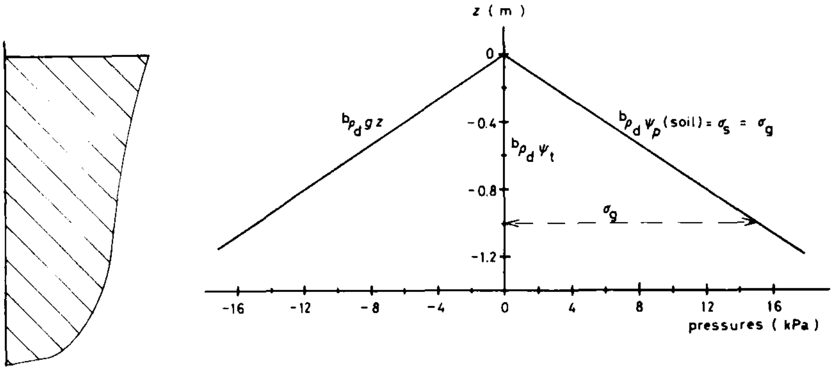
$$\text{Then } h_o = -\frac{\Pi}{\rho_l g} = -\frac{RTc}{\rho_l g} = -\frac{2.4 \times 10^3 \times 100}{10^3 \times 10} \text{ m} = -24 \text{ m}$$

$$\text{Hence, } h_m \text{(sample)} = -25 \text{ m and } pF = {}^{10}\log\left(\frac{-h_m}{\text{cm}}\right) = {}^{10}\log 2500 \approx 3.4.$$

c. Using Equation 3.43, $\ln(p/p_0) = 7.5 \times 10^{-5} \times (-25) \approx -18.8 \times 10^{-4}$.
Hence $p/p_0 \approx 0.998$.

Answer 3.41 Irrespective of the extension of the double layer, the charge of a clay particle is always fully compensated by counterions. So, outside the double layer there is no electrostatic field.

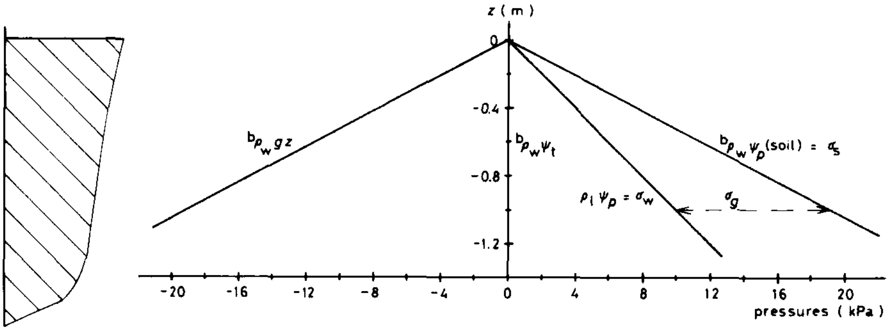
Answer 3.42 The reference point for the gravitational potential is taken at the soil surface, and thus everywhere ${}^b\rho_d\psi_t = 0$.
Then, according to Equations 3.52 and 3.53:
$\sigma_s = {}^b\rho_d\psi_p = -{}^b\rho_d\psi_g = -{}^b\rho_d gz$
With this the soil potentials are determined as indicated in the diagram.



At a depth of 1 m:
$\sigma_s = -{}^b\rho_d gz = -1500 \times 10 \times (-1) \text{ N m}^{-2} = 15 \text{ kPa}$.

Answer 3.43 In a dry soil $\sigma_g = \sigma_s$. Thus at 1 m depth σ_g is also 15 kPa.

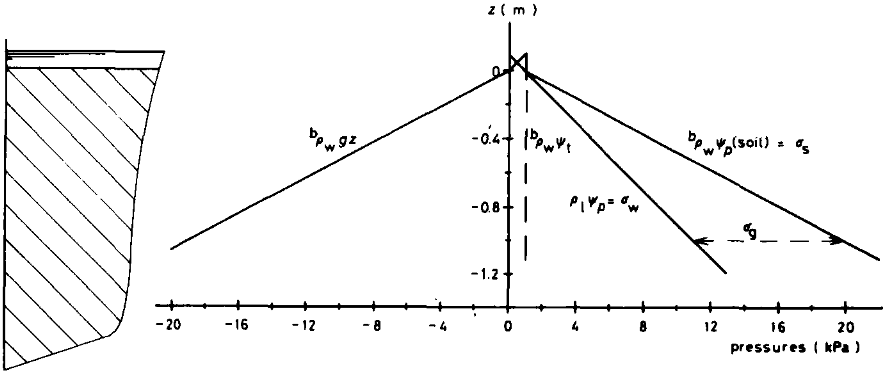
Answer 3.44 a. ${}^b\rho_w = (1500 + 400) \text{ kg m}^{-3} = 1900 \text{ kg m}^{-3}$.
Then, similar to Answer 3.42, at 1 m depth:
$\sigma_s = -{}^b\rho_w gz = -1900 \times 10 \times (-1) \text{ Pa} = 19 \text{ kPa}$.
Also at 1 m depth, $\sigma_w = -\rho_l gz = -1000 \times 10 \times (-1) \text{ Pa} = 10 \text{ kPa}$.
At the soil surface $\sigma_s = \sigma_w = 0$. With this information the diagram can be drawn.



b. According to Equation 3.55, at 1 m depth, $\sigma_g = \sigma_s - \sigma_w = (19 - 10)$ kPa = 9 kPa.

c. Compared with Answer 3.43, the intergranular pressure at 1 m depth has decreased from 15 kPa to 9 kPa. Thus the bearing capacity of the wet soil is lower than that of the dry soil.

Answer 3.45 If there is a 0.10 m layer of water on the soil surface, both σ_s and σ_w increase by 1 kPa. Hence, σ_g remains everywhere the same, as in Answer 3.44. At 1 m depth it is still 9 kPa and the soil surface it is zero. This is shown in the diagram.



Thus, at static equilibrium, the intergranular pressure is zero at the soil surface, both in a completely dry soil and in a submerged soil.

Answer 3.46 At the soil surface, $\sigma_s = 0$ and thus, according to Equation 3.57, $\sigma_g = -\dfrac{\theta}{\phi}\sigma_w$.

Also at the soil surface, the soil is still saturated, so that $\theta/\phi = 1$ and $\sigma_w = -2$ kPa. Hence at the soil surface, $\sigma_g = 2$ kPa.

4 General concepts of transport processes in soil

So far we have considered soils, and the substances and phenomena occurring within them, as being at static equilibrium. This simplification is seldom, if ever, valid. In nature, conditions change continually due to influences from within, as well as from outside the soil. Disturbances of equilibrium conditions tend to be dissipated until new equilibrium is attained. This occurs by so-called transport processes which may involve water, heat, gases, solutes, etc. The prime concern of soil physics is to describe, measure, predict and possibly control these transport processes.

All transport processes have a number of aspects in common. To emphasize this basic similarity we introduce in this chapter concepts which can be used to describe most transport processes occurring in soils. We do this using flow of water as an example, since this forms a logical extension of the subject matter presented in the preceding chapters. We then apply these concepts also to the diffusion of gases and the conduction of heat in soils, respectively. Each of the three transport processes is treated more extensively in subsequent chapters.

4.1 FLOW OF WATER

4.1.1 Movement and accumulation

Flow of water in soils involves two distinct aspects:

- movement, which is the change of position of water with respect to a chosen frame of reference, usually the soil matrix
- accumulation, which is the increase or decrease in time of the amount of water at a given position. Accumulation normally occurs as a result of flow under influence of changing external conditions. It can also be due to exchange with other phases, local production or consumption (negative accumulation), etc.

Question 4.1 Give examples of the latter type of accumulation of water in soils.

4.1.2 Driving force. Hydraulic potential gradient

When a system is at static equilibrium there is, by definition, no transport of matter taking place in that system. Transport occurs when equilibrium conditions are disturbed. A condition for static equilibrium was given in Chapter 2:

$$\Sigma \boldsymbol{F}^{s} = 0 \rightleftharpoons \text{static equilibrium} \tag{2.3}$$

It follows that $\Sigma \boldsymbol{F}^{s}$ can be considered as the driving force for a transport process ensuing from a disturbance from static equilibrium:

$$\text{driving force} = \Sigma \boldsymbol{F}^{s} \tag{4.1}$$

Conditions for static equilibrium can also be expressed in terms of the total potential ψ_t (Equation 2.30).

For the flow of water in soil, in general, only the hydraulic potential ψ_h needs to be considered. As defined in Equation 2.38 the hydraulic potential is the sum of the pressure potential, ψ_p, and the gravitational potential, ψ_g. If Equation 2.24 is applied to $|\Sigma \boldsymbol{F}^{s}|$ (the magnitude of $\Sigma \boldsymbol{F}^{s}$) the corresponding potential in the case of flow of water in soil is ψ_h, and thus

$$\frac{|\Sigma \boldsymbol{F}^{s}|}{m} = -\frac{\partial \psi_h}{\partial s} \tag{4.2}$$

Equations 4.1 and 4.2 indicate that the driving force divided by mass for flow of water in soil is in magnitude equal to the gradient of the hydraulic potential. The minus sign indicates that the driving force, and the ensuing flow of water, are directed towards decreasing values of the hydraulic potential.

In Chapter 2 it was shown that hydraulic potentials can be defined not only as energy divided by mass (ψ_h in J kg^{-1}), but also as energy divided by volume (p_h in J m^{-3} = Pa) or as energy divided by weight (H in J N^{-1} = m). Similarly, hydraulic potential gradients can be expressed per mass, per volume or per weight.

Question 4.2 What is the unit of hydraulic potential gradient expressed per mass, per volume and per weight?

From Answer 4.2 it follows that, similar to Equation 4.2, for flow of water in soil is also true:

$$\text{driving force divided by volume} = -\frac{\partial p_h}{\partial s} \tag{4.3}$$

$$\text{driving force divided by weight} = -\frac{\partial H}{\partial s} \tag{4.4}$$

4.1.3 Flux density

The rate of movement of water in soil is generally expressed as a flux density. The flux density, f_w, of water moving in the direction s is the amount of water passing through a plane perpendicular to the direction s during a time interval, divided by the area of that plane and the magnitude of the time interval. In soil physics the term 'flux' is used often when 'flux density' is meant. Use of the term 'flux' should be restricted to volume of water divided by time.

The amount of water, like potentials, can be expressed as a mass, a volume or a weight. The flux density of water flowing in soil is usually expressed as a volume.

The symbol commonly used for the volume flux density is q.

Question 4.3 Define the flux density on volume basis for the flow of water in soil and state its dimension (unit).

4.1.4 Flux density equation.

The relationship between the driving force and the ensuing flux density depends on the transport mechanism and the conditions under which it takes place. We will call this relationship the flux density equation. Flow of water in soils is dominated by the large friction force per volume of water between the soil and the water. This determines, as will be shown next, the form of the flux density equation.

When a driving force, $\Sigma \boldsymbol{F}^{\mathrm{s}}$, starts acting on a mass of water at rest in a soil, the water will start moving with an acceleration according to Newton's second law of motion (Equation 2.1b). Because of this motion the water will experience drag or friction forces as discussed in Section *2.1.1*.

In Section *2.1.4* the drag force on a spherical particle falling through a liquid at rest was, according to Stokes' law, proportional to the sedimentation velocity. For Stokes' law to be valid the conditions around the falling spherical particle must remain laminar. This means that the velocity must remain so small that turbulence does not occur. Similarly, for small flow velocities of water in soils the drag forces between the water molecules and the solid surfaces will be proportional to the flow velocity. The area of contact between water and soil particles is inherently very large. This means that drag forces divided by mass of water increase very fast with flow velocity. On the other hand, driving forces acting on water in soil are usually not very large. As a result, the drag force acting on a mass of water as it increases with velocity, will soon reach a magnitude equal to that of the driving force. Since these forces act in opposite directions the net force at that instance will become zero. The water will no longer be accelerated but maintain its attained velocity until the balance of forces is disturbed. For all normal flow of water in soil the average 'terminal' velocity, $\bar{v}$, is much smaller than the value at which the drag force would no longer be proportional to the velocity. Thus, when $\Sigma \boldsymbol{F}^{\mathrm{d}} = -\Sigma \boldsymbol{F}^{\mathrm{s}}$,

$$\bar{v} = \text{constant and proportional to } \frac{-\Sigma \boldsymbol{F}^{\mathrm{d}}}{m} = \frac{\Sigma \boldsymbol{F}^{\mathrm{s}}}{m} \tag{4.5}$$

We can derive similar relationships when the forces are expressed per volume or per weight. Using equations 4.3 and 4.4, and the fact that the flux density, q, is proportional to $\bar{v}$, we obtain the flux density equations, also called Darcy's law:

$$q = -K\frac{\partial p_{\mathrm{h}}}{\partial s} = -K\left(\frac{\partial p_{\mathrm{t}}}{\partial s} + \rho_{\mathrm{l}} g \frac{\partial z}{\partial s}\right) \tag{4.6}$$

$$q = -k\frac{\partial H}{\partial s} = -k\left(\frac{\partial h}{\partial s} + \frac{\partial z}{\partial s}\right) \tag{4.7}$$

where K and k are proportionality coefficients.

The last parts of Equations 4.6 and 4.7 are obtained by substituting Equations 3.49 and 2.44, respectively. From the derivation of the latter equations it is obvious that the first terms represent the contribution of the tensiometer pressure potential gradient to the flux density and the second terms that of the gravitational potential gradient. From this it follows that for horizontal flow the second terms vanish, since then the gravitational potential does not change. The flux density equation in terms of ψ_h is omitted, since it is seldom used.

4.1.5 *Transport coefficient. Hydraulic conductivity*

The proportionality coefficients K and k in Equations 4.6 and 4.7 are called hydraulic conductivity, because they are a measure of the ability of the soil to conduct the flow of water. They are special cases of what generally is called the transport coefficient of a medium for a particular transport process. Its dimension depends on the choices of the dimension of the other variables in the flux density equation. The relationship between K and k follows immediately from the second terms on the right hand side of Equations 4.6 and 4.7, since each represents the contribution of the gravitational potential to the flux density:

$$k = \rho_l g K \tag{4.8}$$

Question 4.4 What are the units of K and k?

In some transport processes, e.g. heat conduction in a solid material (Section 4.3), the transport coefficient is nearly independent of the potential of the transported entity. The hydraulic conductivity, however, varies strongly with water content and thus with the matric potential.

Question 4.5 Why does the hydraulic conductivity of a soil vary with the matric potential?

The hydraulic conductivity also differs greatly between soils. The reason is that the friction forces acting on the flowing water depend not so much on the total amount of water in the soil, but on the geometry of the liquid-filled pore space. This, in turn, depends on the pore-size distribution of the soil, and thus on the soil texture and soil structure. Finally, the hydraulic conductivity depends also on the friction between water molecules and thus on the viscosity of the water. Although the viscosity varies significantly with temperature, this factor is usually neglected.

Because the hydraulic conductivity varies over many orders of magnitude between air-dryness and saturation, it is very difficult to solve practical soil water flow problems. This will be discussed further in Chapter 5.

4.1.6 Continuity equation

Transport processes always obey the laws of conservation of matter and energy. Mathematical formulations of transport processes must reflect these laws. Soil water flow problems are usually considered to be isothermal and the law of conservation of energy can then be omitted. The law of conservation of matter is expressed in the so-called continuity equation. For one-dimensional flow of water in soil the continuity equation can be written as:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial s} \tag{4.9}$$

where θ is the volume fraction of water (Section *1.1.5*).

Question 4.6 Derive Equation 4.9, using Figure 4.1.

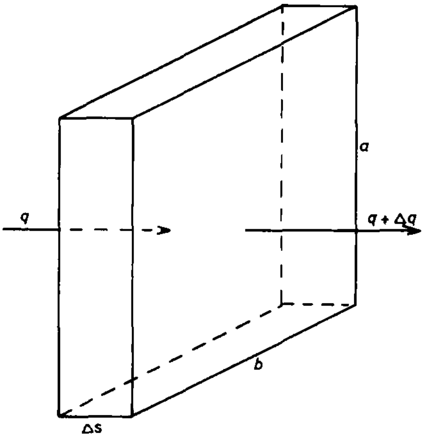


Figure 4.1 Volume element for deriving continuity equation.

Question 4.7 If the flux density at s is q_1 and at $s + \Delta s$ is q_2, fill in either $<$, $=$ or $>$ in the following statements:

a. if $q_2 > q_1$, then $\frac{\partial q}{\partial s} \ldots 0$ and $\frac{\partial \theta}{\partial t} \ldots 0$

b. if $q_2 = q_1$, then $\frac{\partial q}{\partial s} \ldots 0$ and $\frac{\partial \theta}{\partial t} \ldots 0$

c. if $q_2 < q_1$, then $\frac{\partial q}{\partial s} \ldots 0$ and $\frac{\partial \theta}{\partial t} \ldots 0$

In some transport situations, positive and negative accumulation of a component are due not only to changes in flux density in the direction of transport, but also to local additions to or extractions from outside the system (Answer 4.1). For the flow of water in soils the proper continuity equation is then

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial s} + \alpha \tag{4.10}$$

where α is the volume of water added or extracted divided by volume of soil and by time. Often the production term, α, is small relative to the other terms in the continuity equation and can be ignored. This will be done in the rest of this chapter.

Question 4.8 Give a unit analysis of Equation 4.10.

4.1.7 Differential capacity

The flux density equation and the continuity equation must both be satisfied in solving a particular water flow problem. These equations involve the four variables θ, p_t (or h), t and s, while s may be composed of up to three coordinates. Obtaining solutions of water flow problems can often be made easier by eliminating either θ or p_t (or h) by introducing a differential capacity. A familiar example of differential capacity is the electrical capacity of a conductor, which is the ratio of a change in its electrical charge and the resulting change in its electrical potential. Similarly, the differential capacity of soils for water can be defined as

$$C = \mathrm{d}\theta/\mathrm{d}p_m \tag{4.11}$$

or $c = \mathrm{d}\theta/\mathrm{d}h_m$ (4.12)

depending on the form in which the matric potential is used. One cannot use the hydraulic potential for this purpose, because it includes the gravitational potential. The tensiometer pressure potential cannot be used either, because it includes the pneumatic potential. Both potentials have no direct influence on the volume fraction of water (Section *3.1.5*). However, normally the gas phase in the soil is at atmospheric pressure, so that $p_m = p_t$ and $h_m = h$.

Question 4.9 Which relationship can be used to determine the differential water capacity of a soil?

Question 4.10 a. Is the differential water capacity dependent on the volume fraction of water?

b. What value has the differential water capacity of a soil at saturation?

Question 4.11 How can c be determined from the slope of a pF curve?

4.1.8 *Diffusivity*

Transport problems can often be solved more easily by introducing a diffusivity. This is the ratio of the transport coefficient and the differential capacity. The soil water diffusivity is defined as

$$D = \frac{K}{C} = \frac{K}{\dfrac{\mathrm{d}\theta}{\mathrm{d}p_\mathrm{m}}} = K\,\frac{\mathrm{d}p_\mathrm{m}}{\mathrm{d}\theta} \tag{4.13}$$

or $$D = \frac{k}{c} = \frac{k}{\dfrac{\mathrm{d}\theta}{\mathrm{d}h_\mathrm{m}}} = k\,\frac{\mathrm{d}h_\mathrm{m}}{\mathrm{d}\theta} \tag{4.14}$$

Since $p_\mathrm{m} = \rho_\mathrm{l} g h_\mathrm{m}$ and $K = k/\rho_\mathrm{l} g$ (Equation 4.8) the diffusivities in Equations 4.13 and 4.14 are identical.

Question 4.12 Is D a function of the volume fraction of water?
Question 4.13 What is the value of D in a region where the soil is saturated?

If the pressure in the soil gas phase is atmospheric, so that $p_\mathrm{t} = p_\mathrm{m}$ and $h = h_\mathrm{m}$, the flux density equations 4.6 and 4.7 can be rewritten, respectively, as follows:

$$q = -K\left(\frac{\mathrm{d}p_\mathrm{m}}{\mathrm{d}\theta}\frac{\partial\theta}{\partial s} + \rho_\mathrm{l} g \frac{\partial z}{\partial s}\right) = -D\frac{\partial\theta}{\partial s} - \rho_\mathrm{l} g K \frac{\partial z}{\partial s} \tag{4.15}$$

$$q = -k\left(\frac{\mathrm{d}h_\mathrm{m}}{\mathrm{d}\theta}\frac{\partial\theta}{\partial s} + \frac{\partial z}{\partial s}\right) = -D\frac{\partial\theta}{\partial s} - k\frac{\partial z}{\partial s} \tag{4.16}$$

These equations show why the diffusivity is especially useful for horizontal flow problems. Then the second terms vanish and the flux density is proportional to the gradient of the volume fraction of water.

Question 4.14 What does this suggest about the transport mechanism of the flow of water in soils? Is this correct?
Question 4.15 Which flux density equation would you use in an inhomogeneous soil?
Question 4.16 Which effect does hysteresis have on the validity of these flux density equations?

4.1.9 *General flow equations*

To solve a particular water flow problem, the flux density equation best suited for the particular problem can be combined with the continuity equation to obtain a general flow equation. For example, substitution of flux density Equation 4.6 or 4.7 into continuity Equation 4.9 yields, respectively,

$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial s}\left(K\frac{\partial p_{\mathrm{t}}}{\partial s}\right)+\rho_{\mathrm{l}} g \frac{\partial}{\partial s}\left(K\frac{\partial z}{\partial s}\right) \tag{4.17}$$

$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial s}\left(k\frac{\partial h}{\partial s}\right)+\frac{\partial}{\partial s}\left(k\frac{\partial z}{\partial s}\right) \tag{4.18}$$

If the soil air is at atmospheric pressure ($p_{\mathrm{t}} = p_{\mathrm{m}}$ and $h = h_{\mathrm{m}}$) the left hand sides can be written as

$$\frac{\partial \theta}{\partial t}=\frac{\mathrm{d}\theta}{\mathrm{d}p_{\mathrm{m}}}\frac{\partial p_{\mathrm{m}}}{\partial t}=\frac{\mathrm{d}\theta}{\mathrm{d}h_{\mathrm{m}}}\frac{\partial h_{\mathrm{m}}}{\partial t}=C\frac{\partial p_{\mathrm{t}}}{\partial t}=c\frac{\partial h}{\partial t} \tag{4.19}$$

For vertically upward flow, for which $\mathrm{d}s = \mathrm{d}z$, Equations 4.17 and 4.18 become, respectively,

$$C\frac{\partial p_{\mathrm{t}}}{\partial t}=\frac{\partial}{\partial z}\left(K\frac{\partial p_{\mathrm{t}}}{\partial z}\right)+\rho_{\mathrm{l}} g\frac{\partial K}{\partial z} \tag{4.20}$$

$$c\frac{\partial h}{\partial t}=\frac{\partial}{\partial z}\left(k\frac{\partial h}{\partial z}\right)+\frac{\partial k}{\partial z} \tag{4.21}$$

Question 4.17 Show that these equations remain the same for vertically downward flow.

For horizontal flow, $\partial z/\partial s = 0$, and thus the second terms on the right hand side of Equations 4.17 and 4.18 vanish. The first terms represent the contribution of the tensiometer pressure potential gradient, as was already discussed in connection with Equations 4.6 and 4.7.

One can also substitute the diffusivity-type flux density Equations 4.15 or 4.16 into continuity Equation 4.9 and obtain, respectively,

$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial s}\left(D\frac{\partial \theta}{\partial s}\right)+\rho_{\mathrm{l}} g\frac{\partial}{\partial s}\left(K\frac{\partial z}{\partial s}\right) \tag{4.22}$$

$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial s}\left(D\frac{\partial \theta}{\partial s}\right)+\frac{\partial}{\partial s}\left(k\frac{\partial z}{\partial s}\right) \tag{4.23}$$

Again, for horizontal flow the second terms vanish, after which the two equations are identical. The reason for this is of course that, by using the diffusivity, the pressure potential has been eliminated in favour of the volume fraction of water, θ. A substitution similar to Equation 4.19 is not needed, since the number of variables is already reduced to three.

4.1.10 Initial and boundary conditions

The general flow equations discussed in the preceding section are mathematical representations which are valid for most water flow processes in soils. However, they do not yet specify a particular flow problem. The mathematical description of a particular flow problem must include specifications on boundary conditions and, in the case of time-variant problems, also initial conditions. These are usually values of water content, pressure potential or flux density at certain positions and at certain times.

Question 4.18 Give examples of initial and boundary conditions for water flow problems in soil.

The solution of a particular flow problem differs yet from soil to soil, depending on the soil hydraulic properties. Thus, a particular soil water flow problem can only be described completely when also the soil water characteristic (Section *3.1.6*) and the soil hydraulic conductivity or diffusivity are specified as functions of water content or matric potential. Particular soil water flow problems are discussed in Chapter 5.

4.2 GAS DIFFUSION

In soils, the gas pressure generally is constant and equal to atmospheric pressure. Hence, gas components (e.g. O_2, CO_2) move only due to gradients of their partial pressures. Such transport is known as gas diffusion.

The driving force for gas diffusion of a component i can be represented by the partial pressure gradient $\partial p_i/\partial s$, where p_i is the pressure due to gaseous component i. One can also express the driving force as a gradient of the mass concentration, χ_i, which is the mass of component i divided by volume of gas (kg m^{-3}). χ_i is proportional to the partial pressure at low total gas pressures. Experimentally, it is easier to measure χ_i than p_i.

Question 4.19 a. What law describes the proportionality between partial pressure and mass concentration?
b. Derive from this law the relationship between p_i and χ_i.

Using mass concentration gradient as driving force, the flux density equation for gas diffusion is

$$f_i = -D_g \frac{\partial \chi_i}{\partial s} \tag{4.24}$$

Equation 4.24 is called Fick's law, and the transport coefficient D_g is the diffusion coefficient for the particular gaseous component in the particular soil. If the flux density, f_i, is expressed in kg m^{-2} s^{-1}, the unit of D_g is m^2 s^{-1} (check this).

The continuity equation is then

$$\frac{\partial G_i}{\partial t} = -\frac{\partial f_i}{\partial s}(+\alpha) \tag{4.25}$$

where G_i is the amount of component i divided by volume of soil, in kg m^{-3}.

The differential capacity of the soil for component i is:

$$\frac{\mathrm{d}G_i}{\mathrm{d}\chi_i} = \phi_g \tag{4.26}$$

where ϕ_g is the air-filled porosity.

Question 4.20 Derive Equation 4.26.

The general transport equation for gas diffusion in soils is obtained by combining Equations 4.24, 4.25 and 4.26:

$$\frac{\partial \chi_i}{\partial t} = \frac{1}{\phi_g}\frac{\partial}{\partial s}\left(D_g \frac{\partial \chi_i}{\partial s}\right) \tag{4.27}$$

4.3 HEAT CONDUCTION

Heat has no mass and thus the formulation of the driving force for heat conduction cannot be the same as that in Equations 4.2, 4.3 and 4.4. The similarity with material transport can be maintained by choosing as driving force the temperature gradient, $\partial T/\partial s$ (K m^{-1}). As unit of heat flux density, f_h, can be taken J m^{-2} s^{-1} (= W m^{-2}). The flux density equation for heat conduction in soils is then

$$f_h = -\lambda \frac{\partial T}{\partial s} \tag{4.28}$$

The transport coefficient λ is called the heat conductivity and has the unit J m^{-1} s^{-1} K^{-1} (= W m^{-1} K^{-1}). Equation 4.28 is called Fourier's law.

Question 4.21 Check the unit of heat conductivity.

The continuity equation for heat flow is:

$$\frac{\partial Q}{\partial t} = -\frac{\partial f_h}{\partial s}(+\alpha) \tag{4.29}$$

where Q is the heat content divided by volume of the soil in J m^{-3} (check this). Combination of Equations 4.28 and 4.29 and introduction of the differential heat capacity $C_h = \mathrm{d}Q/\mathrm{d}T$ lead to the general heat conduction equation:

$$\frac{\partial T}{\partial t} = \frac{1}{C_h}\frac{\partial}{\partial s}\left(\lambda \frac{\partial T}{\partial s}\right) \tag{4.30}$$

where C_h, normally called volumic heat capacity, is in J m^{-3} K^{-1}.

Temperatures can be measured easily, as well as λ and C_h, since they are nearly independent of temperature. The latter makes heat conduction problems much easier to solve than water flow problems.

If λ is independent of s, and thus also of T (homogeneous soils with uniform water content), this equation can be simplified to

$$\frac{\partial T}{\partial t} = \frac{\lambda}{C_h}\frac{\partial^2 T}{\partial s^2} = D_h\frac{\partial^2 T}{\partial s^2} \qquad (4.31)$$

The heat diffusivity D_h is a measure for the rate at which an imposed temperature gradient in the soil is dissipated by conduction.

Question 4.22 Derive the unit of D_h.

The similarities and differences between the transport processes discussed in this chapter, are summarized in Tables 4.1 and 4.2.

4.4 SUGGESTED LITERATURE

BIRD, R. B., W. E. STEWART and E. N. LIGHTFOOT, 1960. Transport Phenomena. Wiley, 780 p.
CARSLAW, H. S. and J. C. JAEGER, 1959. Conduction of Heat in Solids. Oxford Univ. Press, 510 p.
CRANK, J., 1964. The Mathematics of Diffusion. Oxford Univ. Press, 347 p.

Table 4.1 **Summary of equations for water flow, gas diffusion and heat conduction (without production).**

	water flow	gas diffusion	heat conduction
a. flux density equation	$q = -K\frac{\partial p_\mathrm{h}}{\partial s} = -k\frac{\partial H}{\partial s}$	$f_i = -D_\mathrm{g}\frac{\partial \chi_i}{\partial s}$	$f_\mathrm{h} = -\lambda\frac{\partial T}{\partial s}$
b. continuity equation	$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial s}$	$\frac{\partial G_i}{\partial t} = -\frac{\partial f_i}{\partial s}$	$\frac{\partial Q}{\partial t} = -\frac{\partial f_\mathrm{h}}{\partial s}$
c. general transport equation	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial s}\left(K\frac{\partial p_\mathrm{h}}{\partial s}\right) = \frac{\partial}{\partial s}\left(k\frac{\partial H}{\partial s}\right)$	$\frac{\partial G_i}{\partial t} = \frac{\partial}{\partial s}\left(D_\mathrm{g}\frac{\partial \chi_i}{\partial s}\right)$	$\frac{\partial Q}{\partial t} = \frac{\partial}{\partial s}\left(\lambda\frac{\partial T}{\partial s}\right)$
d. equivalent of c	$C\frac{\partial p_\mathrm{m}}{\partial t} = \frac{\partial}{\partial s}\left(K\frac{\partial p_\mathrm{h}}{\partial s}\right)$ $c\frac{\partial h_\mathrm{m}}{\partial t} = \frac{\partial}{\partial s}\left(k\frac{\partial H}{\partial s}\right)$	$\frac{\partial \chi_i}{\partial t} = \frac{1}{\phi_\mathrm{g}}\frac{\partial}{\partial s}\left(D_\mathrm{g}\frac{\partial \chi_i}{\partial s}\right)$	$\frac{\partial T}{\partial t} = \frac{1}{C_\mathrm{h}}\frac{\partial}{\partial s}\left(\lambda\frac{\partial T}{\partial s}\right)$
e. diffusivity form of c	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial s}\left(D\frac{\partial \theta}{\partial s}\right)$ (horizontal only)	same as above	$\frac{\partial T}{\partial t} = \frac{\partial}{\partial s}\left(D_\mathrm{h}\frac{\partial T}{\partial s}\right)$
f. e for constant D	not applicable	$\frac{\partial \chi_i}{\partial t} = \frac{D_\mathrm{g}}{\phi_\mathrm{g}}\frac{\partial^2 \chi_\mathrm{i}}{\partial s^2}$	$\frac{\partial T}{\partial t} = D_\mathrm{h}\frac{\partial^2 T}{\partial s^2}$

Table 4.2 **Symbols and units for parameters used in equations of Table 4.1**

parameter	water flow		gas diffusion	heat conduction
flux density	$f_\mathrm{w} = q$ $\mathrm{m^3\,m^{-2}\,s^{-1} = m\,s^{-1}}$		f_i $\mathrm{kg\,m^{-2}\,s^{-1}}$	f_h $\mathrm{J\,m^{-2}\,s^{-1} = W\,m^{-}}$
potential	$p_\mathrm{h} = p_\mathrm{t} + \rho_\mathrm{l} g z$ Pa	$H = h + z$ m	χ_i $\mathrm{kg\,m^{-3}}$	T K
transport coefficient	K $\mathrm{m^2\,Pa^{-1}\,s^{-1}}$	k $\mathrm{m\,s^{-1}}$	D_g $\mathrm{m^2\,s^{-1}}$	λ $\mathrm{J\,m^{-1}\,s^{-1}\,K^{-1}}$
content	θ $\mathrm{m^3\,m^{-3}}$ (dimensionless)		G_i $\mathrm{kg\,m^{-3}}$	Q $\mathrm{J\,m^{-3}}$
differential capacity	$C = \frac{\mathrm{d}\theta}{\mathrm{d}p_\mathrm{m}}$ $\mathrm{Pa^{-1}}$	$c = \frac{\mathrm{d}\theta}{\mathrm{d}h_\mathrm{m}}$ $\mathrm{m^{-1}}$	$\phi_\mathrm{g} = \frac{\mathrm{d}G_i}{\mathrm{d}\chi_i}$ $\mathrm{m^3\,m^{-3}}$ (dimensionless)	$C_\mathrm{h} = \frac{\mathrm{d}Q}{\mathrm{d}T}$ $\mathrm{J\,m^{-3}\,K^{-1}}$
diffusivity	$D = \frac{K}{C}$ $\mathrm{m^2\,s^{-1}}$	$D = \frac{k}{c}$ $\mathrm{m^2\,s^{-1}}$	$\frac{D_\mathrm{g}}{\phi_\mathrm{g}}$ $\mathrm{m^2\,s^{-1}}$	$D_\mathrm{h} = \frac{\lambda}{C_\mathrm{h}}$ $\mathrm{m^2\,s^{-1}}$

Answer 4.1 Exchanges of liquid water in soil with other phases can occur as a result of freezing, thawing, evaporation or condensation. An example of local consumption is the uptake of water by plant roots.

Answer 4.2 The unit of hydraulic potential gradient per mass is $\mathrm{J\ kg^{-1}\ m^{-1} = N\ kg^{-1}}$, i.e. force divided by mass.

The unit of hydraulic potential gradient per volume is $\mathrm{J\ m^{-3}\ m^{-1} = N\ m^{-3}}$, i.e. force divided by volume.

The unit of hydraulic potential gradient per weight is $\mathrm{J\ N^{-1}\ m^{-1} = N\ N^{-1}}$, i.e. force divided by weight. It can also be considered as $\mathrm{m\ m^{-1}}$, that is as hydraulic head gradient. Both forms are ultimately dimensionless.

Answer 4.3 The flux density of water on volume basis, q, is the volume of water flowing through a specified cross-section of soil perpendicular to the direction of flow during a certain time interval, divided by the area of the cross-section and the magnitude of the time interval. Thus $[q] = \mathrm{m^3\ m^{-2}\ s^{-1} = m\ s^{-1}}$, which is the dimension of a velocity. However, q is only an equivalent velocity.

Answer 4.4 $[K] = \left[\dfrac{q}{\partial p_\mathrm{h}/\partial s}\right] = \dfrac{\mathrm{m\ s^{-1}}}{\mathrm{J\ m^{-3}\ m^{-1}}} = \dfrac{\mathrm{m\ s^{-1}}}{\mathrm{Pa\ m^{-1}}} = \mathrm{m^2\ Pa^{-1}\ s^{-1}}$

$$[k] = \left[\frac{q}{\partial H/\partial s}\right] = \frac{\mathrm{m s^{-1}}}{\mathrm{m m^{-1}}} = \mathrm{m s^{-1}}$$

Thus k has the dimension of a velocity.

Answer 4.5 As the matric potential decreases, water will be present only in ever smaller pores and in ever thinner films on the soil particles. Thus, the area of contact between the solid phase and the water, and thus the friction forces, per volume of water will increase. This means that the 'terminal' velocity that will be reached with a certain driving force, decreases as the matric potential decreases. Thus the ability of the soil to conduct water flow, the hydraulic conductivity, decreases with the matric potential. While the hydraulic conductivity is a function of the matric potential, it is in the first place a function of the volume fraction of water.

Answer 4.6 The volume of water entering the volume element during time interval Δt is: $qab\Delta t$.

The volume of water leaving the volume element during time interval Δt is: $(q + \Delta q)ab\Delta t$.

The volume of water accumulated in the volume element during Δt then equals $qab\Delta t - (q + \Delta q)ab\Delta t = -ab\Delta q\Delta t$.

This volume of water has accumulated in a volume $ab\Delta s$. Hence:

$$\Delta\theta = \frac{-ab\Delta q\Delta t}{ab\Delta s} \text{ or } \frac{\Delta\theta}{\Delta t} = -\frac{\Delta q}{\Delta s}$$

For infinitesimal time and distance, this finite difference equation yields the continuity equation in differential form:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial s}.$$

Answer 4.7 a. if $q_2 > q_1$, then $\frac{\partial q}{\partial s} > 0$ and $\frac{\partial \theta}{\partial t} < 0$

b. if $q_2 = q_1$, then $\frac{\partial q}{\partial s} = 0$ and $\frac{\partial \theta}{\partial t} = 0$

c. if $q_2 < q_1$, then $\frac{\partial q}{\partial s} < 0$ and $\frac{\partial \theta}{\partial t} > 0$

Answer 4.8 The unit of $\partial\theta/\partial t$ is $m^3\ m^{-3}\ s^{-1} = s^{-1}$
The unit of $\partial q/\partial s$ is $m\ s^{-1}\ m^{-1} = s^{-1}$
The unit of α is $m^3\ m^{-3}\ s^{-1} = s^{-1}$
Thus, the units are all the same, as they should be.

Answer 4.9 The differential water capacity of a soil can be derived from the relationship between the volume fraction of liquid and the matric potential, known as the soil water characteristic. It is the slope (cotangent of inclination) of a graph of this relationship (Figure 3.11).

Answer 4.10 a. The slope of the graph varies with the volume fraction of water. Thus the differential water capacity depends on the volume fraction of water.

b. At saturation, a small change in matric potential does not cause any change of the volume fraction of water. This means that the differential water capacity is zero.

Answer 4.11 From the pF curve one can read $d\theta/d(pF)$, while we have to find $c = d\theta/dh_m$. The relationship between the two differentials can be found using:
$pF = {}^{10}\log(-h_m/\text{cm})$, from which $h_m = -10^{pF}\text{cm}$, and
$d\,10^a = d[e^{\ln 10}]^a = d\,e^{a \ln 10} = \ln 10\, e^{a \ln 10} da = 10^a \ln 10\, da$

$$\text{Thus, } c = \frac{d\theta}{dh_m} = \frac{d\theta}{-1\ \text{cm}\ d\,10^{pF}} = \frac{d\theta}{-1\ \text{cm} \times 10^{pF} \ln 10\, d(pF)} = \frac{1}{h_m \ln 10}\frac{d\theta}{d(pF)} = \frac{0.4343}{h_m}\frac{d\theta}{d(pF)}$$

Answer 4.12 Both K and C are a function of the volume fraction of water, and thus D is also. D usually varies less with volume fraction of water than K.

Answer 4.13 Since, at saturation, the differential water capacity is zero (Answer 4.10b), the diffusivity is infinitely large.

Answer 4.14 These flux density equations suggest that the gradient of the volume fraction of water is the driving force for the flow of water in soils. This would make it similar to the diffusion of salts in solution, for which the concentration gradient can be taken as driving force. From this similarity stems the name diffusivity. This representation suggests that the water moves through the soil by the process of diffusion (see also Section 4.2). This is not correct. Water moves under influence of hydraulic potential gradients. Only when the volume fraction of water is a continuous, singlevalued function of the pressure potential can the chain rule for differentials be used to arrive at equations 4.15 and 4.16.

Answer 4.15 In an inhomogeneous soil, such as at a boundary between two layers, the volume fraction of water is discontinuous (Answer 3.33), making $\mathrm{d}p_\mathrm{m}/\mathrm{d}\theta$ and $\mathrm{d}h_\mathrm{m}/\mathrm{d}\theta$ undefined. Then also D is undefined and Equations 4.15 and 4.16 cannot be used. The water potential is never discontinuous, so Equations 4.6 and 4.7 can always be used.

Answer 4.16 When hysteresis occurs, the soil water characteristic is no longer a singlevalued function. This again makes the diffusivity undefined. Thus, Equations 4.15 and 4.16 are valid only in monotonically drying or wetting, homogeneous soils in which the gas pressure is atmospheric.

Answer 4.17 For downward flow, s is positive downward. In the second term of Equations 4.17 and 4.18, z stands for the gravitational potential. Therefore, z must be taken positive upward and $\mathrm{d}s = -\mathrm{d}z$. In these equations ∂s occurs twice in each term. Upon substitution of $\partial s = -\partial z$, the two minus signs cancel and Equations 4.20 and 4.21 result again.

Answer 4.18 Possible forms of initial and boundary conditions for water flow problems in soil are:

- water content distribution at $t = 0$
- hydraulic potential distribution at $t = 0$
- rainfall intensities as function of time
- rate of evaporation from the soil surface
- zero flux at a specified depth.

Answer 4.19 a. The proportionality is described by the ideal gas law (Equation 2.46).
b. For a gas component i in a gas mixture of total volume V, the relationship between the mass concentration χ_i and the partial pressure p_i is, according to this law,

$$\chi_i = \frac{m_i}{V} = \frac{nM_i}{V} = \frac{M_i}{RT} p_i$$

Answer 4.20 Let A_i be the amount of gas i in a soil with volume V_s, of which the gas-filled volume is V_g. Then

$$G_i = \frac{A_i}{V_\mathrm{s}} \text{ and } \chi_i = \frac{A_i}{V_\mathrm{g}}$$

Therefore, $G_i = \frac{V_\mathrm{g}}{V_\mathrm{s}} \chi_i = \phi_\mathrm{g} \chi_i$

or in differential form:

$$\mathrm{d}G_i = \phi_\mathrm{g} \mathrm{d}\chi_i \quad \text{or} \quad \frac{\mathrm{d}G_i}{\mathrm{d}\chi_i} = \phi_\mathrm{g}$$

Answer 4.21 $[\lambda] = \left[\frac{-f_\mathrm{h}}{\partial T/\partial s}\right] = \frac{\mathrm{J\,m^{-2}\,s^{-1}}}{\mathrm{K\,m^{-1}}} = \mathrm{J\,m^{-1}\,s^{-1}\,K^{-1}}$

Answer 4.22 $\left[\frac{\partial T}{\partial t}\right] = \mathrm{K\,s^{-1}}$

$\left[\frac{\partial^2 T}{\partial\ s^2}\right] = \mathrm{K\,m^{-2}}$

So $[D_\mathrm{h}] = \frac{\mathrm{K\,s^{-1}}}{\mathrm{K\,m^{-2}}} = \mathrm{m^2\,s^{-1}}$

5 Flow of water in soil

5.1 INTRODUCTION

Water is present in every soil profile, but the amount varies with time and place as a result of supply and demand by its environment. If the supply of water at the soil surface (e.g. rainfall, irrigation, dew, flood) during a certain period exceeds the extraction of water from a given soil profile (e.g. evaporation from soil surface, water uptake by plant roots), the excess supply during that period is stored in the soil profile or discharged to lower regions (drainage). The supply at the soil surface can also be smaller than the sum of transpiration and evaporation. In that case, the topsoil will dry out. Upward transport of water from the subsoil (capillary rise) may also take place.

In Chapters 3 and 4 we found:

- at static hydraulic equilibrium:
  p_h (or H) = uniform and $q = 0$
- without static equilibrium,

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial s} \text{ and } q = -K\frac{\partial p_h}{\partial s} = -k\frac{\partial H}{\partial s}$$

In this chapter the subject matter of Chapters 3 and 4 is developed further for vertical flow in soil profiles. Except in one instance (Section *5.2.4*), it is assumed that the soil is homogeneous and isotropic. Three different types of flow will be considered:

1. steady flow in saturated soil:

$$\frac{\partial \theta}{\partial t} = 0 \text{ and } \theta = \phi$$

   q, K (or k) and θ are constant in time and position.

2. steady flow in unsaturated soil:

$$\frac{\partial \theta}{\partial t} = 0 \text{ and } \theta < \phi$$

   q is constant in time and position, but K (or k) and θ only in time.

3. nonsteady flow in unsaturated soil:

$$\frac{\partial \theta}{\partial t} \neq 0 \text{ and } \theta < \phi$$

   q, K (or k) and θ vary with time and position.

Nonsteady flow in saturated soil, in which θ is constant but q varies in time, occurs in drainage processes. This type of flow is usually treated separately and will not be considered here.

5.2 FLOW OF WATER IN SATURATED SOIL

5.2.1 Saturated hydraulic conductivity

A major division in types of flow of water in soil is that between saturated flow and unsaturated flow. The reason is that the hydraulic conductivity of saturated soils is constant in time and, in a uniform soil, also in position. This simplifies transport equations such as Equation 4.18 and makes their solution relatively easy. Flow processes for which the hydraulic conductivity is not constant, are much more complex.

In Chapter 4 two hydraulic conductivities were distinguished. One, K, is associated with the hydraulic potential, $p_h = p_t + \rho_l g z$, expressed as pressure equivalent, and has the unit m^2 Pa^{-1} s^{-1}. The other, k, is associated with the hydraulic potential, $H = h + z$, expressed as head equivalent, and has the unit m s^{-1}. The relationship between the two hydraulic conductivities is:

$$k = \rho_l g K \approx 10^4 \,\mathrm{Pa\,m^{-1}} \times K \tag{5.1}$$

In Chapter 4 it was stated that the hydraulic conductivity of porous media is determined by the internal friction in the water, as characterized by the viscosity, η, and by the friction between the pore walls and the water. To gain a better understanding of the latter, a porous medium can, as a first approximation, be considered as a bundle of capillary tubes.

According to Poiseuille's law (for derivation, see Appendix 1) the rate of discharge, Q, of a capillary tube with radius r is

$$Q = -\frac{\pi r^4}{8\eta}\frac{\partial p_h}{\partial s} \tag{5.2}$$

Suppose a soil column contains cylindrical, parallel pores of one size only, and the number of pores divided by the cross-sectional area perpendicular to the flow is n. Then the flux density through the column is

$$q = nQ \tag{5.3}$$

If the pores, instead of being all the same, are of various size classes, the total flux density is the sum of the flux densities of each of the pore size classes. If n_i represents the number of pores in the i^{th} pore size class with radius r_i divided by total cross-sectional area, and Q_i is the discharge rate per pore in that group, then the total flux density through the soil column is:

$$q = \Sigma n_i Q_i = -\frac{1}{8\eta}\frac{\partial p_\mathrm{h}}{\partial s}\Sigma \pi n_i r_i^4 \tag{5.4}$$

Equation 5.4 is valid if all capillary pores run parallel to the flux direction. In soils, however, they never do. Therefore, a tortuosity factor, τ, must be introduced to account for the lengthened distance over which ∂p_h acts and which the flow must traverse ($\tau \approx 2$). The flux density is then

$$q = -\frac{1}{8\eta}\frac{\partial p_\mathrm{h}}{\tau \partial s}\Sigma \pi n_i r_i^4 \tag{5.5}$$

Since $\pi n_i r_i^2$ is the contribution, $(\Delta\theta)_i$, of the pores with radius r_i to the total volume fraction of water, θ, the flux density is also

$$q = -\frac{1}{8\eta}\frac{\partial p_\mathrm{h}}{\tau \partial s}\Sigma (\Delta\theta)_i r_i^2 \tag{5.6}$$

A comparison of Equation 5.6 with Equation 4.6 yields immediately:

$$K = \frac{1}{8\eta\tau}\Sigma (\Delta\theta)_i r_i^2 \tag{5.7}$$

Thus the hydraulic conductivity of a porous medium can be estimated from the size distribution of the water-filled pores.

Equation 5.7 provides an explanation for the sharp decrease of the hydraulic conductivity with drying and swelling. Since K is a function of the number of water-filled pores and proportional to the square of the pore radius, its value is determined mainly by the number and size of the largest water-filled pores. During drying, the large pores are the first ones to empty and K is determined only by the smaller pores that remain full (Section *5.3.1*). During swelling of clay soils the total porosity increases, but the smaller pores expand at the expense of the larger ones, so that K decreases sharply.

In sandy soils water is largely present in capillary pores. This makes it possible to estimate the pore-size distribution from the soil water characteristic. From the contribution of the different pore-size classes to θ, a rough estimate of K can be made.

Question 5.1 Figure 5.1 is the soil water characteristic of a sandy soil. Estimate the saturated hydraulic conductivity on the basis of the three pore-size classes shown, using the following information:
$\sigma = 7 \times 10^{-2}$ N m^{-1}, $\eta = 10^{-3}$ Pa s and $\tau = 2$.

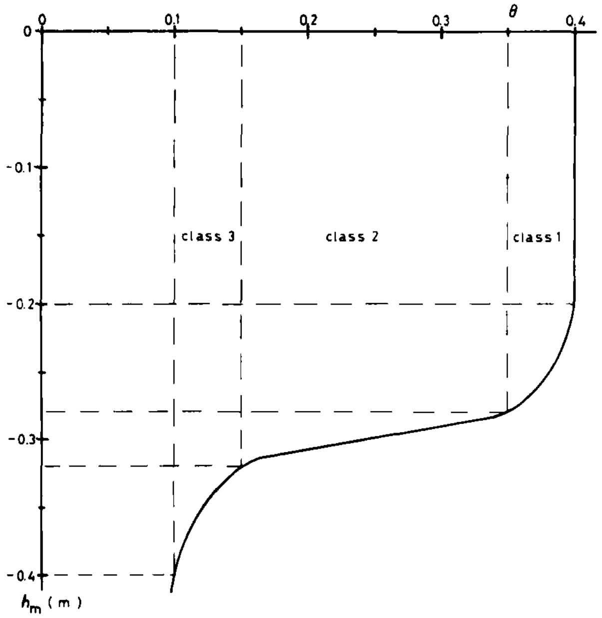


Figure 5.1 Soil water characteristic of sandy soil divided in three pore size classes.

The procedure used in Answer 5.1 gives only a very rough estimate of K, even for a sandy soil. In the literature one can find various procedures to calculate the hydraulic conductivity from the pore-size distribution, but they all lack accuracy.

Saturated hydraulic conductivities vary strongly between soils, due to their different pore-size distributions. Hydraulic conductivities in excess of 1 m d^{-1} are more than adequate for most agricultural purposes, whereas hydraulic conductivities less than 0.01 m d^{-1} may cause problems. Actual hydraulic conductivities can only be judged in relation with prevalent climatic conditions and the use of the soil.

5.2.2 Intrinsic permeability

Equation 5.7 contains the viscosity as the only parameter related to the fluid being transported. This suggests that

$$K_i = \eta K = \frac{1}{8\tau} \Sigma(\Delta\theta)_i r_i^2 \qquad (5.8)$$

is a parameter which depends only on the geometry of the porous medium.

Question 5.2 Check the units of Equation 5.8.

The dimension of m^2 for K_i indicates that it is indeed only a function of the geometric arrangement of the pores and that it is an intrinsic property of a stable porous medium. It is, therefore, called intrinsic permeability.

Equation 5.8 still contains the parameter θ, which is related to water as the permeating fluid. Its presence in Equation 5.8 was obscured in Answer 5.2 because it is dimensionless. The intrinsic permeability of a stable porous medium is the same for gas flow as for water flow and thus a more consistent formulation for K_i is:

$$K_i = \frac{1}{8\tau} \Sigma(\Delta\phi)_i r_i^2 \tag{5.9}$$

where $(\Delta\phi)_i$ is the contribution of all the pores with radius r_i to the porosity, ϕ.

Question 5.3 a. What is the intrinsic permeability of the soil in Question 5.1?
b. What is the intrinsic permeability of highly conducting and poorly conducting soils?

In slaking and swelling soils the pore size distribution varies with wetness. The ratio K_{ia}/K_{iw} can then be used as an index for the degree of slaking or swelling, where K_{ia} and K_{iw} are the intrinsic permeabilities calculated from a measurement with air and water, respectively.

5.2.3 Steady downward flow in homogeneous soil

The flux density equation for the flow of water in saturated soil, with the hydraulic potential expressed on weight basis, was given in Chapter 4:

$$q = -k\frac{\partial H}{\partial s} = -k\left(\frac{\partial h}{\partial s} + \frac{\partial z}{\partial s}\right) \tag{4.7}$$

This relationship is called Darcy's law, after the Frenchman who found this relationship experimentally in 1856 for the flow of water in saturated sand.

In a homogeneous, water-saturated soil k is constant with position and in time. For steady one-dimensional flow, as in the soil column in Figure 5.2, q is also constant in time and with height. From Equation 4.7 it follows then that $\partial H/\partial s$ is also constant with height, or that H varies linearly with height.

The lower end of the soil column in Figure 5.2 is supported by a screen, such that water can flow out freely under atmospheric pressure. This means that at that height, $h = 0$. If the reference level for the gravitational potential is chosen at the same height, then there also $z = 0$ and thus, $H = h + z = 0$.

At the free water surface above the soil column, $z = 0.9$ m and $h = 0$, thus $H = h + z = 0.9$ m. The water in the layer from $z = 0.8$ m to $z = 0.9$ m can be assumed hydrostatic. This means that in this layer H is constant. Thus at the soil surface, $H = 0.9$ m, $z = 0.8$ m and thus $h = 0.1$ m. Since within the soil H, and also h, vary linearly with height, their values are now determined as indicated in Figure 5.2.

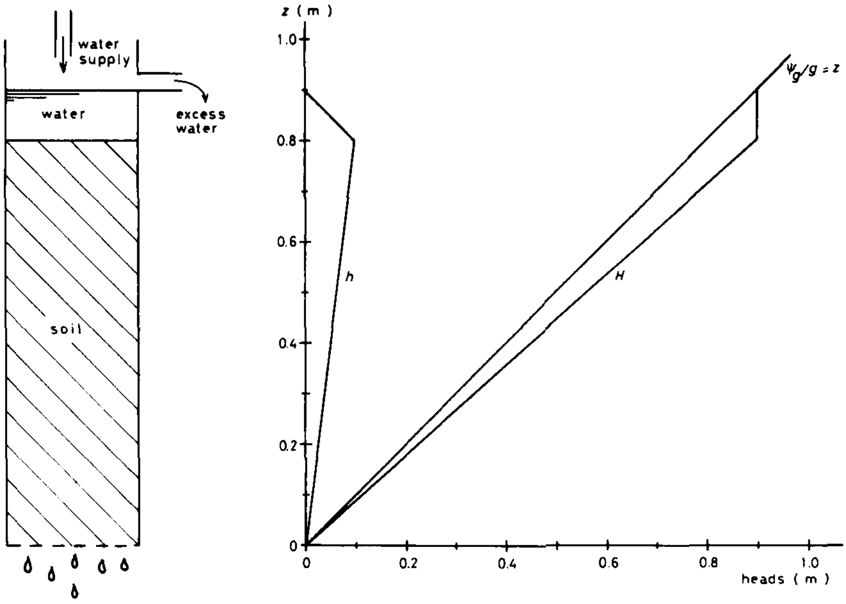


Figure 5.2 Steady saturated flow by inundation.

The system described above and depicted in Figure 5.2 is used to measure the saturated hydraulic conductivity of packed soil columns, as well as undisturbed soil cores taken from the field to the laboratory.

Question 5.4 Calculate the saturated hydraulic conductivity, k, of the soil in Figure 5.2 if the rate of discharge of water at the bottom of the column is 1.62 cm^3 per minute and the internal cross-sectional area of the cylinder is 20 cm^2.

Question 5.5 a. What happens if the cylinder in Figure 5.2 is perforated all around at a height $z = 0.5$ m?

b. What will be the value of h at that height soon after the cylinder is perforated?

c. Do you expect a continuous discharge of water from the perforations? Explain your answer.

d. Make a diagram of the potentials and calculate the flux densities in the upper and lower halves of the column. How much water (cm^3) will be discharged from the perforations in one hour?

5.2.4 Steady downward flow in layered soil

For saturated flow of water in a soil profile or soil column with layers of different hydraulic conductivity, the flux density is the same in all the layers.

Question 5.6 Why is this so?

Because the flux density is everywhere the same, the gradient of H is inversely proportional to the values of k in the different layers. From this H and h can be calculated.

This will be illustrated for the soil column in Figure 5.2 after the bottom 0.2 m is replaced by a layer of which k is one sixth of that in the overlying soil (Figure 5.3). With the subscripts referring to soil 1 and soil 2:

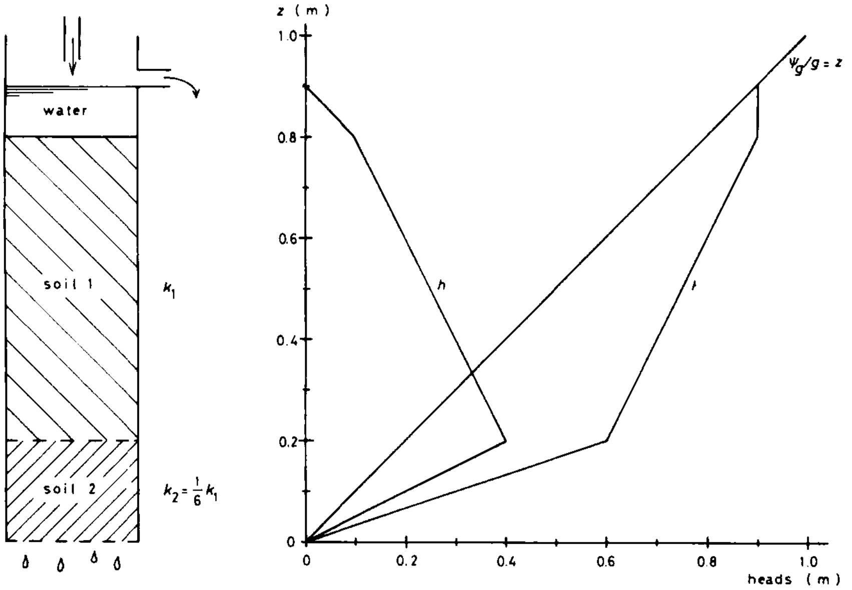


Figure 5.3 Steady saturated flow through two soil layers with different k values.

$$q_1 = -k_1 (\mathrm{d}H/\mathrm{d}z)_1 = -k_2 (\mathrm{d}H/\mathrm{d}z)_2 = q_2 \text{ or} \tag{5.10}$$

$$k_1 \frac{\Delta H_1}{0.6\,\mathrm{m}} = k_2 \frac{\Delta H_2}{0.2\,\mathrm{m}} \tag{5.11}$$

ΔH is the difference of H between the upper and lower boundaries of a layer. With $k_1 = 6k_2$, one then finds $\Delta H_1 = \frac{1}{2}\Delta H_2$. Because $H = 0.9$ m at the upper surface of the soil column and $H = 0$ at the lower surface, $\Delta H_1 + \Delta H_2 = 0.9$ m. It easily follows that $\Delta H_1 = 0.30$ m and $\Delta H_2 = 0.60$ m. The gradients of H in the two layers can now be calculated and the values of h can be found from the equation $h = H - z$.

Question 5.7 Check the potential diagram of Figure 5.3 and calculate the flux density when the hydraulic conductivity of soil 1 is that in Answer 5.4.

A situation similar to that in Figure 5.3 may occur in the field. Excessive implement traffic may compact the soil, while the top layer is kept loose by ploughing. In this way less permeable layers may develop, which hinder, either temporarily or permanently, the downward flow of soil water. When the water stagnates at the interface between the layers and develops a positive hydrostatic pressure, as in Figure 5.3, this causes a so-called perched water table. Since this is detrimental to normal root growth, deep tillage and subsoiling are often applied to break up the compacted layers.

Question 5.8 Assume that the situation in Figure 5.3 represents a flooded rice field. Calculate in mm and m^3 per hectare the daily supply required to maintain a constant water depth at the surface.

Question 5.9 Suppose an effort is made to intercept the water lost to the subsoil b installing closely spaced tile drains at the boundary between layers 1 and 2. (Th water discharged by the drainage system could be pumped to a neighbourin field for reuse.)

a. Draw a new potential diagram.
b. Calculate the flux density through layers 1 and 2.
c. Is the drainage system effective in reducing the 'percolation losses' through layer 2?

Quite a different situation arises when the less permeable layer is on top. This may be caused by loss of soil structure due to puddling or crust formation due to the impact of raindrops. It may also be due to the treading by animals or compaction by heavy machinery.

Question 5.10 The soil profile of a rice field consists of an upper layer of 0.10 m and a lower layer of 0.70 m. Below these layers is coarse gravel. The saturated hydraulic conductivity of the lower layer is 3.5 times that of the puddled layer. The water level in the field is maintained at 0.10 m above the soil surface.

a. Make a schematic drawing of the system and a potential diagram, assuming that the air-entry value is not exceeded.
b. Calculate the flux density to the gravel layer if the lower layer is the same soil as used in Figure 5.2.
c. Why is the assumption 'the air-entry value is not exceeded' necessary?

Figure 5.3 and Answer 5.10 indicate that in stratified soils the flow is mostly influenced by the layer with the lowest hydraulic conductivity.

5.2.5 Steady upward flow

Upward flow of water may occur in a soil profile due to transpiration by vegetation and/or evaporation from the soil surface. If the water is supplied by groundwater, this upward flow will lower the groundwater table, unless the groundwater is replenished by water from elsewhere. For instance, replenishment of groundwater may occur by seepage from higher surroundings, as found in many 'polders' in the Netherlands. Another typical example is seepage from irrigation canals or irrigated fields to the groundwater in neighbouring fields. If the seepage balances the evapotranspiration, the groundwater level remains stable. In the laboratory such a situation can be simulated by a system as depicted in Figure 5.4.

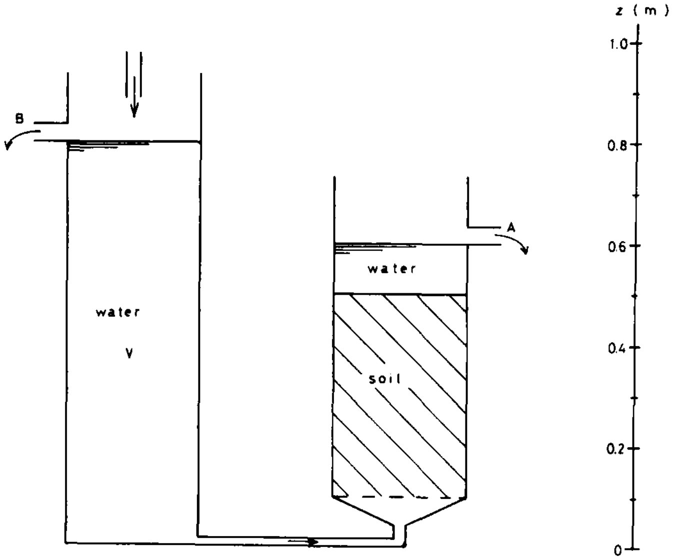


Figure 5.4 Simulation of steady saturated upward flow of water.

Question 5.11 Draw a potential diagram for the upward flow in the homogeneous soil column in Figure 5.4.

Question 5.12 The cross-sectional area of the soil column in Figure 5.4 is 100 cm^2 and the rate of discharge measured at A is 180 cm^3 per hour. Calculate the saturated hydraulic conductivity of the soil column.

5.2.6 Intergranular pressure

In Chapter 3 the relationships between the intergranular pressure, σ_g, the soil pressure, σ_s, and the water pressure, σ_w, were discussed for static equilibrium of the liquid phase. These relationships are illustrated once again in Figure 5.5, which shows the pressure diagram of a sand column with a layer of water on top, in a cylinder closed at the bottom (${}^b\rho_d = 1300$ kg m^{-3} and $\rho_s = 2600$ kg m^{-3}).

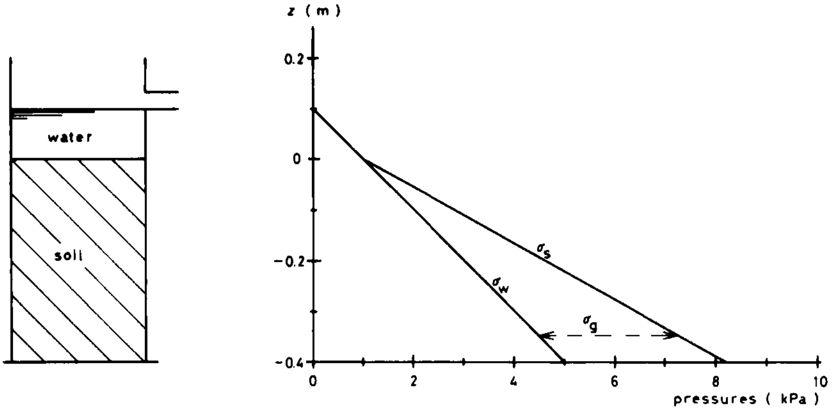


Figure 5.5 Pressures in a soil column at static equilibrium.

Question 5.13 a. Verify the pressure diagram in Figure 5.5.
b. What is the value of σ_g at the bottom?

If the bottom of the cylinder in Figure 5.5 is replaced by a screen and a steady downward saturated water flow is maintained, the pressure distribution will change.

Question 5.14 a. Draw a new pressure diagram for this situation.
b. Determine the intergranular pressure at the bottom of the soil column.

Question 5.15 Suppose the flow conditions in the soil column of Question 5.14 are changed to those in Figure 5.4.
a. Draw a diagram for σ_s and σ_w and determine the intergranular pressure at the bottom of the column.
b. Calculate the intergranular pressure in the soil column if the overflow B is raised to $z = 0.92$ m.

A comparison of Answers 5.14 and 5.15 indicates that the intergranular pressure decreases due to upward water transport. As a result, the bearing capacity of the soil, which is wholly dependent on the friction forces between the grains, is reduced. In extreme cases, i.e. where a strong upward water transport occurs, the intergranular pressure may completely disappear, resulting in quicksand. Such a situation of strong seepage may occur for instance next to dikes in a polder or in deep construction pits.

5.3 FLOW OF WATER IN UNSATURATED SOIL

5.3.1 Unsaturated hydraulic conductivity

In soil physics, soils of which the pore volume is only partially filled with water are called unsaturated. The hydraulic conductivity of unsaturated soils is smaller than that of saturated soils, because only the pores which contain water can contribute to the flow of water. Since the hydraulic conductivity of a pore is proportional to the square of its radius (Equation 5.7), the largest water-filled pores contribute most to that flow. When a saturated soil starts becoming unsaturated, the largest pores are emptied first. As a result, the unsaturated hydraulic conductivity decreases very fast with water content.

It has been found experimentally that Darcy's law (Equation 4.7) is also valid for unsaturated soils. Thus, the flux density for the flow of water in unsaturated soil is

$$q = -k(\theta)\frac{\partial H}{\partial s} = -K(\theta)\frac{\partial p_h}{\partial s} \tag{5.12}$$

where $k(\theta)$ and $K(\theta)$ indicate the functional dependence on the volume fraction of water, θ.

Figure 5.6 shows k as a function of θ for a medium fine sand and a loam. For both soils, k varies over many orders of magnitude. For instance, when in the sand θ decreases from $\theta = 0.34$ to $\theta = 0.07$, k changes from about $k = 1$ m d^{-1} to $k = 10^{-5}$ m d^{-1}.

The relationship between k and θ is not linear, but in many soils $\log k$ is approximately proportional to θ, as indicated in Figure 5.6 by the semi-logarithmic scale. Generally, the k(θ) function is not influenced by hysteresis, in contrast with the $k(h_m)$ function. Therefore, the $k(\theta)$ function is used most of the time.

Question 5.16 The hydraulic conductivity of loam (Figure 5.6B) is lower than that of sand (Figure 5.6A) for the same θ. What is the most probable cause?

Question 5.17 Find the hydraulic conductivity of loam when $h_m = -1.0$ m.

Question 5.18 Explain why a soil can have different k values for a given h_m.

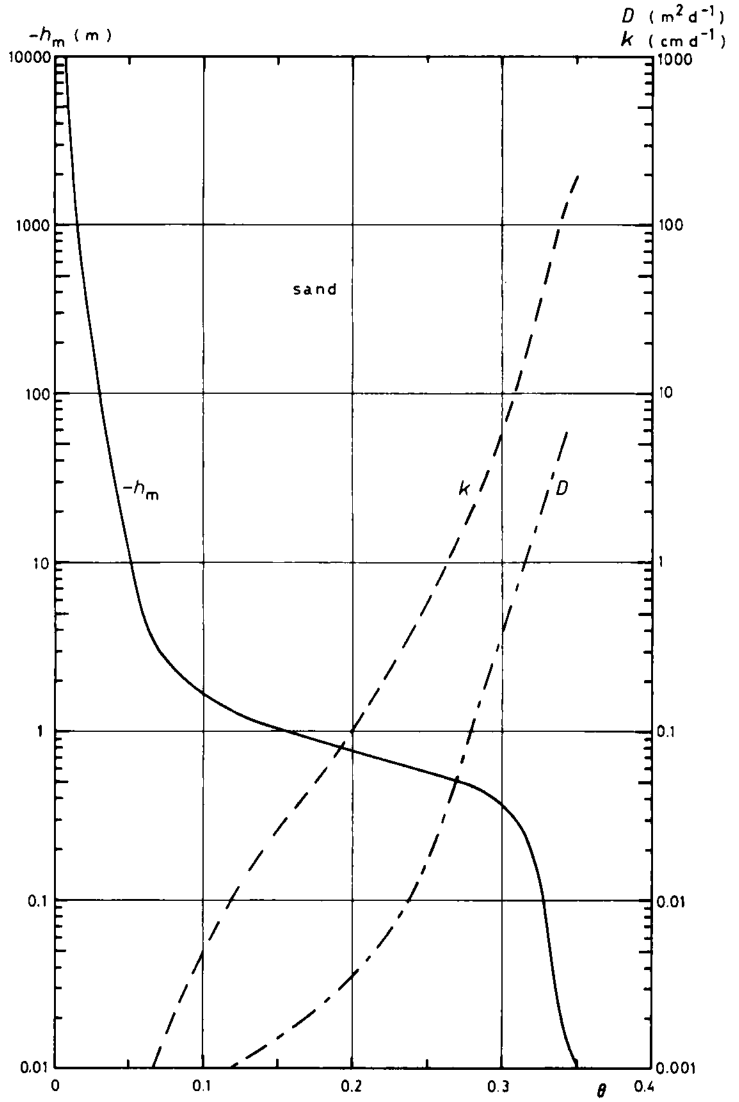


Figure 5.6A Soil water characteristic $h_m(\theta)$ (————), hydraulic conductivity $k(\theta)$ (— — — —) and hydraulic diffusivity $D(\theta)$ (— · — · —) for medium fine sand. (After Rijtema, P. E., 1969. Soil moisture forecasting. Report 513. ICW, Wageningen.)

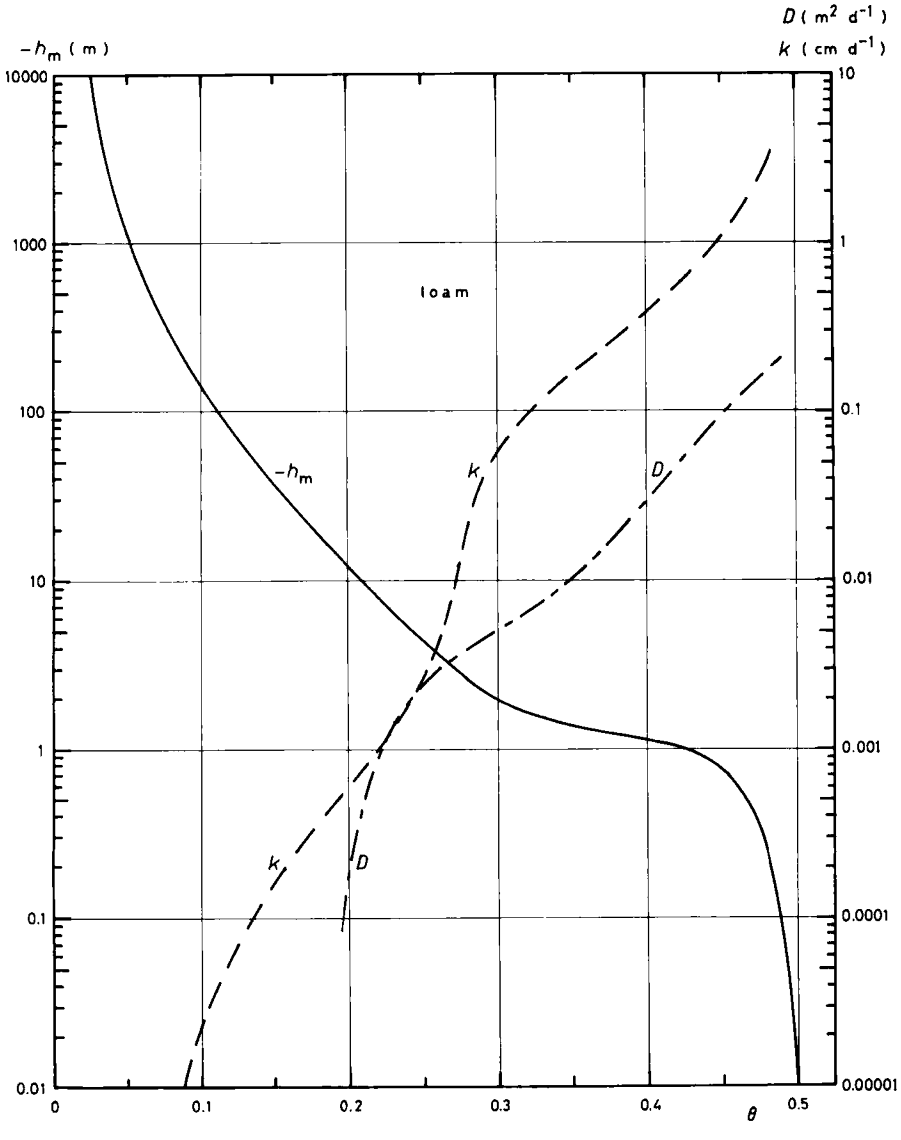


Figure 5.6B Soil water characteristic $h_m(\theta)$ (————), hydraulic conductivity $k(\theta)$ (— — — —) and hydraulic diffusivity $D(\theta)$ (— — — — -) for loam. (After Rijtema, P. E., 1969. Soil moisture forecasting. Report 513. ICW, Wageningen.)

Note (erratum): The function $D(\theta)$ is not correct (don't use it).

5.3.2 Steady flow of water in unsaturated soil

5.3.2.1 Steady upward flow

To study steady, upward flow of water in unsaturated soil, we will first consider a situation as in Figure 5.7. The water in the soil column is in static equilibrium with the simulated groundwater table, because a plastic cover prevents evaporation from the soil surface. Thus the water content profile shown is the same as the soil water characteristic of the soil for the particular range of h_m. Such an equilibrium situation may occur in winter and early spring in a field with a constant, shallow water table.

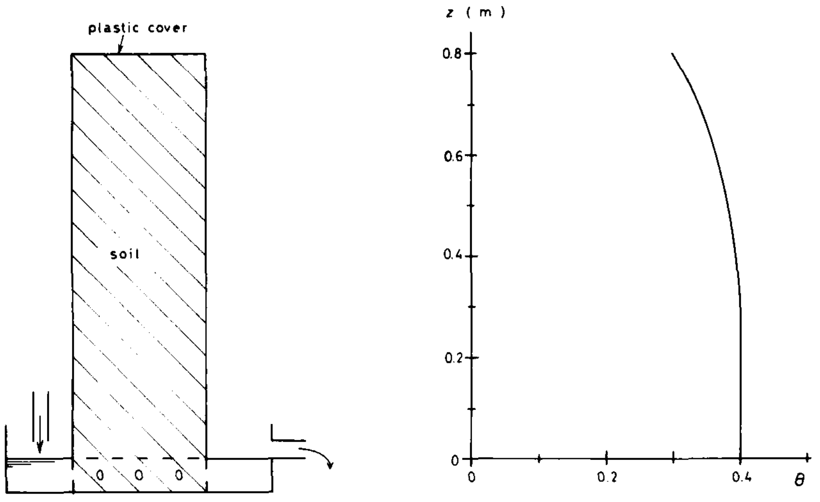


Figure 5.7 Static equilibrium with a groundwater table.

Question 5.19 Draw the potential diagram for Figure 5.7
Question 5.20 What is the air-entry value of the soil in Figure 5.7?

If the plastic cover on top of the soil column in Figure 5.7 is removed, evaporation will start (Figure 5.8). If the evaporation rate is constant and the groundwater table is maintained at its original level, steady upward flow will be established. This means that θ does not vary in time and thus, according to the continuity equation (Equation 4.9),

$$q = -k\frac{dH}{dz} = \text{constant and uniform} \qquad (5.13)$$

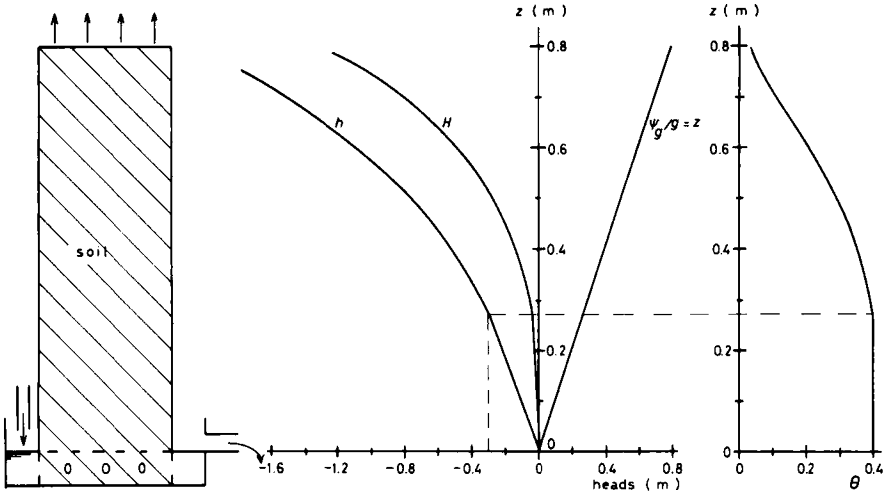


Figure 5.8 Steady upward flow of water, constant evaporation from a stable groundwater table.

For the flow to be directed upwards, H must decrease with height, making $\mathrm{d}H/\mathrm{d}z$ negative and q positive. Thus h, and with it θ, must decrease even more with height than in Answer 5.19. Since k decreases fast with θ (Figure 5.6), it decreases fast with height. To maintain the same flux density at all heights, a certain rate of decrease of k with height must be compensated by the same rate of increase with height of the absolute value of $\mathrm{d}H/\mathrm{d}z$. Thus the H curve in the potential diagram must be increasingly flatter towards the soil surface, as indicated in Figure 5.8.

The above situation is often found in the field during periods of drought, when a rather sharp transition from a dry upper layer to a moist lower zone is developed due to the sharp decrease of k with decreasing water content. The formation of such a sharp transition is the result of a self-accelerating mechanism, which can be explained as follows. When evaporation starts, θ as well as h and H decrease near the surface. But, because k decreases too, the new H gradient is too small to maintain the upward flow needed to satisfy the evaporative demand. Hence, θ decreases still further, causing a further decrease in H, but again not sufficient to offset the accompanying decrease in k, etc. This process continues until the surface layer has become so dry that practically all flow of water ceases. The formation of this dry surface layer is of practical importance because it protects the soil against large evaporation losses. This so-called soil mulch is the principle upon which dry-land farming is practised.

The exact values of H and h in the steady situation represented in Figure 5.8 are difficult to calculate, and then only when the $k(\theta)$ and $h_m(\theta)$ functions are known accurately. The gradients in the lower, water-saturated part of the profile can be calculated easily, provided the saturated k, the evaporation rate and the air-entry value are known.

Question 5.21 Is the relationship of H and h with z curvilinear in the water-saturated part of the profile in Figure 5.8?

Question 5.22 Estimate the evaporation rate in Figure 5.8, assuming a steady state situation, a saturated k value of 0.05 m d^{-1} and $h = -0.22$ m at $z = 0.20$ m.

Question 5.23 a. Determine the height of the saturated zone for the situation in Question 5.22.

b. Does this height remain constant if the evaporation rate changes?

5.3.2.2 Steady downward flow

Figure 5.9 shows the potential diagram for a field situation similar to Figure 5.2. The water at the surface is applied by sprinklers and the groundwater table is stabilized by tile drains. The sprinkling rate is such that, without runoff, the layer of water on the soil surface remains constant.

Question 5.24 Calculate the flux density in Figure 5.9 if k = 10 cm d^{-1} and the water layer is 10 cm deep.

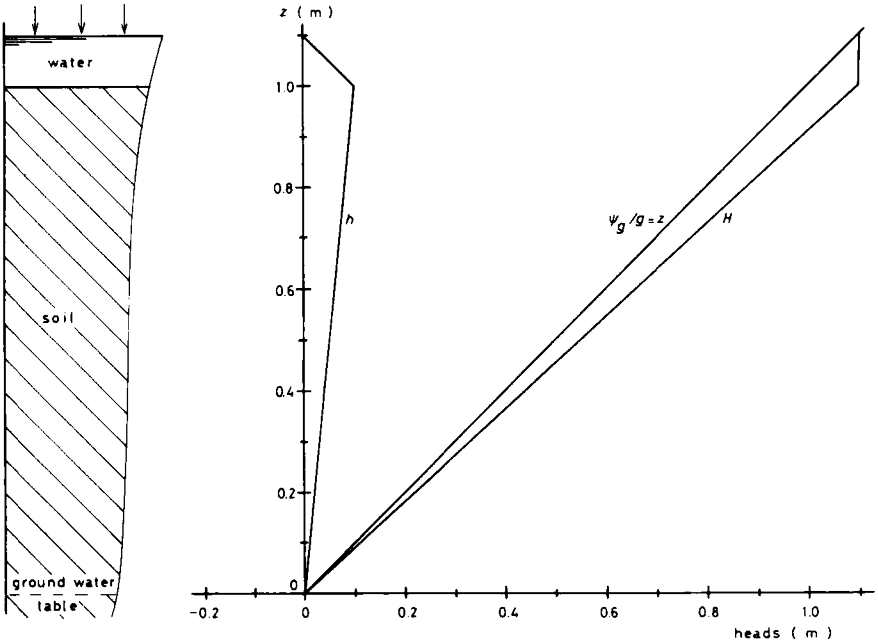


Figure 5.9 Steady downward flow of water at a high sprinkling rate (flooded soil).

If the sprinkling rate is decreased to another constant value, a new steady state situation arises, where the downward flux density still equals the sprinkling rate. This new situation is accompanied by a decrease in the *H* gradient, i.e. both *H* and *h* at the soil surface decrease. If the sprinkling rate falls below a certain critical value, the air-entry value of the soil will be exceeded and an unsaturated zone will develop in the top of the soil profile. Since *k* decreases with θ in the unsaturated zone and *q* is constant, the *H* gradient must increase with height, as shown in Figure 5.10. Consequently, the *h* gradient must also increase with height, which means that the *h* curve becomes steeper towards the soil surface.

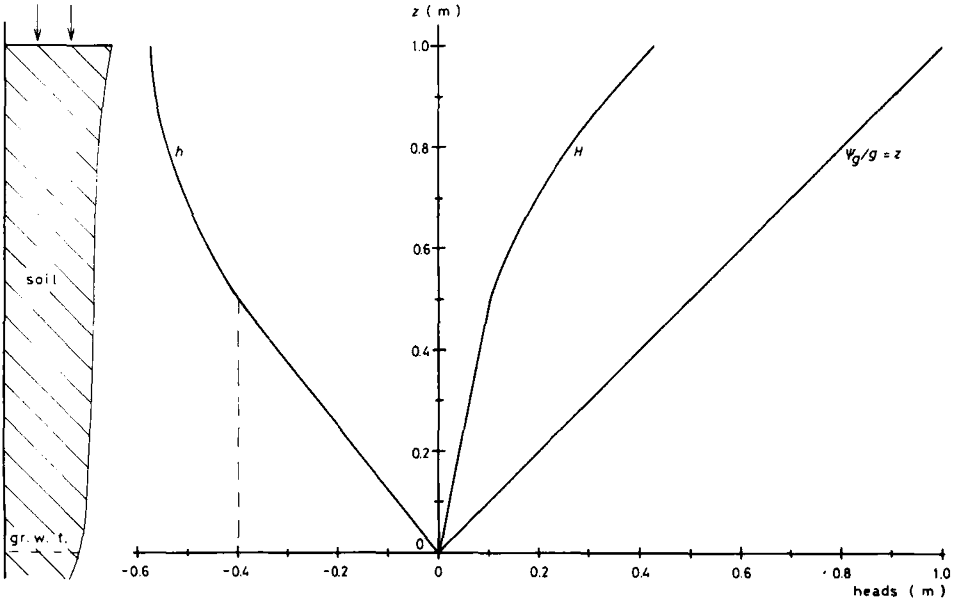


Figure 5.10 Steady downward flow of water at a low sprinkling rate.

Question 5.25 Calculate the sprinkling rate in the steady state situation of Figure 5.9, if the depth of the water layer at the surface is negligible.

Question 5.26. Under what conditions are *H* and *h* linear functions of *z* throughout the profile?

Question 5.27 Assuming that the air-entry value is not exceeded, draw the diagram for the steady state situation with a sprinkling rate of 8 cm d^{-1}.

Question 5.28 Calculate the height of the saturated zone above the groundwater table in the steady state situation if the sprinkling rate is 2 cm d^{-1} and the air-entry value is -0.40 m.

Question 5.29 What is the highest possible *H* gradient that can be attained in a steady state downward flow as shown in Figure 5.10?

Question 5.30 a. Draw as logical as possible the potential diagram for steady downward flow with a sprinkling rate of 1 cm d^{-1}.

b. Why is the phrase 'as logical as possible' needed in a?

Figure 5.11 presents a summary of the h profiles for the various cases of steady vertical flow of water discussed in this Section *5.3.2*. The numbers next to the curves are the values of the flux density. Upward flux densities ($q > 0$) represent evaporation and downward flux densities ($q < 0$) represent drainage. Zero flux density ($q = 0$) is the special case of hydrostatic equilibrium.

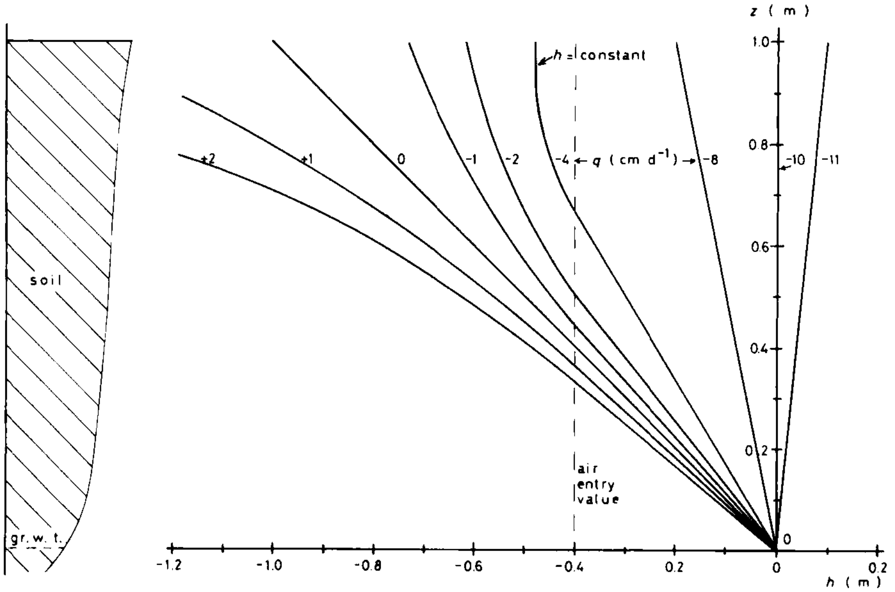


Figure 5.11 Summary of h profiles for steady vertical flow of water with a stable groundwater table.

5.3.3 Nonsteady flow of water in unsaturated soil

5.3.3.1 Analysis of H profiles

In the field, water transport seldom is steady, because precipitation, evapotranspiration, irrigation, drainage, etc. change continually. Transport processes which change with time, are called nonsteady. One can obtain insight into nonsteady water flow processes in the field, qualitatively, by obtaining tensiometer data at different times and depths in the soil profile. With tensiometers one obtains information on the pressure head h, as well as the hydraulic head $H = h + z$ (Section *3.1.4*). From the pressure head at different times one can derive whether the volume fraction of water is increasing or decreasing. From the gradient of H one can deduce in which direction the water is moving, and possibly predict values of θ and directions of flow at future dates.

Question 5.31 Which is the direction of flow if $\partial H/\partial z$ is positive, negative or zero?

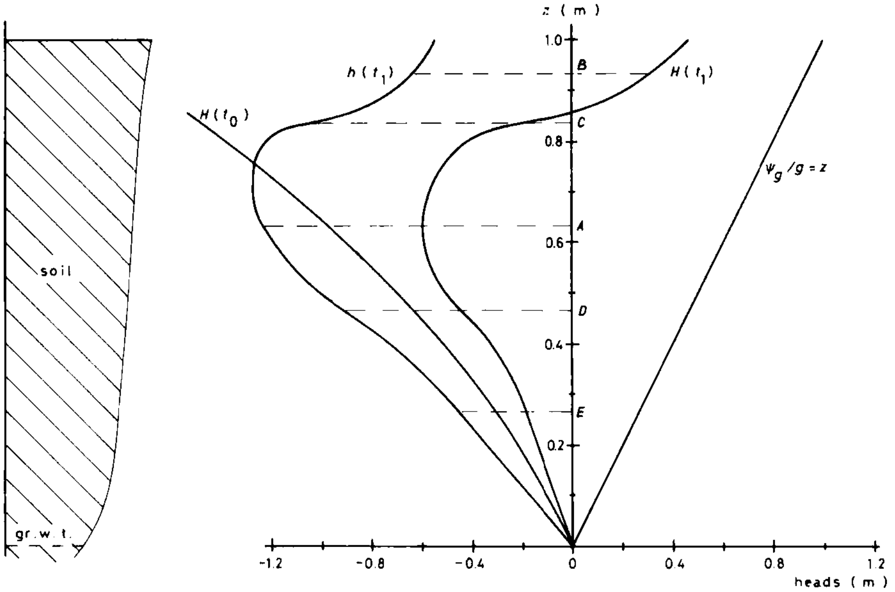


Figure 5.12 H and h profiles in a soil during steady evaporation (t_0) and some time after it started raining (t_1).

Figure 5.12 shows H profiles at two different times. Initially at t_0, there is steady evaporation for which $|\partial H/\partial z|$ increases with height, since θ and k decrease with height (Figure 5.8, with discussion). If then it starts to rain and water infiltrates into the soil profile, the H profile may after some time have changed to that indicated by t_1. From this profile one can tell in which direction the water is moving, and whether $\partial\theta/\partial t > 0$ or $\partial\theta/\partial t < 0$.

Question 5.32 Is there a water layer on top of the soil at time t_1?

Question 5.33 Indicate the depths where there is upward, downward or no movement of water at time t_1.

Question 5.34 By using the continuity equation explain, whether at height A in Figure 5.12, θ decreases, increases or remains constant after t_1.

For height A it is easy to tell whether θ will increase or not, just by looking at the H profile (Answer 5.34). For other heights it is not so easy to predict the changes in θ. Take for instance heights B and C in Figure 5.12. At both heights the flux density is downward.

Since C is the inflection point of the H profile above height A, the H gradient at B is smaller than at C. However, θ is greater at B than at C, as can be interpreted from the h profile. This means that k is greater at B than at C. Therefore, without knowing the exact numerical values of k and $\partial H/\partial z$, it cannot be concluded at which height the greater flux density occurs. This reasoning is true for every height above C. Thus, it is not possible to conclude whether above C, θ will increase or decrease, or even remain constant.

The same problem arises in the interval between heights D and E, where D is the inflection point of the H profile below A, and E is the height where the H profile becomes linear (check this).

In contrast, between A and D, the absolute value of $\partial H/\partial z$ as well as k decrease with height, i.e. $\partial q/\partial z < 0$. Hence, $\partial\theta/\partial t$ is positive at every height in this interval. A similar reasoning can be given for the interval between A and C. Also in this region $\partial q/\partial z$ is negative (check this). Therefore, $\partial\theta/\partial t$ will be positive for every height between C and D.

Question 5.35 Suppose the weather becomes foggy for an extended period of time directly after the rain has stopped.

a. What will be the ultimate shape of the H profile?
b. Compare θ at different heights in this final stage with those at time t_1.

By the preceding reasoning one can acquire only a qualitative understanding of the water flow processes in soil. Since in unsaturated soil k varies with θ, it is not possible to calculate q as a function of height directly from H profiles as shown in Figure 5.12. For this it is necessary to know, in addition, $k(\theta)$ and $\theta(h_m)$ functions such as in Figure 5.6. The procedure that then should be followed to estimate $\partial\theta/\partial t$ is outlined in Figure 5.13. In this figure, the subscript z indicates height, s stands for a small increment in height and $\bar{\theta}$ is the average volume fraction of water.

Question 5.36 Describe in words the steps followed in Figure 5.13 in order to estimate $\partial\theta/\partial t$ at a height z in a soil profile.

The procedure outlined in Figure 5.13 makes use of average values of θ, and not of hydraulic conductivity or pressure head. This amounts to assuming that θ changes linearly with height. This assumption is usually not far from reality and introduces small errors in $\partial\theta/\partial t$. An alternative approach would be to average the k values for each of the pertinent θ values. This amounts to assuming that k changes linearly with θ, which obviously introduces much more serious errors in $\partial\theta/\partial t$ because of the strong nonlinear $k(\theta)$ function (Figure 5.6). An intermediate approach would be to use average h values.

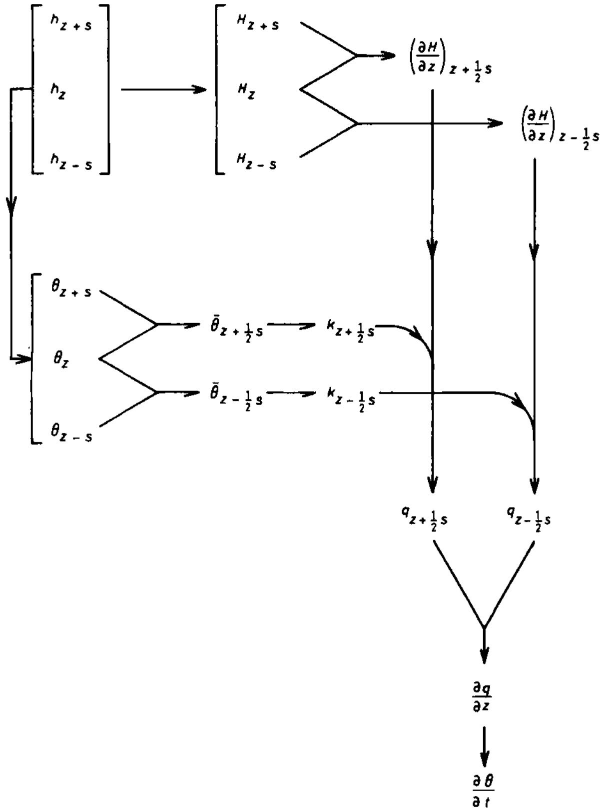


Figure 5.13 Calculation scheme for analysing a θ profile.

Question 5.37 In a medium fine sandy soil, for which the $\theta(h_m)$ and $k(\theta)$ functions are given in Figure 5.6, three tensiometers are installed at depths of 0.30, 0.40 and 0.50 m below the surface. At a certain moment the tensiometers indicate h values of -2.00, -0.80 and -0.40 m, respectively. Estimate $\partial\theta/\partial t$ at a depth of 0.40 m using the scheme given in Figure 5.13. Hysteresis may be neglected.

Question 5.38 a. Also estimate $\partial\theta/\partial t$ at a depth of 0.40 m using average h values. Do the same using average k values.

b. Compare the values for k and $\partial\theta/\partial t$ obtained by the three calculation schemes.

Question 5.39 In a loamy soil profile nine tensiometers are installed at depths of 0, 0.10, 0.20, ... and 0.80 m below the soil surface. Estimate $\partial\theta/\partial t$ at depths 0.10, ... and 0.70 m for the moment that the tensiometers indicate h values of 0, -0.30, -0.80, -1.10, -1.60, -1.00, -0.50, -0.20 and 0 m, respectively. Use the $\theta(h_m)$ and $k(\theta)$ functions in Figure 5.6. Hysteresis may be neglected.

5.3.3.2 General description of infiltration

A typical example of nonsteady unsaturated flow of water is the infiltration of water into soils. When water is applied at the soil surface (e.g. under flood irrigation or inundation), it enters the soil profile and changes the water content distribution with depth. After irrigation has continued for some time, the following zones can usually be distinguished in the water content profile (Figure 5.14):

- saturated zone
- transition zone
- transmission zone
- wetting zone
- wetting front.

The saturated zone is a thin zone at the soil surface. The transition zone is a zone of decreasing water content between the saturated zone and the nearly saturated transmission zone. Its lower end may reach from a few millimeters to a few centimeters below the surface. These upper two zones are not always clearly distinguishable, especially in the laboratory. They are caused by structural changes at the soil surface and by entrapped air.

The transmission zone is the conveyance zone for the infiltrating water. While all the other zones remain nearly constant in thickness, this zone continues to elongate as long as water is supplied at the soil surface. Its water content, though slightly changing with depth, is rather constant and close to saturation.

The wetting zone is the normally thin zone where the water content changes from its initial value to the value of the transmission zone.

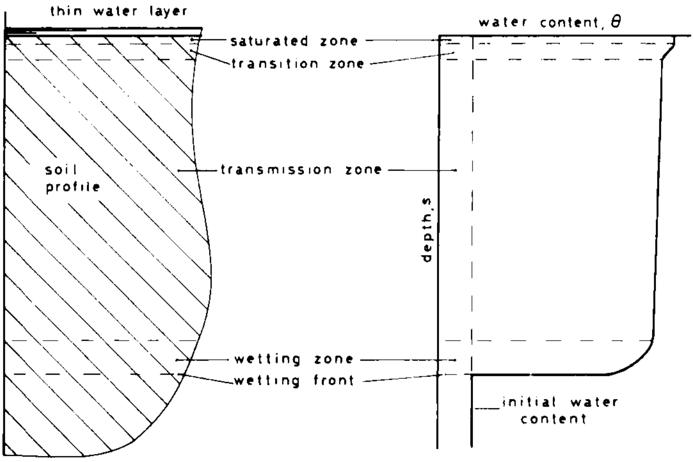


Figure 5.14 Zones of water content profile some time after start of flood irrigation.

The wetting front is the visible limit of water penetration, where the gradient of the pressure head is very large.

If the water, initially present in the soil profile, is assumed at rest, the cumulative infiltration, I, which is the volume of water that has infiltrated into the soil divided by the surface area, is

$$I = \int_{s=0}^{\infty} (\theta - \theta_i)\, \mathrm{d}s \tag{5.14}$$

where s is distance in the direction of flow (here depth in the soil profile), θ is the volume fraction of water at distance s and θ_i is the initial uniform value of θ. The cumulative infiltration is, of course, an increasing function of time.

Question 5.40 Derive Equation 5.14.

The driving forces for the water entering the soil are the gradient of the pressure head between the wetting front and the soil surface, and gravity (Equation 4.7). While the latter is constant, the gradient of the pressure head decreases with time, because of the advancing wetting front. As a result, the flux density through the soil surface, also called the infiltration rate, i, decreases monotonically with time and approaches asymptotically a constant value, as gravity becomes the main driving force (Figure 5.15).

The infiltration rate may be expressed as:

$$i = \frac{\mathrm{d}I}{\mathrm{d}t} \tag{5.15}$$

Question 5.41 Check Equation 5.15.

Integration of Equation 5.15 gives the cumulative infiltration as a function of time:

$$I = \int_{0}^{t} i\mathrm{d}t \tag{5.16}$$

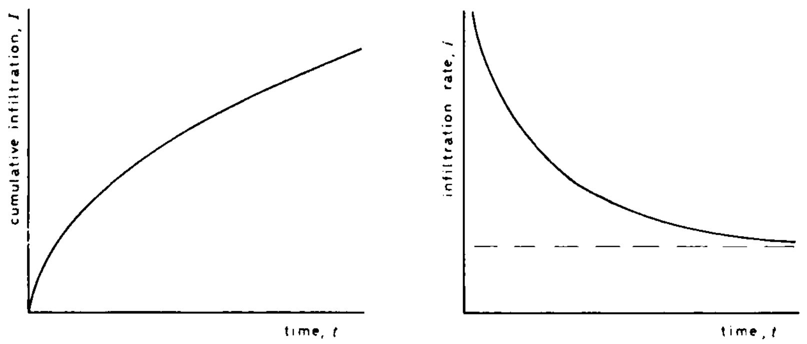


Figure 5.15 Cumulative infiltration, I, and infiltration rate, i, as function of time.

Question 5.42 Derive Equation 5.16 from Equation 5.15.

The cumulative infiltration as a function of time may be measured in the field, but such an experiment does not give any information about the water content distribution or the depth of the wetting front. To obtain the volume fraction of water as a function of depth and time, $\theta(s,t)$, a general flow equation such as

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial s}\left(D\frac{\partial\theta}{\partial s}\right)+\frac{\partial}{\partial s}\left(k\frac{\partial z}{\partial s}\right) \tag{4.23}$$

must be solved, using appropriate initial and boundary conditions.

Equation 4.23 is a nonlinear partial differential equation of order two. It is nonlinear, because both D and k are functions of θ (Figure 5.6). Due to this nonlinearity general analytical solutions of Equation 4.23 are not available.

Water flow problems can be solved numerically with computers, using calculation schemes of the type shown in Figure 5.13 and Answers 5.37, 5.38 and 5.39. Unfortunately, the large computing times required for obtaining solutions for the practical time spans often make it prohibitively expensive. One must then resort to simplifications which make the problem solvable, while retaining the essential characteristics of the flow process. Examples of this will be discussed in the remainder of this chapter.

5.3.3.3 Horizontal infiltration. Boltzmann transformation

For the special case of infiltration from a thin layer of water into a homogeneous soil with uniform initial volume fraction of water, the initial and boundary conditions are:

$$\begin{aligned}&\theta=\theta_i \text{ for } s>0 \text{ and } t=0\\ &\theta=\theta_i \text{ for } s\rightarrow\infty \text{ and } t\geqslant 0\\ &\theta=\theta_0 \text{ for } s=0 \text{ and } t\geqslant 0\end{aligned} \tag{5.17}$$

where θ_0 is the constant (saturated) volume fraction of water at the soil surface ($s = 0$).

In the early stages of vertical infiltration the term $k\,\partial z/\partial s$ in Equation 4.23 is much smaller than the term $D\,\partial\theta/\partial s$ and thus the driving force due to gravity can be neglected. In the case of infiltration into a horizontal soil column in the laboratory, often called absorption, the term $k\,\partial z/\partial s$ is zero at all times, because $\partial z/\partial s$ is zero. For both cases, Equation 4.23 reduces to:

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial s}\left(D\frac{\partial\theta}{\partial s}\right) \tag{5.18}$$

Equation 5.18 may be transformed into an ordinary differential equation by introducing the variable λ defined as:

$$\lambda=st^{-1/2} \tag{5.19}$$

This is known as the Boltzmann transformation. Equation 5.18 then becomes (Appendix 2):

$$-\frac{\lambda}{2}\frac{d\theta}{d\lambda}=\frac{d}{d\lambda}\left(D\frac{d\theta}{d\lambda}\right) \tag{5.20}$$

The boundary conditions 5.17 can also be transformed:

$$\begin{aligned}&\theta=\theta_i \text{ for } \lambda\rightarrow\infty\\&\theta=\theta_0 \text{ for } \lambda=0\end{aligned} \tag{5.21}$$

At this point the absorption problem is expressed entirely in terms of θ and λ. Therefore, one may expect that the solution will be a unique function $\theta(\lambda)$, which for each soil is determined by its particular $D(\theta)$ function and the values of θ_i and θ_0.

Equations 5.20 and 5.21 can be solved analytically if D is assumed constant. This solution is given in detail in Appendix 2. The result is:

$$\theta=\theta_i+(\theta_0-\theta_i)\,\mathrm{erfc}\left(\frac{\lambda}{2\sqrt{D}}\right) \tag{5.22}$$

where erfc stands for complementary error function. A plot of this function is given in Figure 5.16.

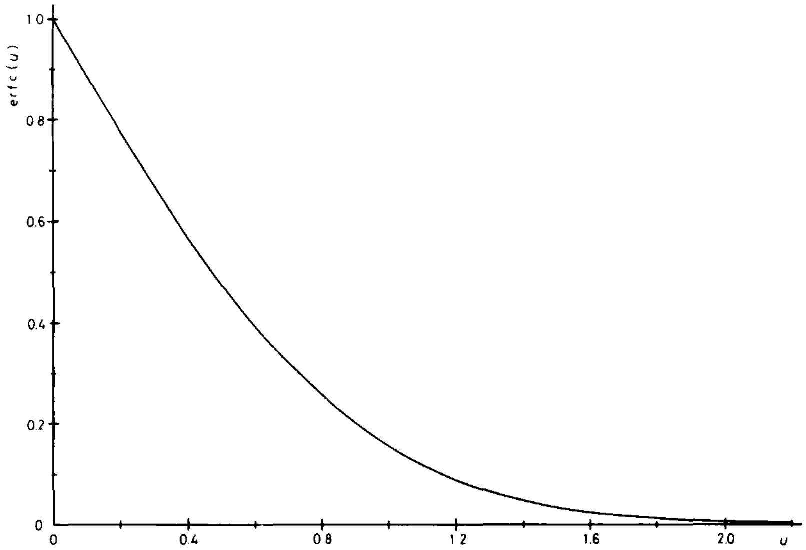


Figure 5.16 The complementary error function erfc (u) for $u \geq 0$.

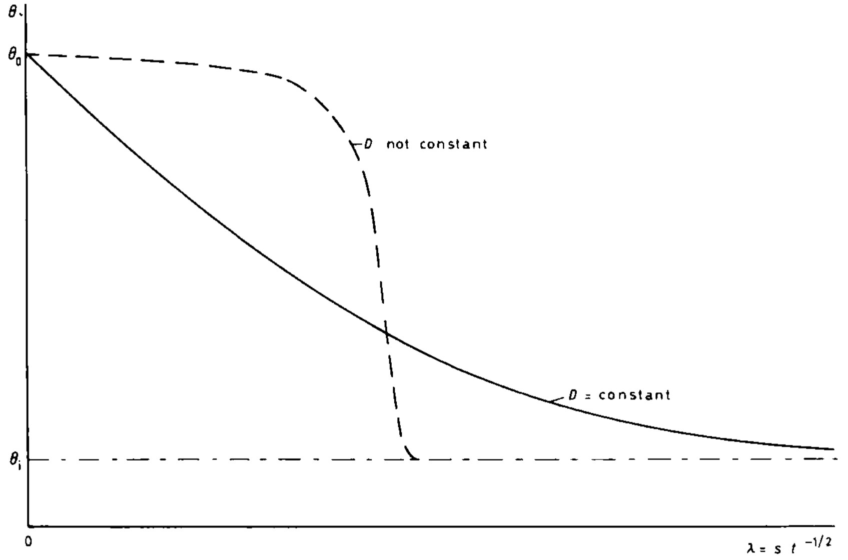


Figure 5.17 Solutions of Equation 5.18 for D = constant and for D of a soil (D increasing with increasing θ).

For a chosen set of values of θ_i, θ_0 and D, Equation 5.22 is plotted in Figure 5.17. When this graph is compared with Figure 5.14, it is obvious that this solution, a gradually decreasing volume fraction of water with distance, is far from reality. The reason for this discrepancy is that D is not constant over the range of θ, but varies with a factor of up to about 10^5.

Equations 5.20 and 5.21 cannot be solved analytically for variable D. However, various techniques can be found in the literature for solving Equations 5.20 and 5.21 by numerical methods. Discussion of these techniques goes beyond the scope of this book. The result of such calculations, for the same values of θ_i and θ_0 but a realistic $D(\theta)$ function, is also plotted in Figure 5.17. This figure shows, as expected, a unique relationship between θ and λ. This implies that $s/\sqrt{t}$ ($= \lambda$) is constant for a given θ.

Question 5.43 Which relationship do you expect between s and t for the visible wetting front and for any other value of θ between θ_i and θ_0?

Combination of Equations 5.14 and 5.19 gives:

$$I = \int_{s=0}^{\infty} (\theta-\theta_i)\mathrm{d}s = \int_{\lambda=0}^{\infty} (\theta-\theta_i)\mathrm{d}\lambda t^{1/2} = t^{1/2} \int_{\lambda=0}^{\infty} (\theta-\theta_i)\mathrm{d}\lambda \qquad (5.23)$$

Introduction of the parameter

$$S = \int_{\lambda=0}^{\infty} (\theta - \theta_i)\mathrm{d}\lambda \tag{5.24}$$

leads to:

$$I = St^{1/2} \tag{5.25}$$

and, consequently,

$$i = \frac{\mathrm{d}I}{\mathrm{d}t} = \tfrac{1}{2}St^{-1/2} = \frac{I}{2t} \tag{5.26}$$

S is called the sorptivity and is a measure for the capacity of a soil to absorb water. It is the cumulative infiltration during the first unit of time (Equation 5.25).

Question 5.44 Explain how S can be found from the $\theta(\lambda)$ curve (for variable D) given in Figure 5.17.

Question 5.45 Show that S can also be given by: $S = \int_{\theta_i}^{\theta_0} \lambda \mathrm{d}\theta$

Question 5.46 Would you expect S to be constant for a given soil?

For initially dry soils ($\theta_i = 0$) the sorptivity varies from about 5×10^{-5} m s$^{-1/2}$ (≈ 0.04 cm min$^{-1/2}$) for a heavy clay soil to about 2×10^{-3} m s$^{-1/2}$ (≈ 1.5 cm min$^{-1/2}$) for a coarse sand.

Question 5.47 a. Calculate the cumulative horizontal infiltration in an initially dry, heavy clay soil during 24 hours.
b. Do the same for a coarse sand.

5.3.3.4 Vertical infiltration

So far, the infiltration process was described only for cases in which the gravitational potential can be neglected. During vertical infiltration, the influence of gravity becomes more important as time progresses. Then Equation 4.23, together with the initial and boundary conditions of Equation 5.17, must be solved. A number of different techniques for solving Equation 4.23 are available. A solution obtained by one of these techniques is

$$s(\theta,t) = a_1t^{1/2} + a_2t + a_3t^{3/2} + a_4t^2 + \ldots \tag{5.27}$$

where a_1, a_2, a_3, a_4, ... are all functions of θ. By means of Equation 5.27, the depth of a given value of θ at time t can be evaluated when a_1, a_2, etc. are known. These coefficients can be evaluated by numerical methods using the $D(\theta)$ and $k(\theta)$ functions.

The cumulative infiltration obtained by the above technique is given by:

$$I = St^{1/2} + At + Bt^{3/2} + Ct^2 + \ldots \tag{5.28}$$

The coefficients of Equation 5.28 are evaluated in a similar way as those of Equation 5.27.

The infiltration rate can be derived from Equation 5.28 by differentiating with respect to t (Equation 5.15):

$$i = \tfrac{1}{2}St^{-1/2} + A + \tfrac{3}{2}Bt^{1/2} + 2Ct + \ldots \qquad (5.29)$$

Equations 5.28 and 5.29 usually are truncated after the first two terms, because for not too large t these series converge rapidly. In that case Equations 5.28 and 5.29 become, respectively,

$$I = St^{1/2} + At \qquad (5.30)$$

and

$$i = \tfrac{1}{2}St^{-1/2} + A \qquad (5.31)$$

S is again the sorptivity defined in Equation 5.24. It is the dominant parameter in the early stage of infiltration. As time progresses, the first term becomes negligible and the importance of A, which represents the main part of the gravitational influence, increases.

Question 5.48 a. Make a graph of the cumulative infiltration, I, and the infiltration rate, i, as a function of time for vertical infiltration during 4 hours in a fine sandy loam for which:

$S = 7.0 \times 10^{-4}\ \mathrm{m\ s^{-1/2}}$
$A = 1.0 \times 10^{-6}\ \mathrm{m\ s^{-1}}$
$B = 1.0 \times 10^{-9}\ \mathrm{m\ s^{-3/2}}$

Use Equations 5.28 and 5.29 (first three terms).

b. Give in the same figure the results when Equations 5.30 and 5.31 are used.

c. Compare the results.

5.3.3.5 Green and Ampt model

A simple but useful model for the infiltration process was initially (in 1911) proposed by Green and Ampt. The advantage of this model is that it leads to equations which can be solved analytically, both for horizontal and vertical infiltration. It is based on the following assumptions (Figure 5.18):

- throughout the wetted zone, the volume fraction of water, θ_t, is uniform and constant with time
- the change of θ_i to θ_t at the wetting front takes place in a layer of negligible thickness
- the pressure head at the wetting front, h_f, has a constant value, independent of the position of the wetting front, s_f.

These assumptions are quite realistic for infiltration into coarse-textured soils with low initial water content, since wetting fronts are, generally, very sharp under those

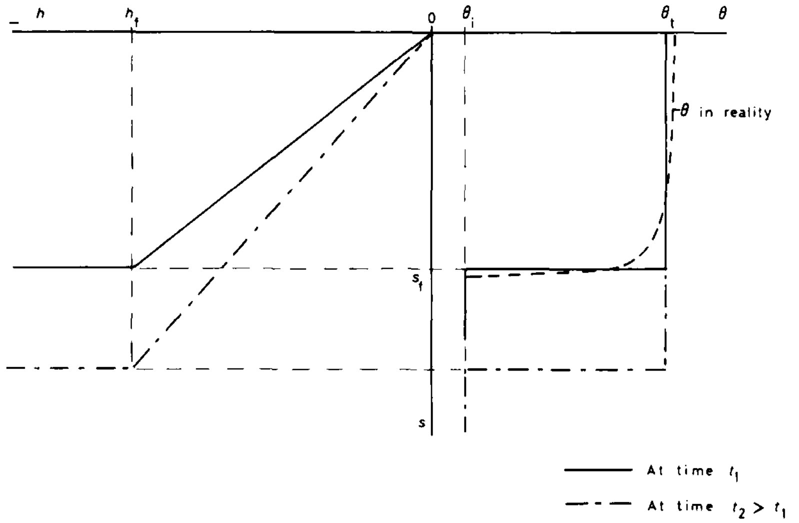


Figure 5.18 Green and Ampt model of infiltration. θ and h profiles at two different times.

conditions.

From the first assumption it follows that throughout the transmission zone the hydraulic conductivity, k_t, has a constant value. Besides, the flux density is the same everywhere in the transmission zone.

Question 5.49 Explain how these conclusions can be made.

Assuming negligible thickness of the water layer on the soil surface, $h = 0$ at $s = 0$, and using the second and third assumption, Darcy's law for this case can be written as:

$$i = q = -k\frac{\partial H}{\partial s} = -k\left(\frac{\partial h}{\partial s} + \frac{\partial z}{\partial s}\right) = -k_t\left(\frac{h_f}{s_f} - 1\right) \tag{5.32}$$

Question 5.50 Explain why one obtains the correct flux density, even though in the above equation h_f/s_f is used instead of $\partial h/\partial s$.

Question 5.51 Is the flux density constant with time?

The cumulative infiltration is:

$$I = \int_{s=0}^{\infty} (\theta_t - \theta_i)\mathrm{d}s = (\theta_t - \theta_i)\, s_f \tag{5.33}$$

and thus:

$$i = \frac{\mathrm{d}I}{\mathrm{d}t} = (\theta_t - \theta_i)\frac{\mathrm{d}s_f}{\mathrm{d}t} \tag{5.34}$$

Equations 5.32 and 5.34 yield:

$$k_t\left(1 - \frac{h_f}{s_f}\right) = (\theta_t - \theta_i)\frac{\mathrm{d}s_f}{\mathrm{d}t} \tag{5.35}$$

When gravity can be neglected (early stage of infiltration or horizontal infiltration), the term $\partial z/\partial s$ in Equation 5.32 vanishes and Equation 5.35 becomes:

$$-k_t\frac{h_f}{s_f} = (\theta_t - \theta_i)\frac{\mathrm{d}s_f}{\mathrm{d}t} \tag{5.36}$$

Rearranging and integrating Equation 5.36 obtains

$$-k_t h_f t = \tfrac{1}{2}(\theta_t - \theta_i)s_f^2 + C \tag{5.37}$$

Since $s_f = 0$ at $t = 0$, the integration constant C is zero, and thus Equation 5.37 can be rearranged to

$$s_f = \left(\frac{-2k_t h_f}{\theta_t - \theta_i}\right)^{1/2} t^{1/2} \tag{5.38}$$

Because k_t, θ_t, θ_i and h_f remain constant during the flow process, Equation 5.38 can be read as $s_f/\sqrt{t} =$ constant, i.e. the depth of the wetting front is proportional with the square root of time (verify that this is the same conclusion as in Question and Answer 5.43).

Combination of Equations 5.33 and 5.38 leads to

$$I = [-2k_t h_f(\theta_t - \theta_i)]^{1/2} t^{1/2} = St^{1/2} \tag{5.39}$$

where the sorptivity is now

$$S = [-2k_t h_f(\theta_t - \theta_i)]^{1/2} \tag{5.40}$$

Question 5.52 a. How can there be a minus sign within the brackets of Equation 5.40, when the square root of a negative number is undefined?

b. Check that S remains constant during the infiltration process.

Question 5.53 Two horizontal columns of the same soil, with initial volume fractions of water θ_1 and θ_2, respectively, are infiltrated with water. Assume θ_t and h_f independent of θ_i. Denote parameters pertaining to sample 1 by subscript 1 and to sample 2 by subscript 2. Using the Green and Ampt model, derive in terms of θ_1, θ_2 and θ_t the ratios of:

a. the distances of the wetting front at time t
b. the cumulative infiltrations at time t
c. the times required for the wetting front to reach s
d. the sorptivities

e. the infiltration rates at time t.

Question 5.54 Paraphrase the relationships between s_1 and s_2, I_1 and I_2, etc. in Question 5.53 if $\theta_1 < \theta_2$.

When gravity cannot be neglected, Equation 5.35 can be written as:

$$\frac{k_t}{\theta_t - \theta_i} \mathrm{d}t = \frac{s_f}{s_f - h_f} \mathrm{d}s_f \tag{5.41}$$

Integration, using $\frac{s_f}{s_f - h_f} = 1 + \frac{h_f}{s_f - h_f}$, yields:

$$\frac{k_t}{\theta_t - \theta_i} t = s_f + h_f \ln(s_f - h_f) + C \tag{5.42}$$

The integration constant C can be found from the condition $s_f = 0$ for $t = 0$:

$$C = -h_f \ln(-h_f) \tag{5.43}$$

The final result is then:

$$t = \frac{\theta_t - \theta_i}{k_t} \left[s_f + h_f \ln \left(\frac{s_f - h_f}{-h_f} \right) \right] \tag{5.44}$$

The physical parameters θ_t, θ_i, k_t and h_f for a given soil must be found experimentally. The values for θ_t and k_t are both not far from the values at saturation. h_f cannot be measured directly, but can be derived from Equation 5.40 by measuring S, k_t and $\theta_t - \theta_i$. Values for h_f vary from about -0.05 m to -0.8 m for different soils. Once these parameters are known, the time needed for the wetting front to reach a certain depth can be calculated directly with Equation 5.44. To find s_f for a certain time t with this equation is a little more difficult, since s_f cannot be expressed explicitly as a function of t. However, it is possible to graph t versus s_f and read s_f for various t from the graph.

Once s_f is known, the cumulative infiltration and the infiltration rate can be found using Equations 5.32 and 5.33.

Question 5.55 For a fine sandy loam $k_t = 1.38 \times 10^{-6}$ m s^{-1}, $\theta_i = 0.1$, $\theta_t = 0.5$ and $h_f = -0.4$ m.

a. Calculate s_f, I and i for horizontal infiltration in this soil at $t = 30$ min.

b. Do the same for $t = 60$ min.

c. Compare the results of the calculations for $t = 30$ min and $t = 60$ min.

Question 5.56 a. Find s_f at $t = 60$ min for vertical infiltration into the soil of Question 5.55, using Equation 5.44. Use the following procedure:

- calculate t using the value found for s_f in Answer 5.55b
- calculate t also for a somewhat larger value of s_f (larger, because gravity is included now)

- find a better guess for s_f by linear interpolation
- calculate again t for the value of s_f just found
- if necessary, repeat these steps.

b. Calculate also I and i using the value found for s_f in a.

c. Compare the results of these calculations with those for horizontal infiltration (Answer 5.55b).

For not too large t, an approximation for s_f can be derived mathematically from Equation 5.44. The logarithmic part of Equation 5.44 can be written as:

$$\ln\left(\frac{s_f - h_f}{-h_f}\right) = \ln\left(1 - \frac{s_f}{h_f}\right) = \ln(1 + y) \tag{5.45}$$

where $y = -s_f/h_f$. For $y^2 < 1$, ln $(1 + y)$ can be written in series form, which leads to the solution (for details see Appendix 3):

$$s_f = f_1 t^{1/2} + f_2 t + f_3 t^{3/2} + \ldots \tag{5.46}$$

$$\text{where } f_1 = \left(\frac{-2k_t h_f}{\theta_t - \theta_i}\right)^{1/2} \tag{5.47}$$

$$f_2 = \frac{2k_t}{3(\theta_t - \theta_i)} \tag{5.48}$$

$$f_3 = \left[-\left(\frac{k_t}{\theta_t - \theta_i}\right)^3 \cdot \frac{1}{162h_f}\right]^{1/2} \tag{5.49}$$

Using Equation 5.33, Equation 5.46 can be transformed into an equation identical to Equation 5.28:

$$I = St^{1/2} + At + Bt^{3/2} + \ldots \tag{5.50}$$

where $S = [-2k_t h_f(\theta_t - \theta_i)]^{1/2}$, as in Equation 5.40, and

$$A = \frac{2}{3}k_t \tag{5.51}$$

$$B = \left(\frac{-k_t^3}{162h_f(\theta_t - \theta_i)}\right)^{1/2} \tag{5.52}$$

Thus the first term in Equation 5.50 is identical to the result for horizontal infiltration (Equation 5.39) and represents the contribution of the pressure head gradient. The following terms represent the contribution of gravity. Of course, the infiltration rate associated with Equation 5.50 is, in form, identical to Equation 5.29.

It should be stressed that these results of the Green and Ampt model are only in form identical to the general solution represented by Equations 5.28 and 5.29. The coefficients of Equation 5.50 have different values and are evaluated in a quite different way. The results can only be used for small times ($y^2 < 1$) and for soils which exhibit sharp wetting fronts.

Question 5.57 Check that for the soil given in Question 5.55, the approximation given above can be used as long as $t < 9$ h 53 min.

Question 5.58 a. Calculate s_f, I and i for vertical infiltration in the soil given in Question 5.55 for $t = 60$ min, using Equations 5.46 and 5.50.

b. Compare the results with those of Answer 5.56.

Question 5.59 a. Calculate s_f for $t = 9$ h 53 min, using Equation 5.46.

b. What is the correct value for s_f when Equation 5.44 is used?

c. Draw a conclusion about the error made.

5.3.3.6 Redistribution

When infiltration of water into a dry soil has ceased, it will be followed by redistribution until the hydraulic potential is uniform throughout the profile. Water present above the wetting front at the time of cessation of infiltration (Figure 5.19 at $t = t_1$) will start to drain due to gravity and the soil below that wetting front will also become wet ($t = t_2$). After some time, the soil below the original wetting front may also dry after it was firstly wetted (see shaded area in Figure 5.19). It should be clear that redistribution is a very complex process, in which hysteresis (Section *3.1.5*) plays an important role. Because of hysteresis, the drying upper soil will, at equilibrium, be wetter than the wetting soil below. The hydraulic potential gradients diminish in time

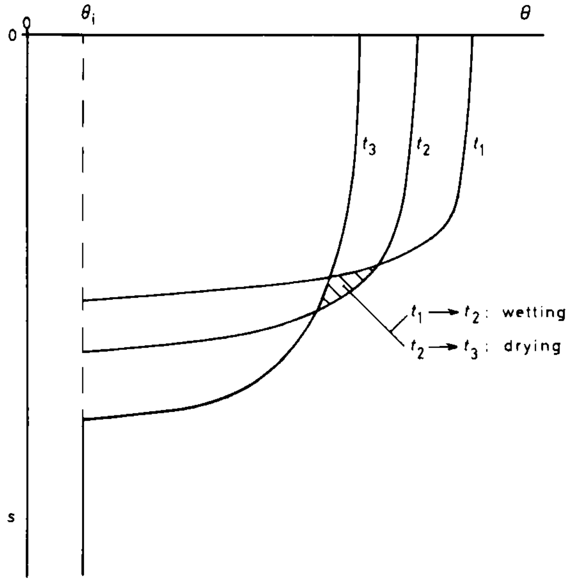


Figure 5.19 Water content profiles at end of infiltration (t_1) and at two later points in time as a result of redistribution.

and may become very small, although equilibrium is hardly ever reached. Thus, hysteresis retards and diminishes redistribution. True equilibrium is only possible when a groundwater table is present (see Figure 5.7). Without such a groundwater table, gravity prevents the water from ever reaching an equilibrium state. Thus, the concept of field capacity (Section *3.1.5*) has no firm theoretical basis, but has practical significance because after one or two days, flux densities usually have become negligible.

5.4 SUGGESTED LITERATURE

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5.5 ANSWERS CHAPTER 5

Answer 5.1 From Figure 5.1 it is obvious that there is no change in water content from $h_m = 0$ to $h_m = -0.20$ m. Thus the air-entry value of the largest pores is -0.20 m. The pores in class 1 have an average air-entry value $h_m = -0.24$ m. The corresponding equivalent pore radius is:

$$r = \frac{2\sigma}{-\rho_l g h_m} = \frac{2 \times 7 \times 10^{-2}\,\mathrm{N\,m^{-1}}}{10^4 \times 0.24\,\mathrm{N\,m^{-2}}} \approx 58\,\mu\mathrm{m}$$

Thus, in Equation 5.7:

$\Delta\theta_1 = 0.05$ and $r_1 \approx 58\,\mu$m.

The same procedure for classes 2 and 3 yields:

$\Delta\theta_2 = 0.20$ and $r_2 \approx 47\,\mu$m ($h_m = -0.30$ m)

$\Delta\theta_3 = 0.05$ and $r_3 \approx 39\,\mu$m ($h_m = -0.36$ m)

A rough estimate of the saturated hydraulic conductivity is then

$$K = \frac{1}{8 \times 10^{-3} \times 2}(0.05 \times 58^2 + 0.2 \times 47^2 + 0.05 \times 39^2) \times 10^{-12}\,\mathrm{m^2\,Pa^{-1}\,s^{-1}}$$
$$= 4.3 \times 10^{-8}\,\mathrm{m^2\,Pa^{-1}\,s^{-1}}.$$

And, therefore, $k = 4.3 \times 10^{-4}\,\mathrm{m\,s^{-1}}$.

One could also first take the average value of θ of each class, read off the corresponding value of h_m, and calculate the value of r for this value of h_m. This yields an only slightly different value of k.

In general, the step sizes in h_m should be kept small, because h_m is directly (not linearly) related to the pore size. The step sizes in θ have no direct influence on the accuracy of the calculations, since they only reflect the number of pores within a certain size range.

Answer 5.2 The unit of $K_i = \eta K$ is Pa s $\times$ m^2 Pa^{-1} s^{-1} = m^2.

The right hand side of Equation 5.8 also has m^2 as unit, because both τ and $(\Delta\theta)_i$ are dimensionless and the unit of r_i^2 is m^2.

Answer 5.3 a. In Answer 5.1, $K = 4.3 \times 10^{-8}\,\mathrm{m^2\,Pa^{-1}\,s^{-1}}$.

With $\eta = 10^{-3}$ Pa s, $K_i = 10^{-3} \times 4.3 \times 10^{-8}\,\mathrm{m^2} = 4.3 \times 10^{-11}\,\mathrm{m^2}$.

b. A highly conducting soil has $k \approx 1\,\mathrm{m\,d^{-1}} \approx 1.2 \times 10^{-5}\,\mathrm{m\,s^{-1}}$ and $K = k/\rho_l g \approx 1.2 \times 10^{-9}\,\mathrm{m^2\,Pa^{-1}\,s^{-1}}$.

With $\eta = 10^{-3}$ Pa s, $K_i \approx 1.2 \times 10^{-12}\,\mathrm{m^2}$.

A poorly conducting soil has $k \approx 0.01\,\mathrm{m\,d^{-1}}$, i.e. 0.01 times the value of a highly conducting soil. Thus, for a poorly conducting soil, $K_i \approx 1.2 \times 10^{-14}\,\mathrm{m^2}$.

Answer 5.4 The flux density is downwards, so it is negative. The rate of discharge is $\dfrac{1.62\,\mathrm{cm^3}}{1\,\mathrm{min}} = \dfrac{1.62 \times 10^{-6}\,\mathrm{m^3}}{60\,\mathrm{s}}$.

The cross-sectional area is 20 cm^2 = $20 \times 10^{-4}\,\mathrm{m^2}$.

$$\text{Hence, } q = -\frac{\text{rate of discharge}}{\text{area}} = -\frac{1.62 \times 10^{-6}\,\mathrm{m^3}}{60 \times 20 \times 10^{-4}\,\mathrm{s\,m^2}} = -1.35 \times 10^{-5}\,\mathrm{m\,s^{-1}}.$$

$$\frac{dH}{dz} = \frac{0.9\ \mathrm{m}}{0.8\ \mathrm{m}} = 1.125\ \mathrm{m\ m^{-1}}$$

Then, according to Darcy's law,

$$k = \frac{-q}{dH/dz} = \frac{1.35 \times 10^{-5}\ \mathrm{m\ s^{-1}}}{1.125} = 1.2 \times 10^{-5}\ \mathrm{m\ s^{-1}}$$

Answer 5.5 a. If the cylinder is perforated, water will squirt out because the water pressure at $z = 0.5$ m is greater than atmospheric pressure.

b. h will become zero.

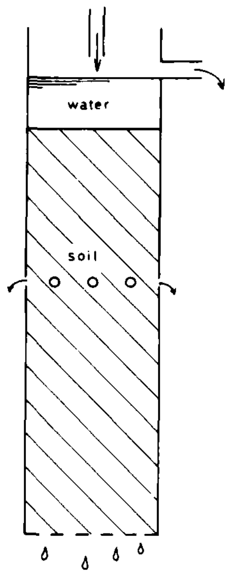
c. Soon after the perforations are made, h is zero both at the top and the bottom of the lower part of the column. This means, that for the lower part $\Delta H = \Delta z$ and $dH/dz = 1$. In the upper part of the column, $dH/dz > 1$. Since both parts are saturated and have the same value of k, the flux density through the upper part will be larger than through the lower part. Thus the lower part will remain saturated, the excess flux density will discharge continuously from the perforations and h will become zero everywhere in the bottom part.

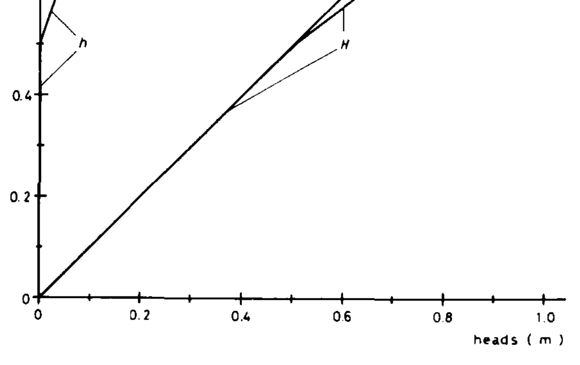
d. From Answer 5.2 and the reasoning above, the diagram below should be self-explanatory. The flux density in the upper part is:

$$q = -k\frac{dH}{dz} = -1.2 \times 10^{-5} \times \frac{0.4}{0.3}\ \mathrm{m\ s^{-1}} = -1.6 \times 10^{-5}\ \mathrm{m\ s^{-1}}$$

In the lower part, $dH/dz = 1$ and thus

$$q = -k = -1.2 \times 10^{-5}\ \mathrm{m\ s^{-1}}$$





The difference between these two flux densities is 0.4×10^{-5} m s^{-1}.

In a soil column with a cross-section of 20 cm^2, this yields a rate of discharge of $0.4 \times 10^{-5} \times 20 \times 10^{-4}$ m^3 s^{-1} or 28.8 cm^3 per hour.

Answer 5.6 The whole column is and remains saturated. Thus, $\partial\theta/\partial t = 0$. Then, according to the continuity equation without production term:

$$\frac{\partial\theta}{\partial t} = -\frac{\partial q}{\partial s} = 0$$

This means that the flux density is constant with height. The hydraulic conductivity and its variation with depth do not enter into the continuity equation and thus have no effect on this result.

Answer 5.7 At $z = 0$, $H = 0$. Since $\Delta H_2 = 0.6$ m, $H = 0.6$ m at $z = 0.2$ m.

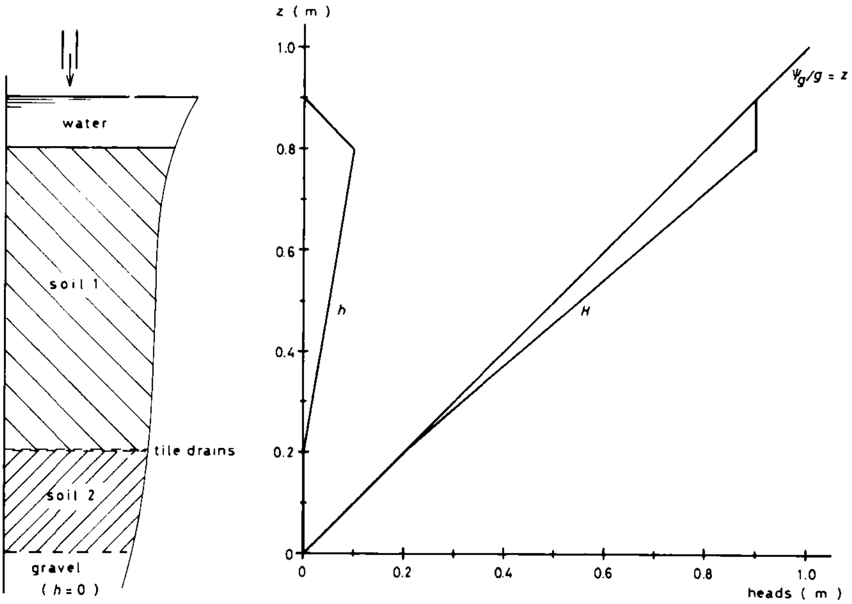
With this the diagram can be drawn.

The flux density in soil 1 is:

$$q = -k\,(\mathrm{d}H/\mathrm{d}z) = -1.2 \times 10^{-5} \times \frac{0.30}{0.60}\,\mathrm{m\,s^{-1}} = -6.0 \times 10^{-6}\,\mathrm{m\,s^{-1}}.$$

Answer 5.8 The flux density is -6.0×10^{-6} m s^{-1} = $-6.0 \times 10^{-6} \times 3600 \times 24$ m d^{-1} ≈ 0.518 m d^{-1}. Therefore, the daily supply should be 518 mm or 5180 m^3 per hectare.

Answer 5.9 a. The drainage system provides atmospheric pressure; hence, $h = 0$ at $z = 0.2$ m. The potential diagram will now be as follows:



b. From the diagram and the hydraulic conductivities, it follows:

$$q_1 = -1.2 \times 10^{-5} \times \frac{0.7}{0.6} \mathrm{m\,s^{-1}} = -1.4 \times 10^{-5} \mathrm{m\,s^{-1}}$$

$$q_2 = -0.2 \times 10^{-5} \times \frac{0.2}{0.2} \mathrm{m\,s^{-1}} = -2.0 \times 10^{-6} \mathrm{m\,s^{-1}}.$$

c. The absolute value of the flux density through the second layer has been reduced from 6.0×10^{-6} m s^{-1} to 2.0×10^{-6} m s^{-1}.

Consequently, the percolation losses have been reduced from 518 mm d^{-1} to 173 mm d^{-1}. Though the latter figure is still very high, the drainage system is effective for recycling purposes.

Answer 5.10 a. Application of Equation 5.10 to this problem gives:

$$\frac{\Delta H_1}{\Delta H_2} = \frac{k_2}{k_1}\frac{\Delta z_1}{\Delta z_2} = 3.5 \times \frac{0.10}{0.70} = 0.5$$

At $z = 0.80$ m, $H = h + z = (0.10 + 0.80)$ m $= 0.90$ m

At $z = 0, H = 0$

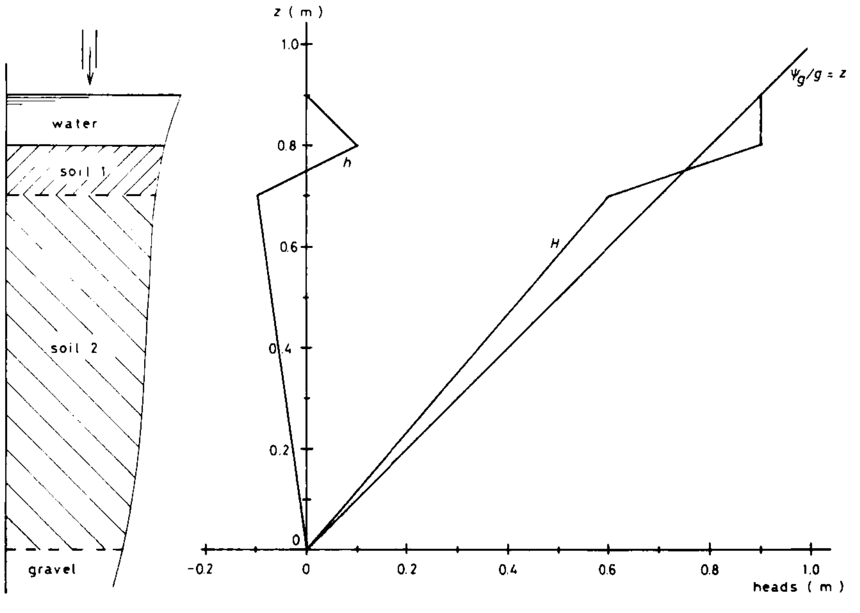
Thus $\Delta H_1 + \Delta H_2 = \Delta H_1 + 2\Delta H_1 = 0.90$ m

Then $\Delta H_1 = 0.30$ m and $\Delta H_2 = 0.60$ m

With this the potential diagram is determined.

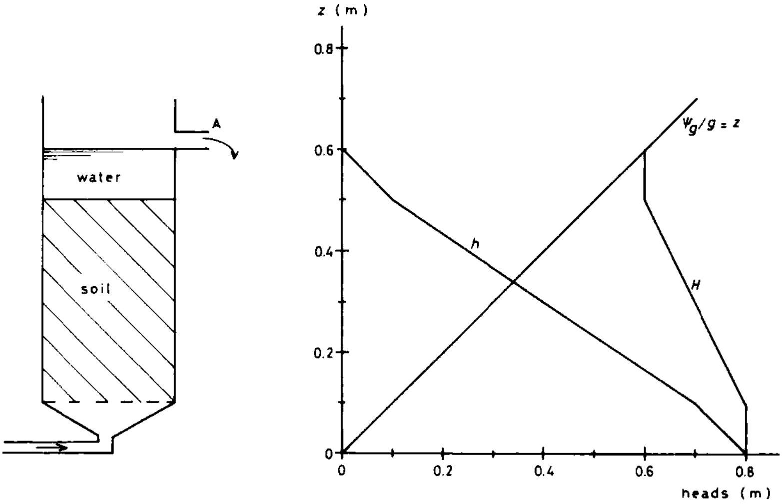
b. The flux density in layer 2 is:

$$q = -k_2 \frac{\Delta H_2}{\Delta z_2} = -1.2 \times 10^{-5} \times \frac{0.60}{0.70} \mathrm{m\,s^{-1}} = -1.03 \times 10^{-5} \mathrm{m\,s^{-1}}$$



c. The potential diagram shows that h is negative in nearly the entire profile. If the air-entry value would be exceeded, the soil layers would no longer be saturated and their hydraulic conductivity would decrease. This would change the flux density and the potential gradients and make the problem undetermined, unless the change in hydraulic conductivity with matric head is specified. Even then, the problem becomes far more complex.

Answer 5.11 At the bottom of the vessel V, $h = 0.80$ m. If the reference level for z is chosen at the same place, $H = 0.80$ m at $z = 0$. The resistance of the connection pipe between the vessel and the bottom of the soil column can be assumed so small, that the hydraulic head loss there is negligible and thus $H = 0.80$ m at $z = 0.10$ m, at the bottom of the soil column. At $z = 0.60$ m, $h = 0$ and thus $H = 0.60$ m. As before, H is constant in the water layer above the soil column. Thus, at $z = 0.50$ m, $H = 0.60$ m and $h = 0.10$ m. With this the potential diagram is determined.



Answer 5.12 The flux density is:

$$q = \frac{180 \times 10^{-6}\,\mathrm{m}^3}{3600\,\mathrm{s} \times 100 \times 10^{-4}\,\mathrm{m}^2} = 5 \times 10^{-6}\,\mathrm{m\,s}^{-1}$$

Across the soil column:

$$\frac{\Delta H}{\Delta z} = \frac{-0.20\,\mathrm{m}}{0.40\,\mathrm{m}} = -0.5\,\mathrm{m\,m}^{-1}$$

$$\text{Thus } k = \frac{-q}{\Delta H/\Delta z} = \frac{5 \times 10^{-6}\,\mathrm{m\,s}^{-1}}{0.5} = 1.0 \times 10^{-5}\,\mathrm{m\,s}^{-1}$$

Answer 5.13 a. Let $z = 0$ at the top of the soil column. Then the groundwater table is at $z = 0.10$ m.

$\phi \approx 1 - \frac{{}^{\mathrm{b}}\rho_{\mathrm{d}}}{\rho_{\mathrm{s}}} = 1 - \frac{1300\ \mathrm{kg\,m^{-3}}}{2600\ \mathrm{kg\,m^{-3}}} = 0.50$

${}^{\mathrm{b}}\rho_{\mathrm{w}} \approx {}^{\mathrm{b}}\rho_{\mathrm{d}} + \phi \times 1000\ \mathrm{kg\,m^{-3}} = 1800\ \mathrm{kg\,m^{-3}}$

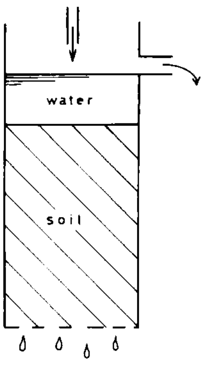
For calculating the soil pressure, two different values for ${}^{\mathrm{b}}\rho_{\mathrm{w}}$ have to be used here:

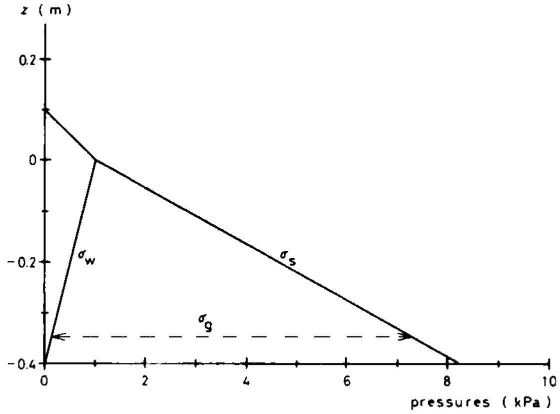
${}^{\mathrm{b}}\rho_{\mathrm{w}} = 1000\ \mathrm{kg\,m^{-3}}$ for $0 < z < 0.10\ \mathrm{m}$
${}^{\mathrm{b}}\rho_{\mathrm{w}} = 1800\ \mathrm{kg\,m^{-3}}$ for $-0.40\ \mathrm{m} < z < 0$

At $z = 0.10$ m, σ_{s} and σ_{w} are zero. At $z = 0$, σ_{s} and σ_{w} are the same and are $1000\ \mathrm{kg\,m^{-3}} \times 10\ \mathrm{N\,kg^{-1}} \times 0.10\ \mathrm{m} = 1.0$ kPa. In the soil, σ_{w} increases with 10 kPa per m depth and σ_{s} increases with $1800\ \mathrm{kg\,m^{-3}} \times 10\ \mathrm{N\,kg^{-1}} \times 1\ \mathrm{m} = 18$ kPa per m depth.

b. At the bottom of the column, $\sigma_{\mathrm{w}} = (1.0 + 0.40 \times 10)$ kPa $= 5.0$ kPa, $\sigma_{\mathrm{s}} = (1.0 + 0.40 \times 18)$ kPa $= 8.2$ kPa and $\sigma_{\mathrm{g}} = \sigma_{\mathrm{s}} - \sigma_{\mathrm{w}} = 3.2$ kPa.

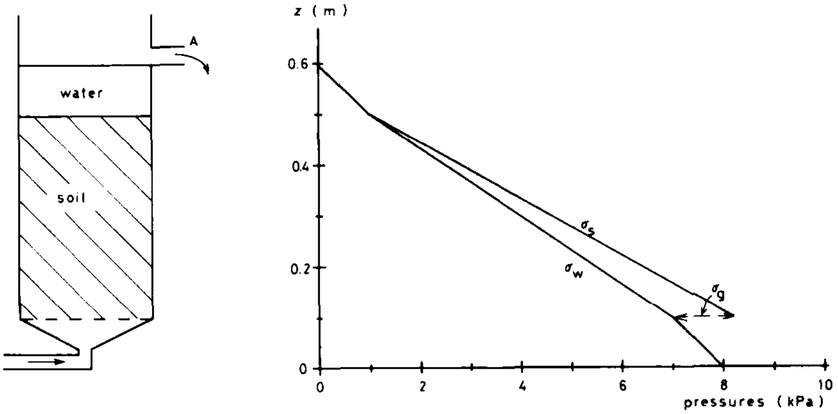
Answer 5.14 a. The whole soil, being static and saturated, has the same value for σ_{s} as in Answer 5.13, but σ_{w} changes. At $z = 0$, σ_{w} is still 1.0 kPa, but at the bottom of the column, σ_{w} is zero, because the water is under atmospheric pressure at $z = -0.40$ m. The new pressure diagram is then as follows:





b. At $z = -0.40$ m, $\sigma_{\mathrm{s}} = 8.2$ kPa and $\sigma_{\mathrm{w}} = 0$. Thus, at that height, $\sigma_{\mathrm{g}} = \sigma_{\mathrm{s}} - \sigma_{\mathrm{w}} = 8.2$ kPa. The decrease in σ_{w} is compensated by the increase in σ_{g}. In general, downward water transport in a soil profile causes an increase of the intergranular pressure relative to the static equilibrium situation.

Answer 5.15 a. σ_{s} remains the same as in Answer 5.14, but the pressure in the water, p', has now the same values as in Answer 5.11. Because in a sand column, $\sigma_{\mathrm{w}} = p'$ (Equation 3.56), σ_{w} at the bottom of the column has increased to 7.0 kPa. Hence, at the bottom of the column, $\sigma_{\mathrm{g}} = (8.2 - 7.0)$ kPa $= 1.2$ kPa.



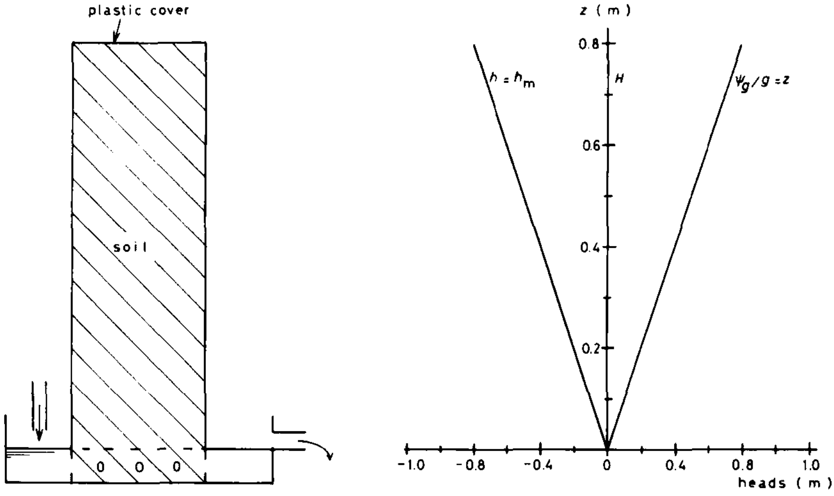
b. If the overflow B is raised from $z = 0.8$ m to $z = 0.92$ m, σ_w at the bottom of the column increases to 8.2 kPa, which equals the value of σ_s. In the whole column then, we find $\sigma_s = \sigma_w$ or $\sigma_g = 0$.

Answer 5.16 At the same water content the water-filled pores of loam are smaller than those of sand. Thus the contact area per volume, and with it the drag force per volume, between the solid phase and the water is the largest for loam. This results in a higher resistance to flow, or a lower hydraulic conductivity.

Answer 5.17 First, use the soil water characteristic to find: at $h_m = -1.0$ m, $\theta \approx 0.425$. Then use the $k(\theta)$ curve to find: at $\theta \approx 0.425$, $k = 6 \times 10^{-3}$ m d^{-1}.

Answer 5.18 For a given value of h_m, the water content may be different due to hysteresis. Since k is in the first place a function of θ, the hydraulic conductivity will be different. Other causes could be entrapped air or structural decay of the soil.

Answer 5.19 The following diagram should be self-explanatory.



Answer 5.20 The soil is saturated below $z = 0.3$ m. Since at $z = 0.3$ m, $h_m = -0.30$ m, the air-entry value is -0.30 m.

Answer 5.21 In the water-saturated part of the profile, k is constant with height. In steady saturated water transport, the flux density is also constant with height. This implies that $\mathrm{d}H/\mathrm{d}z$ is constant with height or that H changes linearly with height, and not curvilinearly. The same is true for h.

Answer 5.22 Since q is constant with height, the evaporation rate at the surface is the same as the flux density in the saturated zone. If $h = -0.22$ m at $z = 0.20$ m, then $H = -0.02$ m at that height, and thus:

$$q = -k\frac{\mathrm{d}H}{\mathrm{d}z} = -0.05\ \mathrm{m\ d^{-1}}\frac{-0.02}{0.20} = 5\ \mathrm{mm\ d^{-1}}$$

Answer 5.23 a. In the saturated zone is

$$\frac{\mathrm{d}H}{\mathrm{d}z} = \frac{q}{-k} = \frac{0.005\ \mathrm{m\ d^{-1}}}{-0.05\ \mathrm{m\ d^{-1}}} = -0.1\ \mathrm{m\ m^{-1}}$$

Then $\frac{\mathrm{d}h}{\mathrm{d}z} = \frac{\mathrm{d}H}{\mathrm{d}z} - \frac{\mathrm{d}z}{\mathrm{d}z} = (-0.1 - 1.0)\ \mathrm{m\ m^{-1}} = -1.1\ \mathrm{m\ m^{-1}}$

or $h = -1.1\,z + C$

Since $h = 0$ at $z = 0$, $C = 0$.

At the top of the saturated zone, h is equal to the air-entry value, or $h = -0.30$ m (Answer 5.20). Thus the height of the saturated zone is

$$z = \frac{h}{-1.1} = \frac{-0.30}{-1.1}\ \mathrm{m} \approx 0.27\ \mathrm{m}$$

b. No. The larger the evaporation rate, the larger the absolute value of the hydraulic potential gradient, and thus also the h gradient, the smaller the height over which the air-entry value will be reached, and thus the smaller the saturated zone.

Answer 5.24 $q = -k\frac{\mathrm{d}H}{\mathrm{d}z} = -0.10\ \mathrm{m\ d^{-1}} \times \frac{1.1}{1.0} = -0.11\ \mathrm{m\ d^{-1}}$

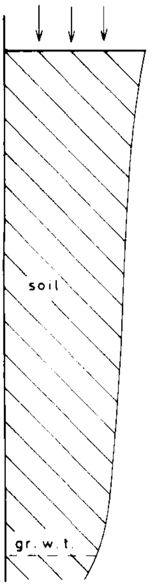
Answer 5.25 In this situation, $h = 0$ at the soil surface and in the whole profile, because $h = 0$ also at $z = 0$ (Answer 5.5d). This means that $H = z$ and $\mathrm{d}H/\mathrm{d}z = 1\ \mathrm{m\ m^{-1}}$.

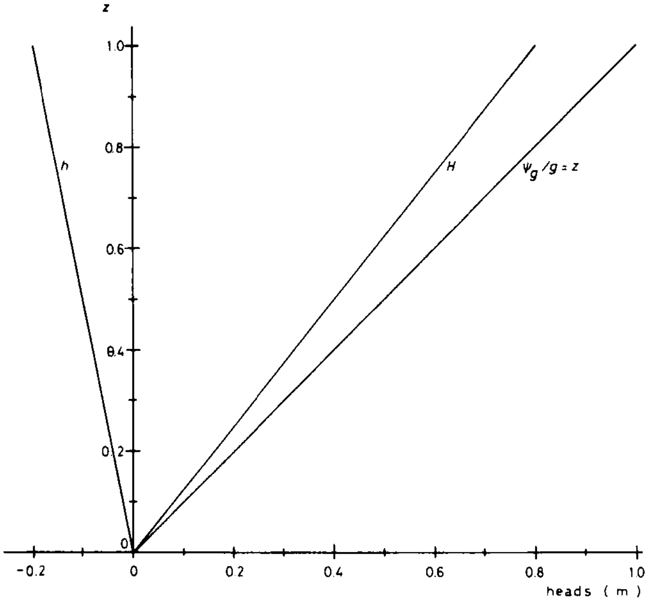
The sprinkling rate then equals the value of the hydraulic conductivity, 10 cm $\mathrm{d^{-1}}$.

Answer 5.26 H and h are linear functions of z only when $\mathrm{d}H/\mathrm{d}z$ = constant, i.e. when k is constant throughout the profile. Because the soil is saturated at the groundwater table, it should be saturated throughout the profile.

Answer 5.27 A flux density of 8 cm $\mathrm{d^{-1}}$ corresponds to $\frac{\mathrm{d}H}{\mathrm{d}z} = -\frac{q}{k} = 0.8\ \mathrm{m\ m^{-1}}$.

Hence, at the soil surface, $H = 0.8$ m and $h = -0.2$ m.





Answer 5.28 The soil will be saturated up to $z = 0.5$ m. See Answer 5.23a for the procedure of solving this problem.

Answer 5.29 In the unsaturated zone, where h is decreasing with height, k is also decreasing with height. Therefore, $\mathrm{d}H/\mathrm{d}z$ must increase with height to satisfy the condition for steady state: $q = -k\,\mathrm{d}H/\mathrm{d}z = \text{constant}$.

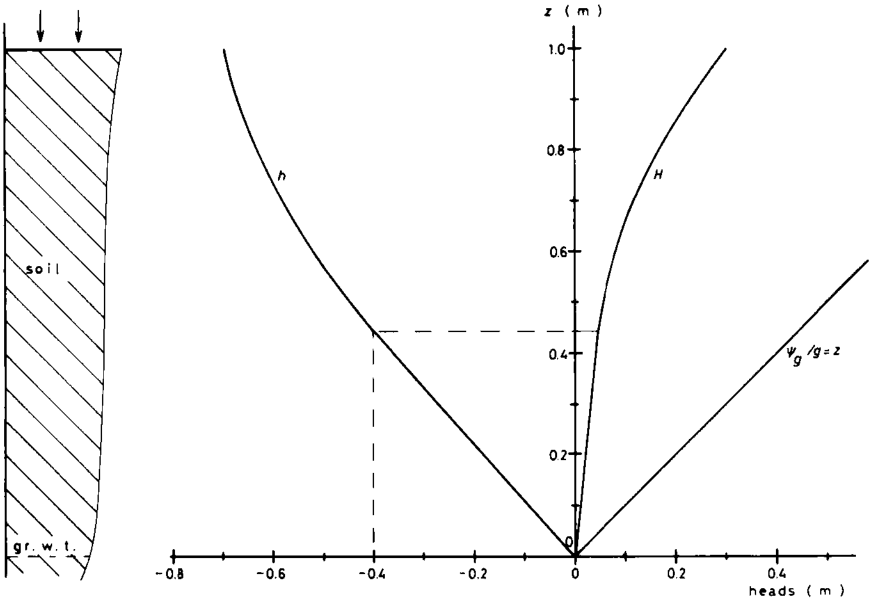
Consequently, $\mathrm{d}h/\mathrm{d}z$ must also increase with height, i.e. become less negative. Now suppose $\mathrm{d}h/\mathrm{d}z$ would become positive at a certain height. Then both h and k would increase with height. To keep the flux density constant, $\mathrm{d}H/\mathrm{d}z$ would have to decrease with height, and thus also $\mathrm{d}h/\mathrm{d}z$. This is in conflict with the supposition made above. Thus, we can conclude that the highest possible value of $\mathrm{d}h/\mathrm{d}z$ is zero and that of $\mathrm{d}H/\mathrm{d}z$ is unity. In this situation h has reached the constant value at which k is equal to the absolute value of the imposed flux density. The water then flows through the soil under the gravitational potential gradient of 1 m m^{-1}.

Answer 5.30 a. From $q = -1.0\ \mathrm{cm\,d^{-1}}$ and $k = 10\ \mathrm{cm\,d^{-1}}$ it follows:

$$\frac{\mathrm{d}H}{\mathrm{d}z} = -\frac{q}{k} = 0.1\ \mathrm{m\,m^{-1}}\ \text{and}$$

$$\frac{\mathrm{d}h}{\mathrm{d}z} = \frac{\mathrm{d}H}{\mathrm{d}z} - \frac{\mathrm{d}z}{\mathrm{d}z} = (0.1 - 1)\ \mathrm{m\,m^{-1}} = -0.9\ \mathrm{m\,m^{-1}}$$

The air-entry value is $h = -0.40$ m (Question 5.28). This value is reached at $z = 0.44$ m (for calculation procedure, see Answer 5.23a). Thus the soil is saturated below $z = 0.44$ m. Above $z = 0.44$ m, the gradient $\mathrm{d}H/\mathrm{d}z$ increases with height,



but cannot exceed the value for which $\mathrm{d}h/\mathrm{d}z = 0$. With these considerations the potential diagram can be drawn.

b. Above $z = 0.44$ m, the soil is unsaturated and θ as well as k vary. Since k is not given in the unsaturated zone, we only can guess the most probable shapes of the H and h profiles. Even if k were known, calculation of these profiles is complex because of the nonlinear nature of the problem.

Answer 5.31 Since z is defined positive upward, a positive flux density is directed upward and a negative flux density downward. Also $q = -k\ \partial H/\partial z$. Thus when $\partial H/\partial z$ is positive, then q is negative, and thus directed downward. Similarly, when $\partial H/\partial z$ is negative, the flux density is directed upward. There is no flow when $\partial H/\partial z = 0$.

Answer 5.32 At the soil surface, $h < 0$. Thus, there can not be free water on the soil surface.

Answer 5.33 Below height A there is still upward movement, because $\partial H/\partial z < 0$. Above height A, the transport is now downward. At height A there is no movement, because $\partial H/\partial z = 0$.

Answer 5.34 At some height z_1 above A the flux density q_1 is negative, because there $\partial H/\partial z$ is positive. At some height z_2 below A the flux density is positive, because $\partial H/\partial z$ is negative. Therefore, at height A,

$$\frac{\partial q}{\partial z} = \lim_{(z_1 \to A \text{ and } z_2 \to A)} \left(\frac{q_1 - q_2}{z_1 - z_2}\right) < 0,$$

because $(q_1 - q_2) < 0$ and $(z_1 - z_2) > 0$.

Thus, at height A, $\frac{\partial\theta}{\partial t} = -\frac{\partial q}{\partial z} > 0$

This means that θ increases with time. Even without the above proof, it is easy to see that the water content at A must increase, because water is moving towards A both from below and from above.

Answer 5.35 a. When there is fog, there is no evaporation. But, downward and upward water flow within the soil profile will continue until finally static equilibrium is attained. Since at static equilibrium $\partial H/\partial z = 0$, H is constant and zero throughout the soil profile, because $h = 0$ at $z = 0$.

b. At equilibrium, h is linear with height and equal to $-z$.

A little above C, $H = 0$ at time t_1, and thus at that height, h at this final stage will be the same as h at time t_1. Therefore, the water content also will remain the same at that height. Above this point, h and θ will decrease compared to their values at t_1. Below this point, both h and θ will increase. At point E and below, the soil is saturated at time t_1. Although there h will increase with time, because the flux will vanish, the water content cannot increase any further.

Answer 5.36 By means of tensiometers, measure h at three different heights, i.e. at z, $z + s$ and $z - s$. Add z to each h to obtain H. Estimate $\partial H/\partial z$ at the heights $z + \frac{1}{2}s$ and $z - \frac{1}{2}s$, assuming that H changes linearly with height in the intervals from z to $z + s$ and from $z - s$ to z. By means of a predetermined soil water characteristic, determine θ that corresponds to h at z, $z + s$ and $z - s$ (assuming $h = h_m$). Assuming that θ changes linearly with height, find θ at the heights $z + \frac{1}{2}s$ and $z - \frac{1}{2}s$. By means of a predetermined $k(\theta)$ function, find the k values at heights $z + \frac{1}{2}s$ and $z - \frac{1}{2}s$. Now,

$$q_{z+\frac{1}{2}s} = -k_{z+\frac{1}{2}s}\left(\frac{\partial H}{\partial z}\right)_{z+\frac{1}{2}s} \text{ and } q_{z-\frac{1}{2}s} = -k_{z-\frac{1}{2}s}\left(\frac{\partial H}{\partial z}\right)_{z-\frac{1}{2}s}$$

Finally, a good estimate of $\left(\frac{\partial\theta}{\partial t}\right)_z$ is

$$\left(\frac{\partial\theta}{\partial t}\right)_z = -\frac{q_{z+\frac{1}{2}s} - q_{z-\frac{1}{2}s}}{(z+\frac{1}{2}s) - (z-\frac{1}{2}s)}$$

Answer 5.37 By reading the appropriate values from Figure 5.6 the following scheme can be set up:

depth	z	h	H	θ	$\bar{\theta}$	k	$\frac{\partial H}{\partial z}$	q	$\frac{\partial \theta}{\partial t}$
m	m	m	m			mm d^{-1}	m m^{-1}	mm d^{-1}	d^{-1}
0.30	0.20	−2.00	−1.80	0.090					
					0.14	0.19	−11	2.1	
0.40	0.10	−0.80	−0.70	0.190					0.105
					0.24	4.2	−3	12.6	
0.50	0	−0.40	−0.40	0.295					

Answer 5.38 a. Using average h values the calculation scheme is:

depth	z	h	H	$\bar{h}$	θ	k	$\frac{\partial H}{\partial z}$	q	$\frac{\partial \theta}{\partial t}$
m	m	m	m	m		mm d^{-1}	m m^{-1}	mm d^{-1}	d^{-1}
0.30	0.20	−2.00	−1.80						
				−1.40	0.11	0.08	−11	0.88	
0.40	0.10	−0.80	−0.70						0.117
				−0.60	0.24	4.2	−3	12.6	
0.50	0	−0.40	−0.40						

Using average k values yields:

depth	z	h	H	θ	k	$\bar{k}$	$\frac{\partial H}{\partial z}$	q	$\frac{\partial \theta}{\partial t}$
m	m	m	m		mm d^{-1}	mm d^{-1}	m m^{-1}	mm d^{-1}	d^{-1}
0.30	0.20	−2.00	−1.80	0.090	0.03				
						0.39	−11	4.29	
0.40	0.10	−0.80	−0.70	0.190	0.75				0.57
						20.4	−3	61.2	
0.50	0	−0.40	−0.40	0.295	40				

b. The results in Answer 5.37 show that the θ values derived from the measured h values vary nearly linearly with depth. We will, therefore, consider these results as standard against which the other two calculation schemes can be evaluated.

A comparison of the results of the first two schemes shows that using average values of h leads to an underestimation of k for the drier soil. The error in the final value of $\partial\theta/\partial t$ is about 10%. The third scheme shows that using average k values leads to a value of $\partial\theta/\partial t$ which is almost six times too large. This illustrates the extreme nonlinear character of unsaturated water transport.

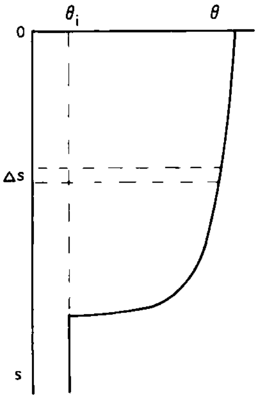
Answer 5.39 The calculation scheme is as follows:

depth m	z m	h m	H m	θ	$\bar{\theta}$	k mm d^{-1}	$\frac{\partial H}{\partial z}$ m m^{-1}	q mm d^{-1}	$\frac{\partial \theta}{\partial t}$ d^{-1}
0	0.80	0	0.80	0.50					
					0.49	50	4.0	−200	
0.10	0.70	−0.30	0.40	0.48					1.10
					0.46	15	6.0	−90	
0.20	0.60	−0.80	−0.20	0.44					0.68
					0.42	5.5	4.0	−22	
0.30	0.50	−1.10	−0.60	0.40					0.10
					0.36	2.0	6.0	−12	
0.40	0.40	−1.60	−1.20	0.32					0.24
					0.37	2.4	−5.0	12	
0.50	0.30	−1.00	−0.70	0.42					0.24
					0.445	9.0	−4.0	36	
0.60	0.20	−0.50	−0.30	0.47					0.04
					0.475	20	−2.0	40	
0.70	0.10	−0.20	−0.10	0.48					0.10
					0.49	50	−1.0	50	
0.80	0	0	0	0.50					

Answer 5.40 A column of soil with surface area A can be divided into n slices, each with thickness Δs. The volume of water absorbed by each slice is then $A(\theta-\theta_i)\Delta s$, where θ is the average volume fraction of water of the slice.

The volume of water absorbed by the whole column then equals
$V = A\sum_n(\theta-\theta_i)\Delta s$

For $\Delta s \to 0$ this can be written as the integral
$$V = A\int_{s=0}^{\infty}(\theta-\theta_i)\,\mathrm{d}s$$
The volume divided by the surface area is then
$$I=\frac{V}{A}=\int_{s=0}^{\infty}(\theta-\theta_i)\,\mathrm{d}s$$



(Whereas θ_i has been assumed constant it could also vary with s, without changing Equation 5.14. The integral would only get another value.)

Answer 5.41 Suppose at time t, the cumulative infiltration is I and at time $t + \Delta t$, $I + \Delta I$. The mean flux density during the time interval Δt is then the increase of the cumulative infiltration ΔI, divided by Δt. i.e. $\Delta I/\Delta t$.

The flux density (infiltration rate) at time t can be found by letting Δt approach zero:

$$i = \lim_{\Delta t \to 0} \left(\frac{\Delta I}{\Delta t}\right) = \frac{\mathrm{d}I}{\mathrm{d}t}$$

Answer 5.42 Equation 5.15 may be written as: $\mathrm{d}I = i\mathrm{d}t$.

Integration with boundary condition $I = 0$ for $t = 0$ gives:

$$I = \int_0^t i\mathrm{d}t$$

Answer 5.43 Since the ratio $s/\sqrt{t}$ is constant for a given value of θ, the distance from the wetting surface to the visible wetting front (or any other value of θ) increases proportional with the square root of time.

Answer 5.44 Since $S = \int_{\lambda=0}^{\infty} (\theta - \theta_i)\mathrm{d}\lambda$, S can be found by evaluating the area between the θ axis, the θ_i line and the $\theta(\lambda)$ curve.

Answer 5.45 Using the product rule $\mathrm{d}(\lambda\theta) = \lambda\mathrm{d}\theta + \theta\mathrm{d}\lambda$ and the boundary conditions $\theta = \theta_i$ for $\lambda \to \infty$ and $\theta = \theta_0$ for $\lambda = 0$,

$$\int_{\lambda=0}^{\infty} (\theta - \theta_i)\mathrm{d}\lambda = \int_{\lambda=0}^{\infty} \mathrm{d}(\theta - \theta_i)\lambda - \int_{\theta_0}^{\theta_i} \lambda\mathrm{d}\theta =$$

$$(\theta - \theta_i)\lambda \Big|_{\lambda=0}^{\infty} - \int_{\theta_0}^{\theta_i} \lambda\mathrm{d}\theta = 0 - 0 - \int_{\theta_0}^{\theta_i} \lambda\mathrm{d}\theta = \int_{\theta_i}^{\theta_0} \lambda\mathrm{d}\theta$$

Also, from Figure 5.17 it is obvious that $\int_{\theta_i}^{\theta} \lambda\mathrm{d}\theta$ and $\int_0^{\infty} (\theta - \theta_i)\mathrm{d}\lambda$

represent the same area.

Answer 5.46 S is not constant for a given soil. The $\theta(\lambda)$ curve is the solution of Equation 5.16 subject to conditions 5.17. Therefore, S depends on θ_i, θ_0 and on the $D(\theta)$ function of the soil. While θ_0 (saturation) and the $D(\theta)$ function normally do not vary for a given soil, S will still depend on θ_i.

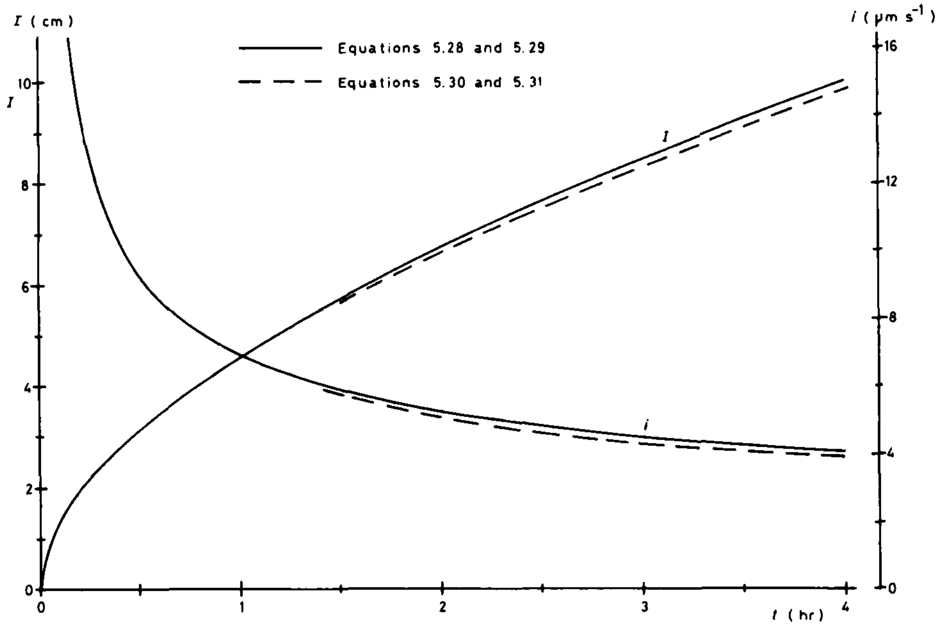
Answer 5.47 a. Using $I = S\sqrt{t}$, we find for heavy clay:

$I = 5 \times 10^{-5}\ \mathrm{m\ s^{-1/2}} \times \sqrt{24 \times 3600\ \mathrm{s}} \approx 1.47 \times 10^{-2}$ m or 1.47 cm

b. For coarse sand:

$I = 2 \times 10^{-3}\ \mathrm{m\ s^{-1/2}} \times \sqrt{24 \times 3600\ \mathrm{s}} \approx 0.59$ m or 59 cm

Answer 5.48 a and b:



c. Truncating Equations 5.28 and 5.29 after the second term introduces an error which increases with time. In this case, the error is still small after 4 hours of infiltration.

Answer 5.49 Since k is only a function of θ, and θ is constant, k must be constant. According to the continuity equation: if θ is constant with time ($\partial\theta/\partial t = 0$), then $\partial q/\partial s$ is zero, i.e. q is independent of s.

Answer 5.50 Since in the wetted zone q and k_t are independent of s, $\partial h/\partial s$ also has the same value throughout this zone, i.e. h is a linear function of s. Thus h decreases uniformly from zero at the soil surface to h_f at the wetting front and $\partial h/\partial s$ may be replaced by h_f/s_f.

Answer 5.51 While k_t and h_f are constant, s_f increases because the wetting front advances. Thus, $q = -k_t\left(\frac{h_f}{s_f} - 1\right)$ decreases with time.

Answer 5.52 a. The quantity between brackets is positive because h_f is negative.

b. Since k_t, θ_t, θ_i and h_f are constant, $S = [-2k_t h_f(\theta_t - \theta_i)]^{1/2}$ is constant.

Answer 5.53 a. For sample 1, $s_1 = \left(\frac{-2k_t h_f}{\theta_t - \theta_1}\right)^{1/2} t^{1/2}$

For sample 2, $s_2 = \left(\frac{-2k_t h_f}{\theta_t - \theta_2}\right)^{1/2} t^{1/2}$

Thus, $\dfrac{s_1}{s_2} = \left(\dfrac{\theta_t - \theta_2}{\theta_t - \theta_1}\right)^{1/2}$

b. $I_1 = (\theta_t - \theta_1)s_1$ and $I_2 = (\theta_t - \theta_2)s_2$

Therefore, $\dfrac{I_1}{I_2} = \dfrac{\theta_t - \theta_1}{\theta_t - \theta_2}\left(\dfrac{\theta_t - \theta_2}{\theta_t - \theta_1}\right)^{1/2} = \left(\dfrac{\theta_t - \theta_1}{\theta_t - \theta_2}\right)^{1/2}$

c. $t_1 = \dfrac{\theta_t - \theta_1}{-2k_t h_f}s^2$ and $t_2 = \dfrac{\theta_t - \theta_2}{-2k_t h_f}s^2$

Thus, $\dfrac{t_1}{t_2} = \dfrac{\theta_t - \theta_1}{\theta_t - \theta_2}$

d. $S_1 = [-2k_t h_f(\theta_t - \theta_1)]^{1/2}, S_2 = [-2k_t h_f(\theta_t - \theta_2)]^{1/2}$

Thus, $\dfrac{S_1}{S_2} = \left(\dfrac{\theta_t - \theta_1}{\theta_t - \theta_2}\right)^{1/2}$

e. $i_1 = \dfrac{S_1}{2\sqrt{t}}$ and $i_2 = \dfrac{S_2}{2\sqrt{t}}$

Therefore, $\dfrac{i_1}{i_2} = \dfrac{S_1}{S_2} = \left(\dfrac{\theta_t - \theta_1}{\theta_t - \theta_2}\right)^{1/2}$

Answer 5.54 When $\theta_1 < \theta_2$, $s_1 < s_2$ for a given t
$I_1 > I_2$ for a given t
$t_1 > t_2$ for a given s
$S_1 > S_2$ independent of s and t
$i_1 > i_2$ for a given t

Answer 5.55 For these calculations we use Equations 5.38, 5.39 and 5.26.

a. At $t = 30$ min:

$$s_f = \left(\frac{-2k_t h_f}{\theta_t - \theta_i}\right)^{1/2} t^{1/2} =$$

$$\left(\frac{-2 \times 1.38 \times 10^{-6}\,\text{m s}^{-1} \times (-0.4\,\text{m})}{0.5 - 0.1}\right)^{1/2} \times (1800\,\text{s})^{1/2} \approx$$

$$1.66 \times 10^{-3}\,\text{m s}^{-1/2} \times 42.43\,\text{s}^{1/2} \approx 0.07048\,\text{m or } 7.05\,\text{cm}$$

$$I = [-2k_t h_f(\theta_t - \theta_i)]^{1/2} t^{1/2} =$$

$$6.645 \times 10^{-4}\,\text{m s}^{-1/2} \times 42.43\,\text{s}^{1/2} \approx 2.819\,\text{cm}$$

$$i = \frac{I}{2t} = \frac{2.819 \times 10^{-2}\,\text{m}}{2 \times 1800\,\text{s}} \approx 7.832 \times 10^{-6}\,\text{m s}^{-1}$$

(The results are not rounded further because they are used again in Answer 5.58.)

b. For $t = 60$ min, similar calculations yield
$s_f \approx 9.968$ cm
$I \approx 3.987$ cm and
$i \approx 5.538 \times 10^{-6}\,\text{m s}^{-1}$

c. Both from the equations used and the results calculated, it follows that s_f and I are proportional with the square root of time, and i is inversely proportional with the square root of time.

Answer 5.56 a. Using $s_f = 9.97$ cm in Equation 5.44 gives $t = 3096$ s. Taking $s_f = 11$

cm gives $t = 3716$ s. By linear interpolation a better guess for s_f is

$$\frac{3600 - 3096}{3716 - 3096} \times (11 - 9.97)\,\mathrm{cm} + 9.97\,\mathrm{cm} = 10.81\,\mathrm{cm}$$

For $s_f = 10.81$ cm, $t \approx 3598$ s, which is accurate enough.

Thus, $s_f = 10.81$ cm.

b. Using Equations 5.32 and 5.33 we find:

$$I = (\theta_t - \theta_i)\, s_f \approx 4.32\,\mathrm{cm}$$

$$i = k_t\left(1 - \frac{h_f}{s_f}\right) = 1.38 \times 10^{-6}\,\mathrm{m\,s^{-1}} \times \left(1 - \frac{-0.4}{0.1081}\right) \approx 6.49 \times 10^{-6}\,\mathrm{m\,s^{-1}}$$

c. Due to the influence of gravity, the values of s_f, I and i at $t = 60$ min for vertical infiltration are all larger than those for horizontal infiltration.

Answer 5.57 The series approximation is valid only for $y^2 = \left(\frac{-s_f}{h_f}\right)^2 < 1$, or for $s_f < |h_f|$. Substituting $s_f = -h_f = 0.4$ m in Equation 5.44, gives: $t \approx 35577$ s or 9 h 52 min and 57 s. Thus the approximation can be used as long as $t < 9$ h 53 min. Beyond that time $s_f > |h_f|$.

Answer 5.58 a. The first terms of Equations 5.46, 5.50 and 5.29 are equal to the right hand side of Equations 5.38, 5.39 and 5.26, respectively. These are already calculated in Answer 5.55. Therefore, we need to calculate only the second and third terms of Equations 5.46, 5.50 and 5.29.

At $t = 30$ min:

$$s_f \approx (7.048 + 0.414 + 0.006)\,\mathrm{cm} \approx 7.47\,\mathrm{cm}$$

$$I \approx (2.819 + 0.166 + 0.002)\,\mathrm{cm} \approx 2.99\,\mathrm{cm}$$

$$i \approx (7.832 + 0.920 + 0.020) \times 10^{-6}\ \mathrm{m\,s^{-1}} \approx 8.77 \times 10^{-6}\ \mathrm{m\,s^{-1}}$$

At $t = 60$ min:

$$s_f \approx (9.968 + 0.828 + 0.017)\,\mathrm{cm} \approx 10.81\,\mathrm{cm}$$

$$I \approx (3.987 + 0.331 + 0.007)\,\mathrm{cm} \approx 4.33\,\mathrm{cm}$$

$$i \approx (5.538 + 0.920 + 0.029) \times 10^{-6}\ \mathrm{m\,s^{-1}} \approx 6.49 \times 10^{-6}\ \mathrm{m\,s^{-1}}$$

b. The differences with the results found in Answer 5.56 are negligible.

Answer 5.59 a. Using Equation 5.46, we find for s_f at $t = 35580$ s:

$$s_f \approx (0.3134 + 0.0818 + 0.0053)\,\mathrm{m} \approx 40.05\,\mathrm{cm}.$$

b. The correct value is, of course, 40 cm (Answer 5.57).

c. Even at this limit for the time of infiltration, for which the approximation can be used, the error is less than 0.14 % and thus more than acceptable for any practical problem.

6 Gas transport in soil

6.1 AMOUNT AND COMPOSITION OF SOIL AIR

The gas phase of soil is characterized by amount or volume and composition. The volume fraction of gas generally decreases with depth in a soil profile due to decreasing porosity as a result of compaction, biological activity, etc. and due to increasing water content. As discussed in Chapter 1, the volume fractions of gas and water are each others complement; together they fill the available pore volume. The critical volume fraction of gas for plant roots depends on the kind of plant and many other factors. A volume fraction of gas of 5–15% is generally considered adequate.

The composition of the soil air also varies with depth and time. Due to free diffusion of the atmospheric components through the soil surface, the composition of the soil air near the surface is almost the same as in the atmosphere, i.e. 80% N_2, with O_2 and CO_2 together forming the remaining 20%. With depth, the concentration of CO_2 increases and that of O_2 decreases because of biological activity in the soil (respiration of plant roots, micro-organisms and soil fauna).

6.2 EQUILIBRIUM CONDITIONS FOR SOIL AIR

Under equilibrium and isothermal conditions, the pressure of air in the gravitational field can be expressed as (Section *2.2.5*):

$$p = p_0 \exp\left(-\frac{M}{RT}gz\right) \tag{2.52}$$

If $z = 1$ m and $T = 290$ K, Equation 2.52 gives $p = 0.99988\ p_0$. Thus, air pressure changes by 0.012% over a height of 1 m. If this small gradient (0.12 mbar m^{-1}) is ignored, air pressure can be regarded as independent of height (i.e. p is uniform). This assumption applies not only to the total gas pressure, but also to the partial pressures of the different gaseous components. Therefore, soil air and the atmosphere at the soil surface are in equilibrium with each other only if the total gas pressure *and* the composition of the soil air are the same as in the atmosphere.

6.3 BULK FLOW OF GAS

Bulk flow of gas in soil results from a gradient of total gas pressure. This can be caused, for instance, by variations in barometric pressure of the atmosphere, displacement of soil air by infiltrating rain or irrigation water, rise of the groundwater table or temperature changes. We can ignore compression of soil air, because of the very small gas-pressure differences encountered in soils. The bulk flow of soil air can then be expressed analogously to the transport of soil water:

$$f_a = -K_a \frac{\partial p}{\partial s} \qquad (6.1)$$

where f_a is the air flux density in $m^3\ m^{-2}\ s^{-1}$, or $m\ s^{-1}$, and $\partial p/\partial s$ is the air pressure gradient in $Pa\ m^{-1}$. Thus, the air conductivity, K_a, has the same unit as the water conductivity ($m^2\ Pa^{-1}\ s^{-1}$).

The air conductivity depends on the volume fraction, size, continuity and geometry of the gas-filled pores. It is, therefore, highly dependent on the volume fraction of water. The air conductivity of a completely dry soil is about 50 times the water conductivity of the same soil when completely saturated, provided the porosity and pore geometry of the soil are the same in both conditions. Of course, for most soils this is not true. The factor 50 is the ratio of the viscosities of water and air ($\eta_w \approx 1.0$ mPa s, $\eta_a \approx 20$ µPa s).

The intrinsic permeability of a soil for air, K_{ia}, can be calculated by an equation similar to that used for water (Section *5.2.2*):

$$K_{ia} = \eta_a K_a \qquad (6.2)$$

The value of K_{ia} depends only on the geometry of the soil pores.

Question 6.1 Give examples of changing porosity and pore geometry of soils upon wetting.

Question 6.2 An undisturbed soil sample is first dried completely to measure the air conductivity and then saturated completely to measure the saturated water conductivity. The values found for K_a and K are 5.6×10^{-8} and 0.11×10^{-8} $m^2\ Pa^{-1}\ s^{-1}$, respectively.

a. Calculate K_{ia} and K_{iw}.
b. What do you conclude about this soil?

Saturated air conductivity can be measured in the laboratory with the apparatus depicted in Figure 6.1. By keeping the counterweight slightly less than the weight of the mobile cylinder, the air in the gas cylinder has a slightly higher pressure than the atmosphere. Thus, air will flow through the soil sample as soon as the stopcock, d, is opened. The saturated air conductivity can then be calculated from the air flux density and the pressure difference between the air in the cylinder and the atmosphere.

Question 6.3 In Figure 6.1:

cross-sectional area of cylinder b = 500 cm^2
pressure difference indicated by manometer = 5 mbar
cross-sectional area of soil sample = 10 cm^2
length of soil sample = 10 cm
velocity of cylinder b = 0.005 $cm\ s^{-1}$

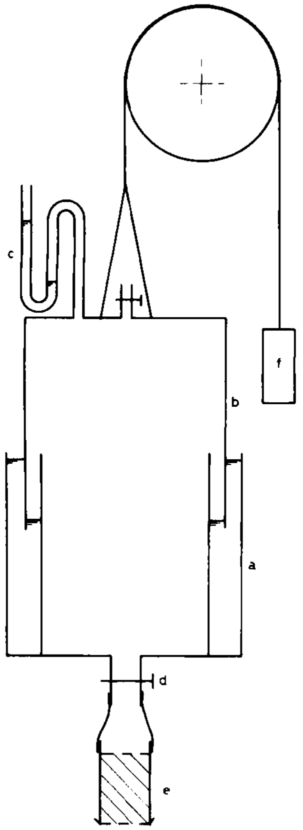


Figure 6.1 Apparatus for measuring air conductivity of a soil sample.
a. Fixed double-walled cylinder with water between the walls; b. mobile cylinder; c. water manometer; d. stopcock; e. soil sample in a cylinder; f. counterweight.

a. Calculate the air flux density through the soil sample.
b. Calculate the pressure gradient in the soil sample.
c. Calculate K_a and K_{ia}.

6.4 GAS DIFFUSION

The composition of soil air changes due to consumption of O_2 and production of CO_2 as a result of respiration of soil organisms and plant roots (Section 6.1). The volume fraction of CO_2 in soil air is more than the 0.03% in the atmosphere. In general, the volume fractions of CO_2 and O_2 together are nearly constant at about 20%. Root growth and other activities of most plants are hindered if the volume fraction of CO_2 is more than about 5% (O_2 fraction < 15%), while the upper limit for plants is about

10% CO_2. Supply of O_2 and removal of CO_2 by exchange with the atmosphere is called soil aeration.

Since bulk flow of soil air contributes very little to the aeration of the root zone (Section 6.3), exchange of O_2 and CO_2 with the atmosphere takes place mainly by diffusion. Diffusion can be described by Fick's law (Section 4.2):

$$f_i = -D_g \frac{\partial \chi_i}{\partial s} \tag{4.24}$$

where D_g is the diffusion coefficient for gases in the particular soil. Gas diffusion in soil due to a partial pressure gradient of one component is always accompanied by counter-current diffusion of other components. The molecules diffusing in opposite directions hinder each other to such a high degree that the friction of the pore walls is negligible compared to the internal friction in the gas. The influence of pore sizes is, therefore, so small that it is possible to estimate the value of D_g from the diffusion coefficient, D_0, of the gas mixture in free air. The value of D_0 only needs to be corrected for the following effects:

- Only a fraction of the cross-sectional area normal to the direction of the flux is available for diffusion. It equals the volume fraction of gas, ϕ_g.
- Diffusion through the pores is not in a straight line. The increase of the effective diffusion distance due to the tortuosity of the pore space and the blocking by water of connections between pores is generally estimated at a factor of about 2, even though this tortuosity factor, τ, will be larger in a wet soil than in a dry soil.

Taking into account the above effects yields the following estimate of D_g:

$$D_g \approx \tfrac{1}{2}\phi_g D_0 \tag{6.3}$$

The flux density equation then becomes:

$$f_i = -\tfrac{1}{2}\phi_g D_0 \frac{\partial \chi_i}{\partial s} \tag{6.4}$$

Question 6.4 Explain why counter-current diffusion takes place during diffusion of gases.

6.5 CO_2 DIFFUSION IN SOIL PROFILES

The CO_2 concentration in the gas phase of a soil generally increases with depth. This is one of the reasons why root development and activity is limited to the topsoil. Under average conditions in the root zone of natural soils, the production rate of CO_2, α_{CO_2}, varies between 0.2 and 0.6 mg $m^{-3}s^{-1}$.

If the production rate and the diffusion coefficient of CO_2 for the soil are known, the CO_2 concentration as a function of depth can be calculated for relatively simple conditions, e.g. when the CO_2 concentration does not change with time (steady state) and the production rate is constant throughout the root zone and zero below the root zone. At steady state, the gradient of the CO_2 concentration at a certain depth must be such that all CO_2 produced below that depth is removed by upward diffusion. Thus, the CO_2 flux density is the highest at the soil surface. It is somewhat easier to solve the problem if a height scale is used, rather than a depth scale, and if $z = 0$ is taken at the bottom of the root zone. The diffusion and continuity equations are then, respectively (Section 4.2):

$$f_{CO_2} = -D_g \frac{\partial \chi_{CO_2}}{\partial z} \qquad (6.5)$$

$$\text{and } \frac{\partial G_{CO_2}}{\partial t} = -\frac{\partial f_{CO_2}}{\partial z} + \alpha_{CO_2} \qquad (6.6)$$

Equation 6.6 can be simplified because, at steady state, $\partial G/\partial t = 0$ and so

$$\frac{df_{CO_2}}{dz} = \alpha_{CO_2} \text{ or } df_{CO_2} = \alpha_{CO_2} dz \qquad (6.7)$$

Note that normal differentials can be used because, at steady state, f and χ are no longer time-dependent, but only a function of height. If α_{CO_2} is assumed constant throughout the root zone, integration of Equation 6.7 yields:

$$f_{CO_2} = \alpha_{CO_2} z + C_1 \qquad (6.8)$$

where the integration constant, C_1, is zero because $f_{CO_2} = 0$ at $z = 0$.

Question 6.5 Why is $f_{CO_2} = 0$ at $z = 0$?

Substitution of $\alpha_{CO_2} z$ for f_{CO_2} in Equation 6.5 yields:

$$\alpha_{CO_2} z = -D_g \frac{\partial \chi_{CO_2}}{\partial z} \text{ or } \alpha_{CO_2} z dz = -D_g d\chi_{CO_2} \qquad (6.9)$$

If D_g is uniform, integration yields:

$$\alpha_{CO_2} z^2 = -2D_g \chi_{CO_2} + C_2 \qquad (6.10)$$

The integration constant, C_2, can be found from the condition that $\chi_{CO_2} = 0$ at the soil surface where $z = z_s$ (because CO_2 in the atmosphere is negligible):

$$C_2 = z_s^2 \alpha_{CO_2}$$

$$\text{So, } \chi_{CO_2} = \frac{\alpha_{CO_2}}{2D_g}(z_s^2 - z^2) \qquad (6.11)$$

Thus, the concentration of CO_2 is a quadratic function of height.

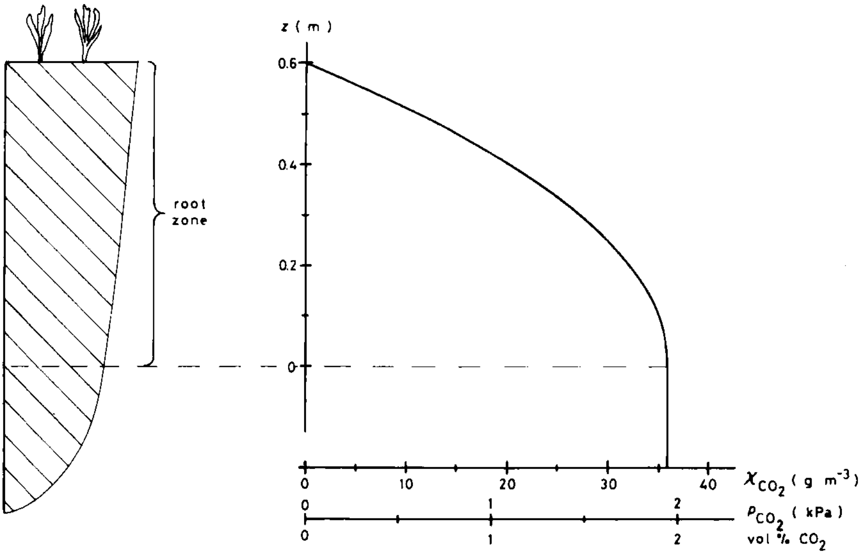


Figure 6.2 Concentration of CO_2 in soil with constant and uniform production of CO_2 and diffusion coefficient.

For a soil with a production rate of CO_2 of 0.6 mg m^{-3} s^{-1}, a 0.6 m root zone and $\phi_g = 0.3$, $D_0 = 2 \times 10^{-5}$ m^2 s^{-1} and $\tau = 2$, Equation 6.11 becomes:

$$\chi_{CO_2} = 0.1\ \mathrm{kg\,m^{-5}}\,[(0.6\ \mathrm{m})^2 - z^2] \tag{6.12}$$

At the lower end of the root zone (where $z = 0$) and below the root zone, the concentration of CO_2 is uniform at 36 g m^{-3}. Figure 6.2 shows the concentration of CO_2 as a function of height as calculated with Equation 6.12.

Question 6.6 Explain why the mass concentration of CO_2 is uniform below the root zone.

Question 6.7 Express Equation 6.11 in terms of depth, s, if $s = 0$ at the soil surface.

Mass concentrations of CO_2 can be converted to partial pressures by the ideal gas law (Answer 4.19):

$$p_{CO_2} = \frac{RT}{M_{CO_2}}\chi_{CO_2} \tag{6.13}$$

Using $RT = 2.4$ kJ mol^{-1} and $M_{CO_2} = 44$ g mol^{-1}, the partial pressure of CO_2 at the bottom of the root zone is about 1960 Pa. This corresponds with a volume fraction of CO_2 of only about 2%, even though the maximum rate of CO_2 production was assumed. Therefore, damaging concentrations of CO_2 can be expected only in soils with extremely low gas-filled porosities, due to compaction, extreme wetness, etc.

Question 6.8 A soil profile has a root zone of 0.7 m. At steady state, the oxygen consumption is 0.4 mg m^{-3} s^{-1} uniform throughout the root zone. Consumption of O_2 below the root zone is negligible. Calculate the partial pressure of O_2 as a function of height with the following information: $\phi_g = 0.32$, $D_0 = 2 \times 10^{-5}$ m^2 s^{-1}, $RT = 2.4$ kJ mol^{-1}, $M_{O_2} = 32$ g mol^{-1} and volume fraction of O_2 in the atmosphere = 20% (partial pressure = 20 kPa).

The above given solution to estimate the CO_2 concentration as a function of height is based on the simplification that ϕ_g and α_{CO_2} are constant throughout the root zone. Besides, it is limited to steady state. In reality, volume fraction of gas is a function of total porosity and volume fraction of liquid, which usually vary with depth. Also root activity is a function of depth. When the production term and the diffusion coefficient can be expressed as relatively simple functions of height, solutions can sometimes be found analytically for steady state conditions. If not, or if they are time-dependent, one has to use numerical methods (and if available, a digital computer) to find a solution.

6.6 SUGGESTED LITERATURE

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6.7 ANSWERS CHAPTER 6

Answer 6.1 The porosity of clay soils changes upon wetting due to swelling, when large pores either disappear or decrease in size. Another example is the slaking of fine-sandy and silty soils, when the porosity and the number of large pores decrease.

Answer 6.2 a. $K_{ia} = \eta_a K_a = 20 \times 10^{-6}$ Pa s $\times 5.6 \times 10^{-8}$ m^2 Pa^{-1} s^{-1} $\approx$ 1.1×10^{-12} m^2

$K_{iw} = \eta_w K = 1.0 \times 10^{-3}$ Pa s $\times 0.11 \times 10^{-8}$ m^2 Pa^{-1} s^{-1} $\approx 1.1 \times 10^{-12}$ m^2

b. The soil does not swell or slake upon wetting.

Answer 6.3 a. Velocity of cylinder b = 5×10^{-5} m s^{-1} and cross-sectional area of cylinder b = 0.05 m^2. Thus, flow rate of air = $5 \times 10^{-5} \times 0.05$ m^3 s^{-1} = 2.5×10^{-6} m^3 s^{-1}.

Cross-sectional area of soil sample = 10^{-3} m^2. Thus, flux density of air,

$$f_a = \frac{2.5 \times 10^{-6}\ \mathrm{m^3\ s^{-1}}}{10^{-3}\ \mathrm{m^2}} = 2.5 \times 10^{-3}\ \mathrm{m\ s^{-1}}.$$

b. If s is taken positive downwards (in the direction of the flux), the pressure gradient $\frac{dp}{ds} = \frac{-5\ \mathrm{mbar}}{0.1\ \mathrm{m}} = \frac{-500\ \mathrm{Pa}}{0.1\ \mathrm{m}} = -5\ \mathrm{kPa\ m^{-1}}$.

c. With Equation 6.1,

$$K_a = -\frac{2.5 \times 10^{-3}\ \mathrm{m\ s^{-1}}}{-5 \times 10^{3}\ \mathrm{Pa\ m^{-1}}} = 5 \times 10^{-7}\ \mathrm{m^2\ Pa^{-1}\ s^{-1}}$$

and with Equation 6.2,

$K_{ia} = 5 \times 10^{-7}$ m^2 Pa^{-1} s^{-1} $\times 20 \times 10^{-6}$ Pa s $= 10^{-11}$ m^2.

Answer 6.4 If the pressure in a gas mixture is to remain constant, the increase in partial pressure of one component must be compensated by an equal decrease in partial pressure of other components. Thus, the partial pressure gradient of the one component is compensated by reverse gradients of other components. These gradients cause the other gas components to move (diffuse) in an opposite direction. That is counter-current diffusion.

Answer 6.5 There is no CO_2 production below $z = 0$. Hence, no CO_2 has to be removed from below $z = 0$. A downward flux also is unlikely, because there usually are layers in the subsoil which block gas diffusion, for instance a groundwater table or rocks. So, below the root zone, $f_{CO_2} = 0$.

Answer 6.6 Below the root zone, there is no flux: $f_{CO_2} = 0$. Therefore, according to Equation 6.5, $\partial \chi_{CO_2}/\partial z = 0$, which means χ_{CO_2} is constant with depth.

Answer 6.7 The continuity equation for the steady state with a production term is:

$$\frac{df_{CO_2}}{ds} = \alpha_{CO_2}$$

For constant α_{CO_2}, integration yields

$f_{CO_2} = \alpha_{CO_2}s + C_1$

Since $f_{CO_2} = 0$ at $s = 0.6$ m, it follows:

$C_1 = -(0.6\,\mathrm{m})\,\alpha_{CO_2}$

and $f_{CO_2} = (s - 0.6\,\mathrm{m})\,\alpha_{CO_2}$

Substitution in the flux density equation yields

$$(s - 0.6\,\mathrm{m})\,\alpha_{CO_2} = -D_g \frac{d\chi_{CO_2}}{ds}$$

and after rearranging and integrating with D_g = uniform:

$$\left[\frac{s^2}{2} - (0.6\,\mathrm{m})s\right]\alpha_{CO_2} = -D_g\chi_{CO_2} + C_2$$

Since $\chi_{CO_2} = 0$ at $s = 0$, $C_2 = 0$. Therefore,

$$\chi_{CO_2} = \frac{\alpha_{CO_2}}{2D_g}[(1.2\,\mathrm{m})s - s^2]$$

Hence, for $\phi_g = 0.3$, $\alpha_{CO_2} = 0.6$ mg m^{-3} s^{-1}, $\tau = 2$ and $D_0 = 2 \times 10^{-5}$ m^2 s^{-1},

$\chi_{CO_2} = 0.1$ kg m^{-5} $[(1.2\,\mathrm{m})s - s^2]$

This equation can also be derived from Equation 6.12 by substituting 0.6 m $- s$ for z, since $s = -z + 0.6$ m. (Check this.)

Answer 6.8 The solution is analogous to that for CO_2, except that the flux is in the opposite direction and the production is negative (consumption). Again $z = 0$ is chosen at the bottom of the root zone, where $f_{O_2} = 0$.

$$\frac{\partial G_{O_2}}{\partial t} = -\frac{\partial f_{O_2}}{\partial z} + \alpha_{O_2} = 0 \text{ and thus } \frac{df_{O_2}}{dz} = \alpha_{O_2}$$

Integration yields $f_{O_2} = \alpha_{O_2}z + C_1$, where $C_1 = 0$ because at $z = 0$, $f_{O_2} = 0$.

Now, the diffusion equation becomes:

$$\alpha_{O_2}z = -D_g\frac{d\chi_{O_2}}{dz} \text{ or } \alpha_{O_2}z\,dz = -D_g\,d\chi_{O_2}$$

After integration: $\alpha_{O_2}z^2 = -2D_g\chi_{O_2} + C_2$

Substituting $\dfrac{M_{O_2}}{RT}p_{O_2}$ for χ_{O_2} (analogous to Equation 6.13) yields

$$\alpha_{O_2}z^2 = -\frac{2D_gM_{O_2}}{RT}p_{O_2} + C_2$$

C_2 can be found from the condition $p_{O_2} = 20$ kPa at $z = 0.7$ m:

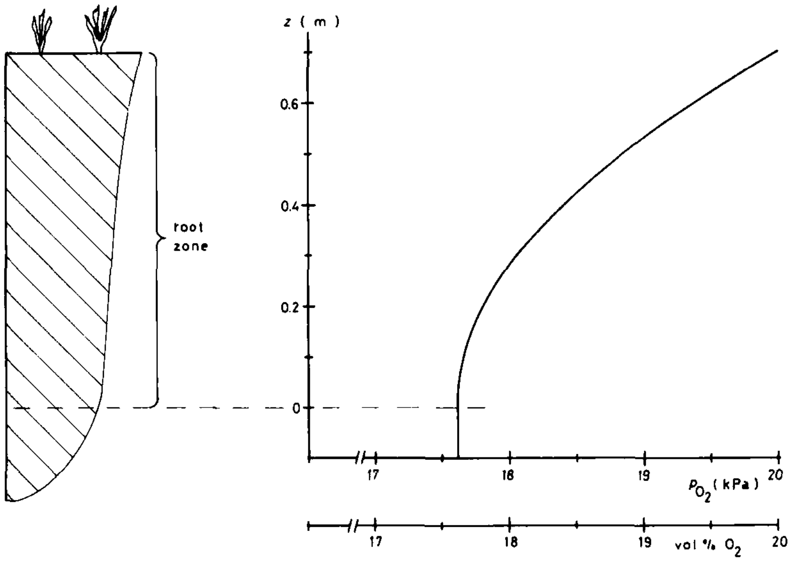
$$C_2 = \frac{2D_gM_{O_2}}{RT} \times 2 \times 10^4\,\mathrm{Pa} + (0.7\,\mathrm{m})^2\,\alpha_{O_2}$$

$$\text{Hence, } [z^2 - (0.7\,\mathrm{m})^2]\,\alpha_{O_2} = \frac{2D_gM_{O_2}}{RT}[2 \times 10^4\,\mathrm{Pa} - p_{O_2}]$$

from which

$$p_{O_2} = -\frac{\alpha_{O_2}}{2D_g}\frac{RT}{M_{O_2}}[z^2 - (0.7\,\mathrm{m})^2] + 2 \times 10^4\,\mathrm{Pa}$$

For $\alpha_{O_2} = -0.4\ \mathrm{mg\ m^{-3}\ s^{-1}}$, $D_0 = 2 \times 10^{-5}\ \mathrm{m^2\ s^{-1}}$, $\phi_g = 0.32$,
$RT = 2.4\ \mathrm{kJ\ mol^{-1}}$ and $M_{O_2} = 32\ \mathrm{g\ mol^{-1}}$:
$p_{O_2} = 4.69\ \mathrm{kPa\ m^{-2}}\ [z^2 - 0.49\ \mathrm{m^2}] + 20\ \mathrm{kPa}$, or
$p_{O_2} = 17.7\ \mathrm{kPa} + 4.69\ \mathrm{kPa\ m^{-2}} \times z^2$
This result is shown in the figure below. The partial pressure of O_2 is a quadratic function of height, but decreases with depth. Below the root zone, the partial pressure is about 17.7 kPa, which corresponds with a volume fraction of O_2 in the soil air of about 17.7%.



7 Heat transport in soil

7.1 INTRODUCTION

An important aspect of the behaviour and use of soils is their thermal regime. This regime is characterized to a large extent by the temperature. Many processes occurring in soils are strongly influenced by temperature. This is true especially for biological processes, such as germination of seeds, plant growth, root development and activity, microbial activity, etc. Physical and chemical processes influenced by temperature are frost heaving, weathering, decomposition of organic matter, etc.

Soil temperatures are determined by the transport processes of heat within the soil and by exchange of heat between the soil and the atmosphere. There are three basically different processes whereby heat can be transported: conduction, convection and radiation. Conduction of heat occurs by transmission of thermal energy of motion from one microscopic particle to another. Transport of heat by a fluid in motion is called convection. Heat convection with accompanying phase changes can increase heat transfer tremendously. This is especially true for water, which has very high values of latent heat of condensation/evaporation and freezing/melting. Radiation is transfer of thermal energy from a body to its surroundings by electromagnetic waves. Thus, in contrast to conduction and convection, radiation can occur through a vacuum.

Transport of heat within soils can occur by conduction and by convection, with or without latent heat transport. Heat conduction is governed by the thermal soil properties, volumic heat capacity and heat conductivity. The thermal exchange processes at the soil surface are dominated by the meteorological conditions and occur by radiation, conduction and convection, with or without phase changes. The thermal soil properties are strongly dependent on water content. A complete description of the thermal regime of soils is very complex and falls outside the scope of this text. Only the main aspects will be presented.

7.2 THERMAL SOIL PROPERTIES

7.2.1 Volumic heat capacity

The change of heat content of soil divided by volume and by change of temperature is called volumic heat capacity, C_h. It is expressed in J m^{-3} K^{-1} (Section 4.3). The volumic heat capacity of a soil can be obtained by summing the contributions of the different soil components:

$$C_h = \sum_i C_{h,i}\,\phi_i = \sum_i \rho_i\, c_i\, \phi_i \qquad (7.1)$$

where $C_{h,i}$ and c_i are the volumic and specific heat capacities, and ϕ_i and ρ_i are the volume fraction and density of component i. The volumic heat capacities of various soil components are presented in Table 7.1.

Table 7.1 Thermal properties of soil components at 10°C

	c kJ kg^{-1} K^{-1}	C_h MJ m^{-3} K^{-1}	λ J m^{-1} s^{-1} K^{-1} (W m^{-1} K^{-1})
quartz	0.76	2.0	8.8
clay minerals	0.73	2.0	2.9
organic matter	1.8	2.5	0.25
water	4.2	4.2	0.57
ice (0°C)	2.1	1.9	2.18
air (saturated with water vapour)	1.0	0.0013	0.025

Question 7.1 a. Derive an expression for the volumic heat capacity of a mineral soil (without organic matter) as a function of ϕ and ϕ_l, using Equation 7.1.

b. What can you conclude about the contribution of the soil air to the volumic heat capacity of the soil?

Answer 7.1 shows that the heat capacity of a soil is a linear function of the water content θ ($= \phi_l$) (Figure 7.1).

7.2.2. *Heat conductivity*

The heat conductivity, λ, of a soil is defined as the heat flux density by conduction through the soil divided by the temperature gradient. It is expressed in J m^{-1} s^{-1} K^{-1} (Section 4.3). The heat conductivity has no simple relationship with the heat conductivities of the individual soil components, because the conduction of heat takes place through all kinds of sequences of the conducting materials, in series and in parallel. The value of λ depends highly on the way in which the best conducting mineral particles are interconnected by the less conducting water phase and are separated by the poorly conducting gas phase.

Figure 7.1 shows the heat conductivity of a soil as a function of volume fraction of liquid. At very low water contents, λ is generally smaller than 0.5 J m^{-1} s^{-1} K^{-1}. The heat transport then takes place mainly through the narrow points of contact between the soil particles. The contribution of the soil air is very small due to the very low heat conductivity of air. A small increase in water content of a dry soil causes only a modest increase of λ, because this water forms thin films around the soil particles. Further increases in water content cause a sharp increase of the heat conductivity, because water has a much higher heat conductivity than air and this water collects

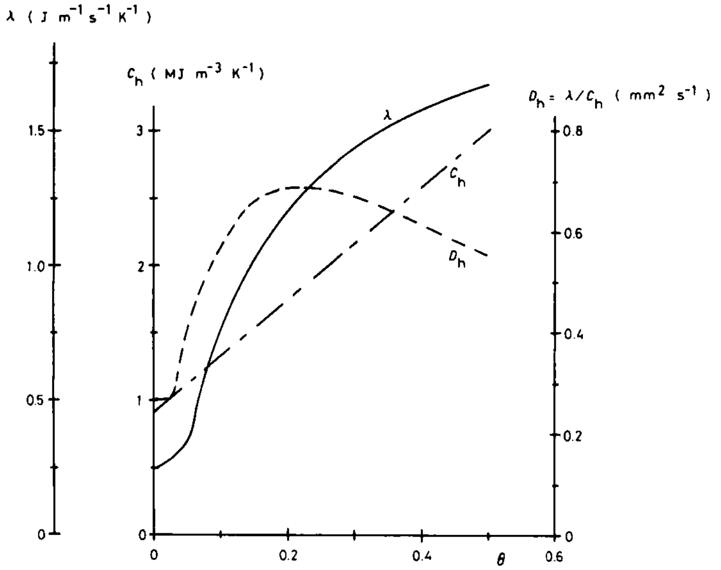


Figure 7.1 Thermal soil properties as a function of volume fraction of liquid.

around the contact points between the soil particles. Water around contact points forms very effective 'bridges' for conduction of heat. Upon further increase of the water content, the value of λ increases ever more gradually, because the conducting cross-sectional area of the water 'bridges' increases ever more slowly. The maximum value of λ is reached at water saturation. For mineral soils this value is generally between 1.5 and 2.0 J m^{-1} s^{-1} K^{-1}.

7.3 HEAT CONDUCTION IN DRY SOIL

In Section 4.3, one-dimensional conduction of heat was described with the equation

$$\frac{\partial T}{\partial t} = \frac{\lambda}{C_h} \frac{\partial^2 T}{\partial s^2} \tag{4.31}$$

This equation is valid if λ is independent of position, which is usually not true. The following sections contain solutions of this equation for three simple situations, all involving dry soil for which Equation 4.31 is valid.

7.3.1 Steady heat conduction

With steady heat conduction the temperature, per definition, does not change in time, i.e. $\partial T/\partial t = 0$. Since λ and C_h are constant and non-zero for dry soil, Equation

4.31 reduces to:

$$\frac{d}{ds}\left(\frac{dT}{ds}\right) = 0 \qquad (7.2)$$

Integrating twice yields:

$$T = C_1 s + C_2 \qquad (7.3)$$

The integration constants, C_1 and C_2, can be determined if the temperature is known for two values of s.

Question 7.2 In a steady situation the temperature in a homogeneous dry soil profile is 30 °C at the soil surface ($z = 0$) and 15 °C at a depth of 1 m ($z = -1$ m).

a. Express the temperature as a function of height.
b. Calculate the temperature gradient.

7.3.2 Cyclic variation of surface temperature

In nature, soil temperatures fluctuate due to more or less cyclic variations of the surface temperature, $T_{0,t}$. These variations are the diurnal (day-night) and the annual (summer-winter) cycles. Both can be approximated by a sinus function (Figure 7.2). The mathematical derivation of the solution of Equation 4.31 for these cyclic boundary conditions will not be given here. The solution for a soil with a constant 'average' temperature, T_{av}, is:

$$T_{s,t} = T_{av} + A_0 \exp(-s/d) \sin(\omega t - s/d) \qquad (7.4)$$

where $T_{s,t}$ is the temperature at depth s and time t, A_0 is the amplitude of the sinusoidal temperature variation at the soil surface, ω is the angular frequency of the temperature variation (wave), t is time and d is the damping depth. The angular frequency is

$$\omega = \frac{2\pi}{t_c} \qquad (7.5)$$

where t_c is the time needed to complete one cycle of the wave.

Question 7.3 Make a graph of the temperature at the soil surface, $T_{0,t}$, as function of time if $T_{av} = 20$ °C, $A_0 = 10$ °C and $\omega = 2\pi/24$ radians per hour ($t_c = 24$ h).

The sinusoidal temperature fluctuation at the surface penetrates into the soil by heat conduction. However, due to the finite heat diffusivity of the soil, the amplitude of the heat wave decreases with soil depth (Figure 7.2). In Equation 7.4, this phenomenon is accounted for by the introduction of the so-called damping depth, d, which is the depth where the amplitude of the temperature fluctuation has decreased to $A_0/e \approx 0.37 \times A_0$.

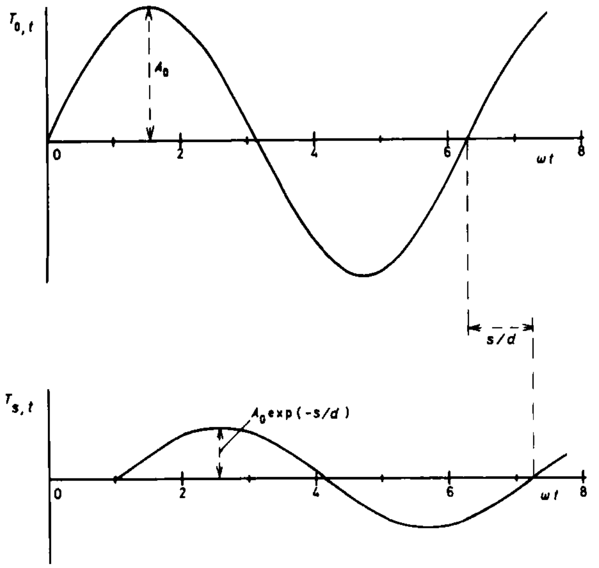


Figure 7.2 Cyclic variation of temperature at the soil surface and at depth $s = d$.

Question 7.4 Check this!

The damping depth depends on the thermal properties of the soil and on the angular frequency of the temperature variation according to

$$d = \sqrt{\frac{2\lambda}{\omega C_h}} = \sqrt{\frac{2D_h}{\omega}} \tag{7.6}$$

The finite heat diffusivity of the soil also causes a phase shift between the heat wave at the soil surface and at a certain depth s (Figure 7.2). This is represented in Equation 7.4 by the variable $-s/d$ in the sinus function ($-s/d$ is the phase angle).

Question 7.5 A soil has the following thermal properties:
$\lambda = 0.6\ \mathrm{J\,m^{-1}\,s^{-1}\,K^{-1}}$
$C_h = 1.6\ \mathrm{MJ\,m^{-3}\,K^{-1}}$
Calculate the damping depth for a diurnal and for an annual temperature fluctuation.

Question 7.6 a. Calculate the amplitude of the daily temperature fluctuation at 0.3 m depth in the soil above, if the amplitude at the soil surface is 10 K.
b. Make a graph of the temperature wave and compare it with the graph in Answer 7.3 (take $T_{av} = 20\,°\mathrm{C}$).

Question 7.7 What is the phase difference between the temperature fluctuations at the soil surface and at a depth $s = d$?

7.3.3 *Step-increase of surface temperature*

If a soil profile has a uniform initial temperature ($T = T_i$ for $s \geqslant 0$ and $t = 0$) and the surface temperature is suddenly increased and then maintained at that higher temperature ($T = T_0$, for $s = 0$ and $t > 0$), heat will penetrate into the soil from the surface. This so-called step-increase hardly occurs in nature; it is artificial. However, with the boundary conditions just stated, Equation 4.31 can be solved analytically. The solution is similar to that for infiltration of water in soil (Section 5.3.3.3 and Appendix 2):

$$\frac{T - T_i}{T_0 - T_i} = \operatorname{erfc}\left(\frac{s}{2\sqrt{(\lambda/C_h)t}}\right) \tag{7.7}$$

where $\frac{T - T_i}{T_0 - T_i}$ is the temperature change ($T - T_i$) at time t and distance s expressed as a fraction of the temperature change at the surface ($T_0 - T_i$). Since T can vary only between T_i and T_0, it can easily be checked that this fraction can only vary between 0 and 1. It is, therefore, often called dimensionless temperature. The value of erfc (u) for any positive value of u can be found from Figure 5.16. Consistent with the above conclusion, erfc (u) varies between 0 and 1.

Question 7.8 A soil with the thermal properties stated in Question 7.5 has a constant initial temperature. At $t = 0$ the temperature is raised by 10 K. Calculate the increase of the temperature at 0.10 m depth after 5 h and 40 min.

Question 7.9 At what time will the same temperature increase occur at 0.20 m depth and at 0.30 m depth?

Note: Determine this in a simple way, without complicated calculations.

Question 7.10 What is, obviously, the relationship between the depth s and the time t at which a certain increase of temperature is found?

7.4 THE HEAT BALANCE OF THE SOIL SURFACE

As mentioned in Section 7.3, the temperature at the soil surface varies periodically. During daylight, most of the incoming direct and indirect short-wave radiation is absorbed by the soil surface, the remainder is reflected back into the atmosphere. During darkness as well as daylight, the soil surface loses energy by emitting long-wave heat radiation. The net radiation flux density, f_n (J m^{-2} s^{-1} = W m^{-2}), is the difference between the unreflected incoming short-wave radiation and the outgoing long-wave heat radiation. Normally, during the day $f_n > 0$, and at night $f_n < 0$. The value of f_n can be measured directly or estimated from meteorological data.

The net radiation flux density is used for:

- evaporation of water at the soil surface (f_e)
- transpiration of water by the vegetative cover (f_t)

- heating of the soil (f_s)
- heating of the atmospheric air by conduction at the soil surface (f_a).

The heat balance of the soil can thus be described by:

$$f_n = f_e + f_t + f_s + f_a \qquad (7.8)$$

The distribution of f_n over f_e, f_t, f_s and f_a is important, because f_e and f_t determine the amount of water that can be evaporated, whereas f_s determines the soil temperature and, thus, the growth of crops.

Generally, it is not easy to calculate the distribution of f_n over the four terms in Equation 7.8, because the four processes are strongly interrelated. An estimate of this distribution is more easily made for the following two special situations:

- A rather dry, bare topsoil. Here, transpiration is absent and evaporation from the soil surface is negligible. Thus Equation 7.8 can be simplified to $f_n = f_s + f_a$. The distribution of f_n over f_s and f_a can then be estimated on the basis of calculations of heat conduction in dry soil and turbulent heat transport in the atmosphere. Such calculations yield, on average, a ratio of 1 : 1 between f_s and f_a. The exact value of this ratio depends on the thermal properties of the soil and the wind velocity.
- A rather wet soil. Here, evaporation of water consumes a significant part of the net radiation flux density. The soil acts mainly as a heat reservoir, absorbing heat during the day and discharging it again at night. As a result, the soil temperature changes little over a complete diurnal cycle, making f_s negligible over that interval. The net radiation flux density during one daily cycle thus will be distributed over f_e, f_t and f_a. The relative magnitudes of f_e, f_t and f_a can be estimated theoretically, but this will not be discussed here further.

Question 7.11 The net radiation flux density at a bare, dry soil surface ($C_h = 0.8$ MJ m^{-3} K^{-1}) is 20 W m^{-2}. Assuming $f_s/f_a = 1$, calculate:
a. the thickness of the surface layer in which the temperature is raised, on average, 1 K per hour;
b. the thickness of the layer of air ($C_h = 1.3$ kJ m^{-3} K^{-1}) above the soil surface that can be heated, on average, 1 K per hour.

7.5 THERMAL REGIME OF UPPER SOIL LAYERS

The temperature fluctuations in the soil profile resulting from the cyclic temperature variations at the soil surface were discussed in Section *7.3.2*. The influence of λ and C_h on these fluctuations was also described. This insight, together with the heat balance concept, can be used to characterize the thermal regime of the upper soil layers.

In dry soil, evaporation consumes very little energy from the net radiation, leaving nearly all energy available for heating the soil. At the same time, the heat penetrates into the soil only slowly, due to the low values of the heat diffusivity and damping depth (Figure 7.1). This combination of circumstances causes a large accumulation of heat in the surface layer with a relatively low heat capacity, resulting in a rapid rise of the soil surface temperature during daytime. At night, the surface of a dry soil cools down more rapidly than a wet soil surface. Thus, a dry soil is subject to much more stronger temperature variations in the upper layer than a wet soil. Due to the heat exchange between the soil and the atmosphere, the air layers above dry soil also will have large temperature fluctuations. This explains the large differences between day- and night-temperatures in deserts.

In temperate climates, dry soils will be warmer than wet soils during day-time. In early spring, this offers more favourable conditions for the germination of seeds and early development of crops. On the other hand, dry surfaces will cool down more strongly during the night and have a higher incidence of night frost. Thus, for a given climate, differences in thermal regime of the surface layers are determined to a large extent by differences in water content.

Question 7.12 Explain why much incoming radiation is required to heat up the surface layer of a wet soil.

7.6 COMBINED HEAT AND WATER VAPOUR TRANSPORT

7.6.1 Isothermal water vapour diffusion

Water vapour is one of the components of the gas phase of the soil. Like the other components, it can be transported both by bulk flow and by diffusion. Since bulk flow of soil air is generally negligible (Section 6.3), the principal transport mechanism of water vapour in soil is diffusion. The water vapour flux density by diffusion, f_v, according to Fick's law is (Section 6.4):

$$f_v = -D_g \frac{\partial \chi_v}{\partial s} \tag{4.24}$$

The diffusion coefficient is the corrected form of the molecular diffusion coefficient of water vapour in free air:

$$D_g \approx \tfrac{1}{2}\phi_g D_0 \tag{6.3}$$

The concentration gradient can be replaced by the vapour pressure gradient according to the ideal gas law (Answer 4.19).

In Chapter 3, it was shown that the relative humidity of the soil air is 99.3% at $pF = 4$ and approximately 93% at $pF = 5$. This means that differences in vapour pressure between, for instance, a very dry topsoil and a moist subsoil are very small, provided the temperature is constant throughout the profile. Therefore, isothermal water vapour diffusion can generally be neglected.

7.6.2 Non-isothermal water vapour diffusion

The saturated vapour pressure depends on temperature, as shown in Table 7.2. Since the soil atmosphere is normally saturated with water vapour, a temperature gradient in the soil will induce a considerable vapour pressure gradient.

Table 7.2 Saturated vapour pressure of water

T(°C)	0	5	10	15	20	25	30	35	40	45
p_0 (kPa)	0.61	0.87	1.23	1.70	2.33	3.16	4.23	5.61	7.36	9.56

Question 7.13 Does water vapour diffusion take place in the direction of higher or lower temperatures?

Question 7.14 What is the vapour pressure gradient induced by a temperature gradient of 1 K cm^{-1} at 15°C?

Question 7.15 What is the concentration gradient of the water vapour for the pressure gradient of Answer 7.14? ($RT \approx 2.4$ kJ mol^{-1}).

Question 7.16 Calculate the water vapour flux density resulting from the concentration gradient of Answer 7.15, if $D_0 \approx 2 \times 10^{-5}$ $\text{m}^2\ \text{s}^{-1}$ and $\phi_g = 0.30$.

Question 7.17 Convert the water vapour flux density in Answer 7.16 to an equivalent liquid water flux density.

Upon partially drying to, e.g., $pF = 4$, a poorly conducting soil may attain a hydraulic conductivity as low as 10^{-6} m d^{-1}. Such drying, generally, coincides with large hydraulic head gradients of the order of 10^2 to 10^3 m m^{-1}. Under those conditions, the liquid flux density still will be of the order of 0.1 to 1 mm d^{-1}. This liquid transport is about 5 to 50 times as high as the maximum equivalent water flux density calculated in Answer 7.17. Under wetter conditions, the vapour flux density will be even smaller relative to the liquid flux density. Thus, the contribution of the vapour flux density to the total water transport in soils is generally negligible. Moreover, temperature gradients that cause vapour diffusion tend to be dissipated by that diffusion because of the large latent heat transport associated with it.

Question 7.18 Explain this last statement.

7.6.3 Evaporation

The upward transport of water vapour away from the soil surface into the atmosphere is called evaporation. Except in a very thin laminar boundary layer, this transport is turbulent. Air turbulance is much more effective in transporting water vapour than diffusion. The energy required at the soil surface to sustain evaporation is provided mainly by the net solar radiation. This is the limiting factor for evaporation from a bare, wet soil.

In a bare, dry soil, the limiting factor for evaporation is the supply of water to the soil surface. As shown in Section 5.3.2.1, water in the soil is conserved by the formation of a very dry topsoil with a low water conductivity. In Section *7.6.2*, it was shown that the combined liquid and vapour transport of water in a topsoil at $pF = 4$ may produce a total flux density of no more than approximately 1 mm d^{-1}.

7.6.4 Evapotranspiration

When the soil is covered by vegetation, the situation is quite different. Water can then be transported from anywhere in the root zone, via the roots, stems and leaves into the atmosphere, the so-called soil-plant-atmosphere-continuum (SPAC). The loss of water from soil via plants is called transpiration. Transpiration is mostly a passive process. While leaf stomates are open for the exchange of CO_2 and O_2 for the process of photosynthesis, water vapour is lost to the atmosphere. The deficit is replenished by water uptake by the roots. The driving force for the transpiration stream is the large potential difference between the water in the soil and in the atmosphere. Again, except for a thin boundary layer, the transport of water vapour from the leaves into the atmosphere is turbulent. The stomates close when the roots cannot take up enough water to satisfy the evaporative demand. Then, with the transpiration, photosynthesis is also interrupted, which will result in yield reductions.

Under a full vegetative cover, evaporation from the soil can, generally, be neglected. When the soil is partially covered with vegetation, water is lost by both evaporation and transpiration, which usually are lumped together as evapotranspiration.

Evapotranspiration is a very complex process, which is intimately connected with the discipline of micrometeorology. The shorter the time span over which one wants to know the amount of evapotranspiration, the more complex the problem becomes. Evapotranspiration can be much higher than the evaporation from a bare soil, up to 10 mm d^{-1}. The determining factors are, in general:

- the turbulent transport of water vapour in the atmosphere, as determined by the wind velocity
- the supply of energy for evaporation from the leaves and the soil surface, mainly provided by solar radiation
- the availability of water in the soil and the flow to individual roots
- the type of crop and its growth stage.

7.7 SUGGESTED LITERATURE

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7.8 ANSWERS CHAPTER 7

Answer 7.1 a. The volumic heat capacity is composed of the following contributions

minerals:	$(1-\phi)\times 2.0$	$\mathrm{MJ\,m^{-3}\,K^{-1}}$
water:	$\phi_l \times 4.2$	$\mathrm{MJ\,m^{-3}\,K^{-1}}$
air:	$(\phi-\phi_l)\times 0.0013$	$\mathrm{MJ\,m^{-3}\,K^{-1}}$
total ≈	$2.0\,(1-\phi)+4.2\,\phi_l$	$\mathrm{MJ\,m^{-3}\,K^{-1}}$

b. The contribution of soil air to the volumic heat capacity of a soil is negligible due to the low volumic heat capacity of air.

Answer 7.2 a. To express the temperature as a function of height, substitute z for s in Equation 7.3:

$T = C_1 z + C_2$

Since $T = 30\,^\circ\mathrm{C} = 303\ \mathrm{K}$ at $z = 0$, $C_2 = 303\ \mathrm{K}$ and $T = C_1 z + 303\ \mathrm{K}$.

Substitution of $T = 15\,^\circ\mathrm{C} = 288\ \mathrm{K}$ and $z = -1.0\ \mathrm{m}$ gives:

$288\ \mathrm{K} = -1\ \mathrm{m}\times C_1 + 303\ \mathrm{K}$ or $C_1 = 15\ \mathrm{K\,m^{-1}}$.

Thus $T = 15\ \mathrm{K\,m^{-1}}\times z + 303\ \mathrm{K}$.

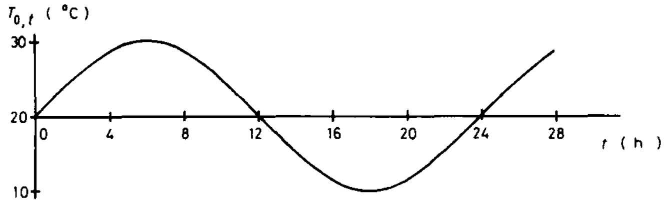
b. The temperature gradient is:

$$\frac{\mathrm{d}T}{\mathrm{d}z} = C_1 = 15\ \mathrm{K\,m^{-1}}$$

Answer 7.3 For the soil surface, where $s = 0$, Equation 7.4 reduces to:

$$T_{0,t} = T_{\mathrm{av}} + A_0 \sin \omega t = 20\,^\circ\mathrm{C} + 10\,^\circ\mathrm{C}\sin\left(\frac{\pi t}{12\ \mathrm{h}}\right)$$

$$\begin{aligned}
t &= 0 &&\rightarrow T_{0,0} = (20+10\sin 0)\,^\circ\mathrm{C} &&= 20\,^\circ\mathrm{C}\\
t &= 6\ \mathrm{h} &&\rightarrow T_{0,6} = (20+10\sin\tfrac{\pi}{2})\,^\circ\mathrm{C} &&= 30\,^\circ\mathrm{C}\\
t &= 12\ \mathrm{h} &&\rightarrow T_{0,12} = (20+10\sin\pi)\,^\circ\mathrm{C} &&= 20\,^\circ\mathrm{C}\\
t &= 18\ \mathrm{h} &&\rightarrow T_{0,18} = (20+10\sin\tfrac{3\pi}{2})\,^\circ\mathrm{C} &&= 10\,^\circ\mathrm{C}\\
t &= 24\ \mathrm{h} &&\rightarrow T_{0,24} = (20+10\sin 2\pi)\,^\circ\mathrm{C} &&= 20\,^\circ\mathrm{C}
\end{aligned}$$



Answer 7.4 At depth $s = d$, the equation is:

$$T_{d,t} = T_{\mathrm{av}} + A_0 \exp(-1) \sin(\omega t - 1)$$

Thus the amplitude of the temperature fluctuation at $s = d$ is A_0/e.

Answer 7.5 For the daily fluctuation, $\omega = \dfrac{2\pi}{24 \times 3600}\ \mathrm{s}^{-1}$.

$$\text{Thus, } d = \sqrt{\frac{2 \times 0.6 \times 24 \times 3600}{2\pi \times 1.6 \times 10^6}\ \mathrm{m}^2} = 0.10\ \mathrm{m}$$

For the annual wave, $\omega = \dfrac{2\pi}{365 \times 24 \times 3600}\ \mathrm{s}^{-1}$

$$\text{Thus, } d = \sqrt{\frac{2 \times 0.6 \times 365 \times 24 \times 3600}{2\pi \times 1.6 \times 10^6}\ \mathrm{m}^2} = 1.94\ \mathrm{m}$$

As Equations 7.5 and 7.6 indicate, d is proportional to $\sqrt{t_c}$.

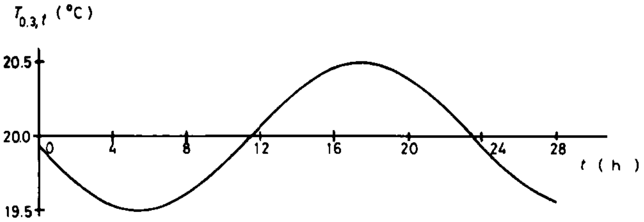
Answer 7.6 a. At 0.3 m depth the amplitude equals:

$$A_0 \exp(-s/d) = 10\ \mathrm{K} \times \exp(-0.3/0.1) \approx 0.50\ \mathrm{K}$$

b. The phase shift with the soil surface equals:

$$-s/d = \frac{-0.3}{0.1} = -3\ \mathrm{rad} \approx -172^\circ$$

A comparision with Answer 7.3 shows that the temperature at 0.3 m depth attains its minimum value when the surface temperature is near its maximum value.



Answer 7.7 According to the equation in Answer 7.4 the phase difference is -1 radian.

Answer 7.8 Substitution in Equation 7.7 of $T_0 - T_i = 10$ K, $s = 0.1$ m, $\lambda = 0.6\ \mathrm{J\ m^{-1}\ s^{-1}\ K^{-1}}$, $C_h = 1.6\ \mathrm{MJ\ m^{-3}\ K^{-1}}$ and $t = 5$ h 40 min $= 20400$ s gives:

$$\frac{T - T_i}{10\ \mathrm{K}} = \mathrm{erfc}\ \frac{0.1}{2\sqrt{\dfrac{0.6 \times 20400}{1.6 \times 10^6}}} = \mathrm{erfc}\ (0.57)$$

With the aid of Figure 5.16 we find:

$$\frac{T - T_i}{10\ \mathrm{K}} = \mathrm{erfc}\ (0.57) \approx 0.4 \text{ and } T - T_i \approx 4\ \mathrm{K}$$

Answer 7.9 The same increase of the dimensionless temperature means that $\dfrac{T - T_i}{T_0 - T_i}$

again equals erfc (0.57) (Answer 7.8), and thus

$$\frac{s}{2\sqrt{(\lambda/C_h)t}} = 0.57$$

If $s = 0.20$ m, which is twice the value of s in Question 7.8, then also $\sqrt{t}$ must be twice as large and t must be four times as large. Hence, t = 22 h 40 min. Similarly, for $s = 0.30$ m, t must be nine times as large, or 51 h.

In both cases t can also be computed by substituting the appropriate values for s, λ and C_h.

Answer 7.10 Obviously, for a certain increase of temperature, we find that $s/\sqrt{t}$ is constant. The reason is, that a certain value of $\frac{T - T_i}{T_0 - T_i}$ yields only one particular value of $\frac{s}{2\sqrt{(\lambda/C_h)t}}$, because erfc ($u$) is a singlevalued function. If λ and C_h are constant, then $s/\sqrt{t}$ must be constant.

Answer 7.11 At a bare, dry soil surface, $f_e = f_t = 0$. Therefore, $f_n = f_s + f_a = 20$ W m^{-2}. Assuming $f_s/f_a = 1$, $f_s = f_a = 10$ W m^{-2} = 10 J m^{-2} s^{-1}.

a. Recalling the definition of differential heat capacity (Table 4.2), this radiation flux density heats the soil according to

$$\frac{36000\ \mathrm{J\ m^{-2}\ h^{-1}}}{0.8 \times 10^6\ \mathrm{J\ m^{-3}\ K^{-1}}} = 4.5 \times 10^{-2}\ \mathrm{m\ K\ h^{-1}}$$

Thus, the soil layer heated 1 K in one hour is 4.5×10^{-2} m or 4.5 cm.

b. Similarly, for air:

$$\frac{36000\ \mathrm{J\ m^{-2}\ h^{-1}}}{1.3 \times 10^3\ \mathrm{J\ m^{-3}\ K^{-1}}} = 27.7\ \mathrm{m\ K\ h^{-1}}$$

Thus, the air layer heated 1 K in one hour is 27.7 m thick.

Answer 7.12 The surface layer of a wet soil requires much incoming radiation to heat up, because

a. much of the incoming radiation is used for evaporation

b. the remaining heat is distributed over a large depth, due to the large damping depth

c. the heat remaining in the topsoil causes a relatively small increase of temperature, due to the high value of the heat capacity.

Answer 7.13 Water vapour diffusion occurs from higher to lower vapour pressures, thus from higher to lower temperatures.

Answer 7.14 At 15 °C, a temperature difference of 1 K induces a water vapour pressure difference of approximately 0.1 kPa (Table 7.2). Thus a temperature gradient of 1 K cm^{-1} produces a water vapour pressure gradient of about 10 kPa m^{-1}.

Answer 7.15 Since $\chi_v = \frac{M_v}{RT} p_v$, a pressure gradient of 10 kPa m^{-1} corresponds with a concentration gradient

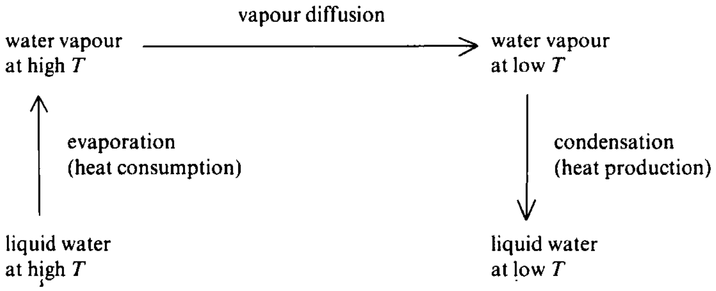
$$\frac{\partial \chi_v}{\partial s} = \frac{M_v}{RT}\frac{\partial p_v}{\partial s} \approx \frac{18\,\mathrm{g\,mol^{-1}} \times 10^4\,\mathrm{Pa\,m^{-1}}}{2.4 \times 10^3\,\mathrm{J\,mol^{-1}}} = 7.5 \times 10^{-2}\,\mathrm{kg\,m^{-4}}$$

Answer 7.16 The absolute value of the water vapour flux density is:

$$|f_v| \approx \tfrac{1}{2}\phi_g D_0 \frac{\partial \chi_v}{\partial s} = \tfrac{1}{2} \times 0.3 \times 2 \times 10^{-5} \times 7.5 \times 10^{-2}\ \mathrm{kg\,m^{-2}\,s^{-1}} = 2.25 \times 10^{-7}\ \mathrm{kg\,m^{-2}\,s^{-1}}$$

Answer 7.17 Since $\rho_w \approx 10^3\ \mathrm{kg\,m^{-3}}$, the equivalent liquid water flux density is $2.25 \times 10^{-10}\ \mathrm{m\,s^{-1}}$ or about $0.02\ \mathrm{mm\,d^{-1}}$.

Answer 7.18 The latent heat of evaporation of water is high. Therefore, much heat is consumed where water vapour is formed and produced where it condenses. The net effect of this is an often large heat flux density in the direction of the vapour flux, as follows:



One might say that the effective heat conductivity has increased. This heat transport associated with the water vapour diffusion will eventually wipe out the temperature gradient, unless it is maintained by outside influences.

8 Epilogue

G. H. Bolt

The preceding text was intended to convey an understanding of only the most basic aspects of physical processes occurring in soil. This epilogue appraises the additional understanding needed for analysing and managing the functions of soil in the real world.

Several such functions come to mind, ranging from the classic 'habitat for plants', accidental storage and/or passing station for unwanted spills, assigned storage reservoir for water or energy, to that of foundation of structures and other engineering functions. Of these functions, the first one mentioned requires the broadest insight, because it involves the support of life. It is traditionally one of the major areas of interest of Soil Physics. Accordingly, most of the following considerations are directed towards that function. The other functions usually pose simpler, though not always simple, problems. The role of soil as storage/passing station for water-carried solutes involves the retardive and dispersive soil properties. These phenomena, also referred to as miscible displacement, are often treated under the heading of Soil Chemistry (see, e.g., Bolt and Bruggenwert (1), chapter 7). Under conditions of flooding the effect of cracks in soil may well be decisive for the break-through of surface-added pollutants into the groundwater. Storage of heat in soil and heat production around power cables and gas mains with the ensuing transport of water away from the heat source, may be considered as soil engineering problems. A useful extension on this subject, beyond the introductory material in Chapter 7, can be found in Kirkham and Powers (5), chapter 10. Freezing phenomena represent another form of coupled heat and water transport in soils. A review of past and present work on this subject is given by Miller (Hillel (3), chapter 11). A general introduction to Soil Physics from the engineering point of view is given by Yong and Warkentin (8).

Focussing now on the ecological significance of soil with respect to plant growth and crop production, a starting point can be that, with a few exceptions, plant roots require an adequate supply of water and oxygen. These two components are complementary: maximum presence of the one implies absence of the other. Therefore, plant roots can function only in the so-called 'unsaturated zone', making transport and retention of water and diffusion of gases of prime importance. Water enters soil mostly from the surface with a climate- or human-controlled intermittent pattern. The water retentivity and conductivity functions (see Figure 3.12 and 5.6, respectively) of a soil determine which fraction of the water will percolate beyond the root zone, and at which rate this will occur. The remaining water is available for evaporation from the soil surface and uptake by plant roots. Transpiration is mostly a passive process. While leaf stomates are open for the exchange of CO_2 and O_2 for the process of photosynthesis, water vapour is lost to the atmosphere. When the roots cannot take up enough water to replenish the plant and satisfy the evaporative demand, the stomates close and photosynthesis is interrupted. As a result, there is a relationship between the amount of water transpired and the amount of dry matter produced. Plant roots must respire to sustain their physiological functions, and thus they consume O_2 and produce CO_2. These gases must be exchanged with the atmosphere through the same pore system that must ensure a satisfactory water supply.

The above simplistic sketch clearly indicates the intimate connections between crop productivity and physical soil properties, within constraints of climatic conditions, human activity, etc. It also hints at the difficulties to be encountered when attempting to switch from *understanding* an observed crop growth pattern for a given set of conditions, to *predicting* such a pattern. Predictive use of soil physical knowledge is much more complicated than explanatory use. As is often the case in nature, the whole crop production system is one of checks and balances, a rather well-buffered system. Before the computer era, solutions of the second-order, nonlinear, partial differential flow equations (e.g., Equations 4.18 and 4.23) were possible only for simple boundary conditions (see, e.g., Philip (6)). These solutions are inadequate for predicting the distribution of water content in time and depth within the unsaturated zone following a 'normal' pattern of intermittent water supply.

When computers became available in the sixties, the possibilities for predictive applications of soil physics appeared to increase tremendously. In the early years, computers were used primarily to find numerical solutions to the partial differential flow equations for complex boundary conditions and multilayered columns (for a rather complete survey of such methods, see Remson, Hornberger and Molz (7)). When the ever increasing capacity and speed of computers made it possible to solve more and more complex problems, other 'limiting' factors became apparent. For instance, computation of water infiltration in a layered soil column requires complete information on the soil hydraulic functions for all layers. Such information is hardly ever available with any degree of completeness. The moment one admits alternating wetting and drying periods – mandatory for simulating crop growth under field conditions – one must include complications due to hysteresis, which requires 'families' of water retention curves for each layer (see Figure 3.14). The corresponding $k(\theta)$ functions, to a first approximation, can be assumed nonhysteretic. Not so much the difficulties of coping with such families of curves when performing predictive calculations, but the lack of information on the precise shape of such families for each layer in the soil profile prevents detailed studies of this nature. Swelling and shrinkage of clay soils present even more vexing problems. In short, calculation facilities very soon exceed the possibilities of obtaining experimental information on relevant parameters with a detail commensurate with the complexity of the calculation models.

Towards the end of the seventies, two new developments took place. The first one was the introduction of dynamic simulation models for more complete systems. In the context of the present discussion on soil as a physical habitat for plants, for instance, one would attempt to model the entire growth process of a crop during a season using simplified assumptions (see, e.g., Feddes, Kowalik and Zaradny (2)), rather than try to model precisely the transport of water in a single soil column. In such simulation models, water supply at the soil surface, climatic conditions, distributed water uptake by plant roots and other pertinent factors are simulated, according to assumed interactive relationships, to predict the rate of crop growth. These relationships should be derived from the best available theoretical and experimental information on the various processes. In the earlier stages of simulation-modeling, quite extreme simplifications of many simultaneously considered processes were needed. Increased sophistication of computer hardware and software now makes it possible to cope with a high degree of complexity of subprocesses. Quite generally, the predictive value of computer programs now being developed for simulating crop production, involving soil physical processes, depends critically on the insight of the model builder to separate important aspects of these physical processes from subordinate ones. Even if the hard to obtain, detailed information on soil physical parameters is available, clear insight in the basic processes is still indispensable.

The other development that took place in soil physics, en route towards predicting field behaviour, was the increasing awareness of the so-called spatial variability of soil properties and the consorted efforts to cope with it (for a review by Warrick and Nielsen, see Hillel (3), chapter 13). Two aspects of spatial variability of soil properties require special attention. The first aspect is: how should one sample, within the realm of practicability, for determining a particular soil property? This problem concerns the statistics of so-called regionalised variables or geostatistics. The second aspect of spatial variability is the need for acceptable averaging methods to predict the physical behaviour of three-dimensional field units rather than that of one-dimensional soil columns. Transport processes in soil involve 'series' as well as 'parallel' conductive pathways, which require different methods of averaging. Perhaps a clear example of this is the obvious dominance, for vertical transport of water through soil, of the very small pores in a *horizontal* clay pan, in contrast to that of a single big 'pore' provided by a *vertical* crack. In general, the field-average *effect* of a certain soil physical process is not likely to be similar to that of a homogeneous soil column with field-average *values* of the material properties of concern. Here again, soil physicists who must decide on sampling procedures and/or select parameter values suitable for modeling soil physical processes on a field scale, need a clear insight in the basic features of these processes.

Recently, remote sensing with multispectral radiation, active and passive radar, or radiation temperature appears able to provide relatively cheap, bird's-eye view information on the spatial variability of some physical soil properties (Holz (4)). Admitting that still much remains to be done on the interpretation of such data, combination of dynamic simulation, geostatistical analysis and remote sensing may well lead, eventually, to the much needed predictive insight in soil physical processes on a field scale.

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Appendix 1

Laminar flow of liquid in a cylindrical tube. Poiseuille's law.

The rate of discharge, Q, of a liquid in a cylindrical capillary tube with fixed radius, R (Figure A1.2), depends on the driving force acting on the liquid and on the internal friction forces between liquid molecules, which are characterized by the viscosity of the liquid, η.

The driving force per volume is the gradient of the hydraulic potential, $-\mathrm{d}p_\mathrm{h}/\mathrm{d}x$. This driving force will be considered constant from here on.

The viscosity can be demonstrated with the aid of a layer of liquid between two parallel plates of a solid material (Figure A1.1).

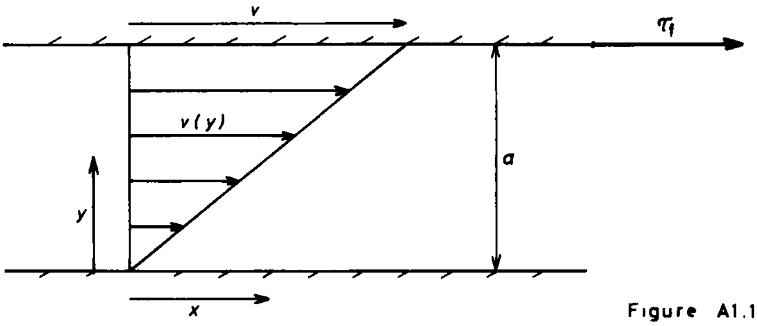

Figure A1.1

If the upper plate is moved relative to the lower plate with a constant velocity, v, the velocity of the liquid will be zero at the boundary with the lower plate and equal to v at the boundary with the upper plate, due to adhesion. In between the plates the velocity $v(y)$ increases linearly with the distance y to the lower plate.

The force divided by area of the upper plate that must be applied to maintain the velocity, v, is equal and opposite to the internal friction force per area, τ_f, in the liquid. This force appears to be inversely proportional to the distance, a, between the plates. This means that τ_f is proportional to the gradient of the velocity $\frac{\mathrm{d}v}{\mathrm{d}y}$. The proportionality factor is the viscosity, η:

$$\tau_\mathrm{f} = \eta \frac{\mathrm{d}v}{\mathrm{d}y} \qquad \text{(A1.1)}$$

The rate of discharge can be calculated when the velocity distribution in the tube is known as a function of r.

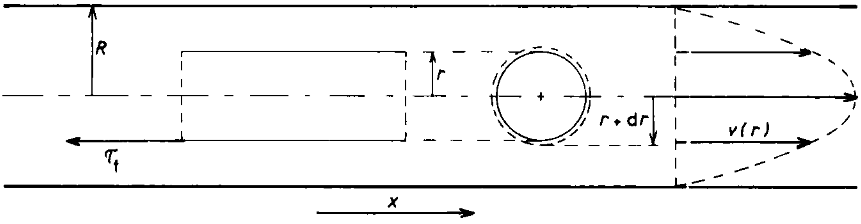

Figure A1.2

For laminar flow the velocity of the liquid is zero at the wall of the tube and has its maximum value in the centre of the tube. Intermediate velocities depend only on the radial distance from the centre, r, and thus are constant in concentric cylindrical surfaces around the centre.

There is a friction force between adjoining concentric liquid layers with different velocities. Consider the liquid cylinder with radius r and length l. The friction force on the cylindrical liquid body per volume is then

$$F^{\mathrm{d}} = \tau_{\mathrm{f}} \frac{2\pi r l}{\pi r^2 l} = \tau_{\mathrm{f}} \frac{2}{r} \tag{A1.2}$$

At mechanical equilibrium (steady laminar flow) the sum of the forces acting on the cylindrical body equals zero:

$$-\frac{\mathrm{d}p_{\mathrm{h}}}{\mathrm{d}x} + \tau_{\mathrm{f}} \frac{2}{r} = 0 \tag{A1.3}$$

Substitution of $\eta \frac{\mathrm{d}v}{\mathrm{d}r}$ for τ_{f} yields:

$$\eta \frac{2}{r} \frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\mathrm{d}p_{\mathrm{h}}}{\mathrm{d}x} \tag{A1.4}$$

$$\text{or } \mathrm{d}v = \frac{1}{2\eta} \frac{\mathrm{d}p_{\mathrm{h}}}{\mathrm{d}x} r \mathrm{d}r \tag{A1.5}$$

Integration yields:

$$v = \frac{1}{4\eta} \frac{\mathrm{d}p_{\mathrm{h}}}{\mathrm{d}x} (r^2 + C) \tag{A1.6}$$

where C is an integration constant. From the boundary condition $v = 0$ for $r = R$ follows: $C = -R^2$.

Thus the velocity of the liquid at radial distance r is:

$$v_r = \frac{1}{4\eta} \frac{\mathrm{d}p_{\mathrm{h}}}{\mathrm{d}x} (r^2 - R^2) \tag{A1.7}$$

The velocity of the liquid in an infinitely thin cylindrical layer between r and $r + \mathrm{d}r$ is then also v_r. The surface area of the cross-section of this cylindrical layer is $2\pi r\mathrm{d}r$.

The discharge rate of the cylindrical layer is then:

$$\mathrm{d}Q = 2v_r\pi r\mathrm{d}r = \frac{\pi}{2\eta}\frac{\mathrm{d}p_\mathrm{h}}{\mathrm{d}x}(r^2 - R^2)\, r\mathrm{d}r \tag{A1.8}$$

and the total discharge rate of the whole tube is

$$Q = \int_0^R \mathrm{d}Q = \frac{\pi}{2\eta}\frac{\mathrm{d}p_\mathrm{h}}{\mathrm{d}x}\int_0^R (r^3 - R^2 r)\, \mathrm{d}r \tag{A1.9}$$

$$\text{or } Q = -\frac{\pi R^4}{8\eta}\frac{\mathrm{d}p_\mathrm{h}}{\mathrm{d}x} \tag{A1.10}$$

Equation A1.10 is known as Poiseuille's law.

Appendix 2

Boltzmann substitution.

We seek the solution of the partial differential Equation 5.18

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial s}\left(D\frac{\partial\theta}{\partial s}\right) \tag{5.18}$$

with initial and boundary conditions

$$\begin{aligned}&\theta=\theta_i \text{ for } s>0 \text{ and } t=0\\&\theta=\theta_i \text{ for } s\rightarrow\infty \text{ and } t\geqslant 0\\&\theta=\theta_0 \text{ for } s=0 \text{ and } t\geqslant 0\end{aligned} \tag{5.17}$$

The partial differential equation can be transformed to an ordinary differential equation by means of the so-called Boltzmann substitution. This substitution consists of the introduction of a new independent variable λ defined as:

$$\lambda=st^{-1/2} \tag{5.19}$$

The substitution leads to a solution of the equation, since the boundary conditions can also be expressed unambiguously in the variable λ:

$$\begin{aligned}&\theta=\theta_i \text{ for } \lambda\rightarrow\infty\\&\theta=\theta_0 \text{ for } \lambda=0\end{aligned} \tag{5.21}$$

The transformation of Equation 5.18 can be done as follows. From Equation 5.19 it follows that:

$$\frac{\partial\lambda}{\partial s}=t^{-1/2} \text{ and } \frac{\partial\lambda}{\partial t}=-\tfrac{1}{2}st^{-3/2}=-\tfrac{1}{2}\lambda t^{-1}$$

Hence, using the chain rule, the left-hand side of Equation 5.18 can be written as

$$\frac{\partial\theta}{\partial t}=\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\frac{\partial\lambda}{\partial t}=-\tfrac{1}{2}\lambda t^{-1}\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} \tag{A2.1}$$

Again, using the chain rule,

$$D\frac{\partial\theta}{\partial s}=D\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\frac{\partial\lambda}{\partial s}=t^{-1/2}D\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}$$

and thus the right-hand side of Equation 5.18 can be written as

$$\frac{\partial}{\partial s}\left(D\frac{\mathrm{d}\theta}{\mathrm{d}s}\right)=\frac{\partial}{\partial s}\left(t^{-1/2}D\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)=t^{-1/2}\frac{\partial}{\partial s}\left(D\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)=$$

$$t^{-1/2}\frac{\partial\lambda}{\partial s}\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(D\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)=t^{-1}\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(D\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right) \tag{A2.2}$$

Combination of Equations 5.18, A2.1 and A2.2 gives

$$-\frac{\lambda}{2}\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}=\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(D\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right) \tag{5.20}$$

Equation 5.20 can be rearranged to

$$-\frac{\lambda}{2}\mathrm{d}\lambda=\frac{\mathrm{d}\lambda}{\mathrm{d}\theta}\mathrm{d}\left(D\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)$$

If D is a constant:

$$-\frac{\lambda}{2}\mathrm{d}\lambda=D\frac{\mathrm{d}\lambda}{\mathrm{d}\theta}\mathrm{d}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)=\frac{D}{\mathrm{d}\theta/\mathrm{d}\lambda}\mathrm{d}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)$$

Integration yields

$$-\frac{\lambda^2}{4}=D\ln\left(\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)+C$$

or $-\dfrac{\lambda^2}{4D}=\ln\left(C_1\dfrac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)$

or $\exp(-\lambda^2/4D)=C_1\dfrac{\mathrm{d}\theta}{\mathrm{d}\lambda}$ $\quad$ (A2.3)

where C and C_1 are constants.
Rearranging Equation A2.3 gives:

$$\mathrm{d}\theta=\frac{\exp(-\lambda^2/4D)}{C_1}\mathrm{d}\lambda=\frac{2\sqrt{D}\exp(-(\lambda/2\sqrt{D})^2)}{C_1}\mathrm{d}(\lambda/2\sqrt{D})$$

Substitution of $(\lambda/2\sqrt{D})=u$ gives:

$$\mathrm{d}\theta=\frac{2\sqrt{D}}{C_1}\exp(-u^2)\,\mathrm{d}u$$

or after integration

$$\theta=\frac{2\sqrt{D}}{C_1}\int\exp(-u^2)\,\mathrm{d}u \tag{A2.4}$$

According to the initial and boundary conditions, λ (and thus also u) varies between 0 and ∞, while θ varies from θ_i to θ_0. Thus Equation A2.4 can be written as

$$\theta(u) = \frac{2\sqrt{D}}{C_1}\int_0^u \exp(-\beta^2)\,\mathrm{d}\beta + C_2 \tag{A2.5}$$

where C_2 is an integration constant and β is a dummy variable for u.

The initial condition $\theta = \theta_i$ for $\lambda \to \infty$, and thus $u \to \infty$, gives

$$\theta_i = \frac{2\sqrt{D}}{C_1}\int_0^\infty \exp(-u^2)\,\mathrm{d}u + C_2$$

and, using $\int_0^\infty \exp(-u^2)\,\mathrm{d}u = \frac{\sqrt{\pi}}{2}$, it follows:

$$C_2 = \theta_i - \frac{\sqrt{\pi D}}{C_1}$$

and thus Equation A2.5 becomes:

$$\theta(u) = \theta_i + \frac{2\sqrt{D}}{C_1}\int_0^u \exp(-\beta^2)\,\mathrm{d}\beta - \frac{\sqrt{\pi D}}{C_1} \tag{A2.6}$$

The boundary condition $\theta = \theta_0$ for $\lambda = 0$, and thus $u = 0$, gives:

$$\theta_0 = \theta_i - \frac{\sqrt{\pi D}}{C_1} \text{ and thus } C_1 = \frac{-\sqrt{\pi D}}{\theta_0 - \theta_i}$$

Substitution of C_1 in Equation A2.6 gives

$$\theta(u) = \theta_i + (\theta_0 - \theta_i)\left(1 - \frac{2}{\sqrt{\pi}}\int_0^u \exp(-\beta^2)\,\mathrm{d}\beta\right) \tag{A2.7}$$

The function $\frac{2}{\sqrt{\pi}}\int_0^u \exp(-\beta^2)\,\mathrm{d}\beta$ is known as the error function erf (u), while $1 - \text{erf}(u)$ is the complementary error function, erfc (u).

Using this function and substituting $\lambda/2\sqrt{D}$ for u, one finally finds θ as a function of λ:

$$\theta(\lambda) = \theta_i + (\theta_0 - \theta_i)\,\text{erfc}\,(\lambda/2\sqrt{D}) \tag{A2.8}$$

or, by substituting $s/\sqrt{t}$ for λ, as a function of s and t:

$$\theta(s,t) = \theta_i + (\theta_0 - \theta_i)\,\text{erfc}\,(s/2\sqrt{Dt}) \tag{A2.9}$$

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Appendix 3

Green and Ampt model. Vertical infiltration.

This appendix shows the derivation of Equation 5.46 in detail.
The logarithmic term of Equation 5.44, can be written as

$$\ln\left(\frac{s_f - h_f}{-h_f}\right) = \ln\left(1 - \frac{s_f}{h_f}\right) = \ln(1 + y) \tag{5.45}$$

where $y = -\frac{s_f}{h_f}$. For $y^2 < 1$, $\ln(1 + y)$ can be expanded as

$$\ln(1 + y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots \tag{A3.1}$$

Thus, Equation 5.44 becomes

$$t = \frac{\theta_t - \theta_i}{k_t}\left(s_f - s_f - \frac{s_f^2}{2h_f} - \frac{s_f^3}{3h_f^2} - \frac{s_f^4}{4h_f^3} - \ldots\right) =$$

$$-\frac{\theta_t - \theta_i}{2k_t h_f} s_f^2 \left(1 + \frac{2s_f}{3h_f} + \frac{s_f^2}{2h_f^2} + \ldots\right) \tag{A.3.2}$$

To find s_f as a function of t we first take the square root of Equation A3.2:

$$t^{1/2} = \left(-\frac{\theta_t - \theta_i}{2k_t h_f}\right)^{1/2} s_f \left(1 + \frac{2s_f}{3h_f} + \frac{s_f^2}{2h_f^2} + \ldots\right)^{1/2} \tag{A3.3}$$

In general, if $P = a + bx + cx^2 + dx^3 + \ldots$, then

$$P^{1/2} = a^{1/2}\left[1 + \frac{b}{2a}x + \left(\frac{c}{2a} - \frac{b^2}{8a^2}\right)x^2 + \ldots\right] \tag{A3.4}$$

Using this in Equation A3.3 and rearranging terms leads to:

$$\left(\frac{-2k_t h_f}{\theta_t - \theta_i} t\right)^{1/2} = s_f + \frac{1}{3h_f} s_f^2 + \frac{7}{36\,h_f^2} s_f^3 + \ldots \tag{A3.5}$$

Again, in general, if $Q = ax + bx^2 + cx^3 + \ldots$, where $a \neq 0$, then

$$x = AQ + BQ^2 + CQ^3 + \ldots \tag{A3.6}$$

where $A = \frac{1}{a}$, $B = -\frac{b}{a^3}$, $C = \frac{1}{a^5}(2b^2 - ac)$, etc.

This can be used to reverse Equation A3.5. The result will give s_f as a function of t in series form:

$$s_f = f_1 t^{1/2} + f_2 t + f_3 t^{3/2} + \ldots \tag{5.46}$$

where $$f_1 = \left(\frac{-2k_t h_f}{\theta_t - \theta_i} \right)^{1/2} \tag{5.47}$$

$$f_2 = \frac{2k_t}{3(\theta_t - \theta_i)} \tag{5.48}$$

$$f_3 = \left(\frac{-k_t^3}{162(\theta_t - \theta_i)^3 h_f} \right)^{1/2} \tag{5.49}$$

This series is convergent for t smaller than a certain value, which depends on the value of the different coefficients f_1, f_2, f_3, etc.

Subject index