Journal of Geochemical Exploration 108 (2011) 220–232

Contents lists available at ScienceDirect

Journal of Geochemical Exploration

journal homepage: www.elsevier.com/locate/jgeoexp

Delineation of mineralization zones in porphyry Cu deposits by fractal concentration–volume modeling

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ARTICLE INFO

Article history:
Received 18 December 2009
Accepted 2 March 2011
Available online 11 March 2011

Keywords:
Mineralization zones
Porphyry deposits
Concentration–volume (C–V) fractal model
Sungun
Chah-Firuzeh
FCV software

ABSTRACT

The purpose of this study was to identify the various mineralization zones especially supergene enrichment and hypogene in two different Iranian porphyry Cu deposits, based on subsurface data and by using the proposed concentration–volume (C–V) fractal method. The Sungun and Chah-Firuzeh porphyry Cu deposits, which are situated in NW and SE Iran, respectively, were selected for this study. Straight lines fitted through log–log plots showing C–V relations for Cu were employed to separate supergene enrichment and hypogene zones from oxidation zones and barren host rocks in the two deposits and to distinguish a skarn mineralized zone from the hypogene zone in Sungun deposit. In the proposed C–V fractal method, the identification of mineralization zones is based on power–law relationships between Cu concentrations and the volume of rocks hosting porphyry Cu mineralization. Separate subsurface data from the two deposits were analyzed by C–V fractal method and the results have been compared with geological models which included alteration and mineralogical models. The comparison shows that the interpreted zones based on the C–V fractal method are consistent with the geological models. The proposed C–V method is a new approach to defining zones in a mineral deposit and there was no commercial software available to perform the relevant calculations; therefore, a fractal concentration–volume (FCV) software was designed by the authors to achieve this goal.

1. Introduction

Recognition of supergene enrichment and hypogene zones from oxidation zones and barren host rocks is one of the main aims in exploration of porphyry Cu deposits. Customized geological methods for zone detection and recognition in porphyry deposits are usually based on mineralographical, petrographical and alteration assemblages of alterated minerals, specifically in potassic and phyllic alteration types (Beane, 1982; Berger et al., 2008; Schwartz, 1947; Sillitoe, 1997). A conceptual model of lateral and vertical variations in mineralogy within alteration zones was first proposed by Lowell (1968) and, later, its application was extended to many other deposits in the North American Cordillera (Lowell and Guilbert, 1970). Based on this model, similar models were developed and introduced with potassic alteration usually located at the central and deeper parts of porphyry Cu deposits (Cox and Singer, 1986; Melfos et al., 2002; Sillitoe and Gappe, 1984). In addition, fluid inclusion studies (e.g., Asghari and Hezarkhani, 2008; Nash, 1976; Roedder, 1971; Ulrich et al., 2001) and investigations of ^{34}S isotopes (e.g., Wilson et al., 2007) are other methods that have been used for recognition of different zones in porphyry Cu deposits. All the methods are inadequate, however, if variations in grade of the principal ore element are not considered in exploration of porphyry Cu deposits. It is a fact that ore grades vary with changes in the geological properties, such as mineralogy and alterations, of different zones in a porphyry Cu deposit. Different geological interpretations could be presented for defining boundaries of different zones in porphyry Cu deposits, which may also lead to different results if the ore element grade distribution is not taken into consideration.

Euclidian geometry identifies geometrical shapes with an integer dimension say 1D, 2D, 3D, etc. However, there are many other shapes or spatial objects, whose dimensions cannot be mathematically described by integers but by real numbers or fractions. These spatial objects are called fractals. In abstract form, fractals describe complexity in data distribution by estimation of their fractal dimensions. Various geochemical processes can be described based on differences in fractal dimensions obtained from analysis of relevant geochemical data. The fractal theory, which was developed by Mandelbrot (1983), has been widely applied in the geosciences since the 1980s up to the present, (e.g., Agterberg et al., 1993; Ali et al., 2007; Cheng et al., 1994; Goncalves

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doi:10.1016/j.gexplo.2011.03.005

et al., 2001; Sim et al., 1999; Turcotte, 1986; Wei and Pengda, 2002). Methods of fractal analysis also serve to illustrate relationships of geological, geochemical and mineralogical settings with spatial information derived from analysis of mineral deposit occurrence data (Carranza, 2008; Carranza et al., 2009; Goncalves et al., 2001). However, good knowledge of geological and geochemical environmental controls on mineralization is essential in the recognition and classification of geochemical populations based on methods of fractal analysis (Carranza, 2009; Carranza and Sadeghi, 2010; Cheng, 1999; Goncalves et al., 2001; Li et al., 2003; Sim et al., 1999). Fractals are characterized by a scaling law that relates two variables: the scale factor and the object being measured. This scaling relationship is described by a power law function, which in turn describes the inherent physical attributes of the object being analyzed (Lauwerier, 1991; Ortega et al., 2006; Takayasu, 1990). The exponent of the power law function refers to the fractal dimension. Fractal dimensions in geological and geochemical processes correspond to variations in physical attributes such as rock type, vein density or orientation, fluid phase, alteration phenomena, structural feature or dominant mineralogy, and so on (Sim et al., 1999). It has been shown that hydrothermal mineral deposits, such as porphyry Cu deposits, exhibit non-Euclidean variations in ore element concentrations in rocks and related surface materials such as water, soil, stream sediment, till, humus, and vegetation (Cheng, 2007; Cheng and Agterberg, 2009).

Therefore, fractal dimensions of variations in geochemical data can provide useful information and applicable criteria to recognize and classify mineralized and barren zones within a study area. Various log–log plots in fractal methods tools are proper methods for separation of geological recognizing and classifying populations in geochemical data because threshold values can be recognized and determined are indicated as breakpoints in those plots. These geochemical threshold values recognized though fractal analysis is usually correlated with explainable by geological field observations features or processes. These investigations are the basis for this proposed fractal method application and usage for zones separation in porphyry deposits.

The aim of this study was to develop and apply a concentration–volume (C–V) fractal method to distinguish supergene enrichment and hypogene zones, from oxidation zones and barren host rocks, based on the distribution of Cu grades, in the Sungun and Chah-Firuzeh porphyry Cu deposits located, respectively, in NW and SE Iran. A fractal concentration–volume (FCV) software was developed in Visual Basic for application (VBA) in Excel to implement the C–V fractal method. Subsequently, a discussion is presented whereby zonations defined by Cu threshold values are correlated with explained by relevant geological data, specifically alteration data. In addition, we attempted to utilize the main results of the C–V fractal analysis and the interpretations derived in this study in 3D analysis of geochemical distribution.

2. Fractal concentration–volume method

Cheng et al. (1994) proposed the fractal concentration–area (C–A) model for separating geochemical anomalies from background in order to characterize the distribution of major, minor and trace element concentrations in relation to the Mitchell-Sulphurets porphyry system in British Columbia (Canada). This model has the general form:

$$A(\rho \le \upsilon) \propto \rho^{-a1};\ A(\rho \ge \upsilon) \propto \rho^{-a2} \tag{1}$$

where $A(\rho \le \upsilon)$ and $A(\rho \ge \upsilon)$ denote areas (A) with concentration values ρ that are, respectively, smaller and greater than contour value ρ defining that areas (υ represents the threshold), which define those areas and a1 and a2 are characteristic exponents for both criteria. In log–log plots of concentration contours versus areas, certain concentration contours representing breakpoints in the plots are considered threshold values separating geochemical populations in the data. Zou et al. (2009) applied the fractal C–A method to characterize the vertical distribution of element concentrations in the Qulong copper deposit, Tibet, western China. Fractal models have been used to identify the vertical distribution properties of Cu concentration values in mineralized and non-mineralized zones. Cheng (2008) described hydrothermal processes in the Earth's crust associated with ore deposits, such as porphyry ore deposits, which are characterized by high metal concentrations of metals with having fractal or multifractal properties.

Cheng et al. (1994) and Zou et al. (2009) have suggested that the fractal C–A model, are applicable in volume or can be extended to volumetric extensions because element distributions in horizontal or vertical directions are in accordance with fractal models. Different forms of the C–A model expressed in Eq. (1) can be rewritten as:

$$A_h(\rho \le \upsilon) \propto \rho^{-a1};\ A_h(\rho \ge \upsilon) \propto \rho^{-a2} \tag{2}$$

$$A_v(\rho \le \upsilon) \propto \rho^{-a1};\ A_v(\rho \ge \upsilon) \propto \rho^{-a2} \tag{3}$$

where $A_h(\rho \le \upsilon)$, $A_h(\rho \ge \upsilon)$, $A_v(\rho \le \upsilon)$ and $A_v(\rho \ge \upsilon)$ denote two areas with concentration values smaller and values greater than the contour value ρ defining that area respectively for all variables are the same as those in Eq. (1), but the subscripts h and v denote areas described in horizontal and vertical section directions, respectively. Consequently, the relationships defined in Eqs. (2) and (3) suggest that element distributions in a volume also follow a fractal model. The proposed fractal C–V model can be expressed, therefore, in the following general form:

$$V(\rho \le \upsilon) \propto \rho^{-a1};\ V(\rho \ge \upsilon) \propto \rho^{-a2} \tag{4}$$

where, $V(\rho \le \upsilon)$ and $V(\rho \ge \upsilon)$ denote volumes (V) with concentration values (ρ) that are, respectively, smaller and values greater than contour values (υ), which defines those volumes and a1 and a2 are characteristic exponents. Based on this kind of characterization, it is assumed hypothesized that different zones in porphyry Cu deposits have fractal properties, and they can occur where as described by power–law relationships between their ore element concentrations and volumetric extensions. In log–log plots of concentration contours versus volumes, certain concentration contours representing breakpoints in the plots are considered threshold values separating geochemical populations in the data. To calculate $V(\rho \le \upsilon)$ and $V(\rho \ge \upsilon)$ enclosed by a concentration contour in a 3D model, in this study, the original borehole data of ore element concentrations were interpolated by using the geostatistical and inverse distance weighted (IDW) method. The interpolated 3D block model was used for the purpose of this study. Volumes $V(\rho \le \upsilon)$ and $V(\rho \ge \upsilon)$ are equal to the unit volume of a voxel (or volume cell) multiplied by the number of voxels with concentration values (ρ) that are, respectively, smaller and greater than a certain concentration value (υ). Log–log plots of the concentration contours versus the corresponding volumes [$V(\rho \le \upsilon)$ and $V(\rho \ge \upsilon)$] follow a power–law relationship.

Breaks between straight-line segments in those log–log plots represent threshold values separating populations of geochemical concentration values representing lithological and mineralogical zonations due to distinct geochemical processes. In porphyry Cu deposits, zones of high Cu concentrations comprise relatively few voxels in a 3D block model, whereas zones of low Cu concentrations comprise numerous voxels. Therefore threshold values in this recognized by applying the proposed fractal C–V model likely represent boundaries between different ore zones and barren wall rocks.

3. Geological setting of the case study areas

3.1. Sungun porphyry Cu deposit

The Sungun porphyry Cu deposit is situated about 100 km NE of Tabriz, Eastern Azerbaijan province, NW Iran. The porphyry stock associated with the deposit intruded into the Sungun anticline and into Cretaceous limestone and Eocene volcanic-pyroclastic rocks of andesitic to trachytic composition. Based on petrography and geochemical studies (Hezarkhani and Williams-Jones, 1998), the Sungun deposit is a composite stock comprising an early monzonite/quartzmonzonite assemblage and a later diorite/granodioritic phase. Copper mineralization was accompanied by both potassic and phyllic alterations. Field observations and petrographic studies (Hezarkhani and Williams-Jones, 1998), reveal that emplacement of the Sungun stock took place in several intrusive pulses, each with an associated hydrothermal activity. Early hydrothermal alteration produced a potassic assemblage, mainly orthoclase-biotite, in the central parts of the stock; propylitic alteration developed simultaneously with potassic alteration, but in the peripheral parts of the stock, and phyllic alteration occurred later, overprinting the earlier alterations (Hezarkhani and Williams-Jones, 1998). The Sungun deposit, with over 500 Mt resources of Cu–Mo sulfide ore, is located in the NW part of the NW–SE trending main Iranian Cenozoic magmatic belt of Urumieh-Dokhtar or Sahand-Bazman, which is one of the subdivisions of the Zagros orogenies (Alavi, 1994; Berberian and King, 1981; Stocklin, 1977). Almost all of the important Iranian porphyry deposits such as Sarcheshmeh, Sungun, Meiduk and Darrehzar occur within this belt (Shahabpour and Doorandish, 2008). Based on the geological map of Sungun deposit (Fig. 1), relationships exist among the various subtypes of Sungun intrusive rocks and the periphery of the mineralized zone. The porphyry quartz monzonites surround all other porphyry phases to the south and west, whereas Cretaceous limestones and the associated skarns surround the porphyry phases to the north and east. Andesitic dykes are distributed in the northern and western parts of the deposit, mainly outside the stock. Skarn type

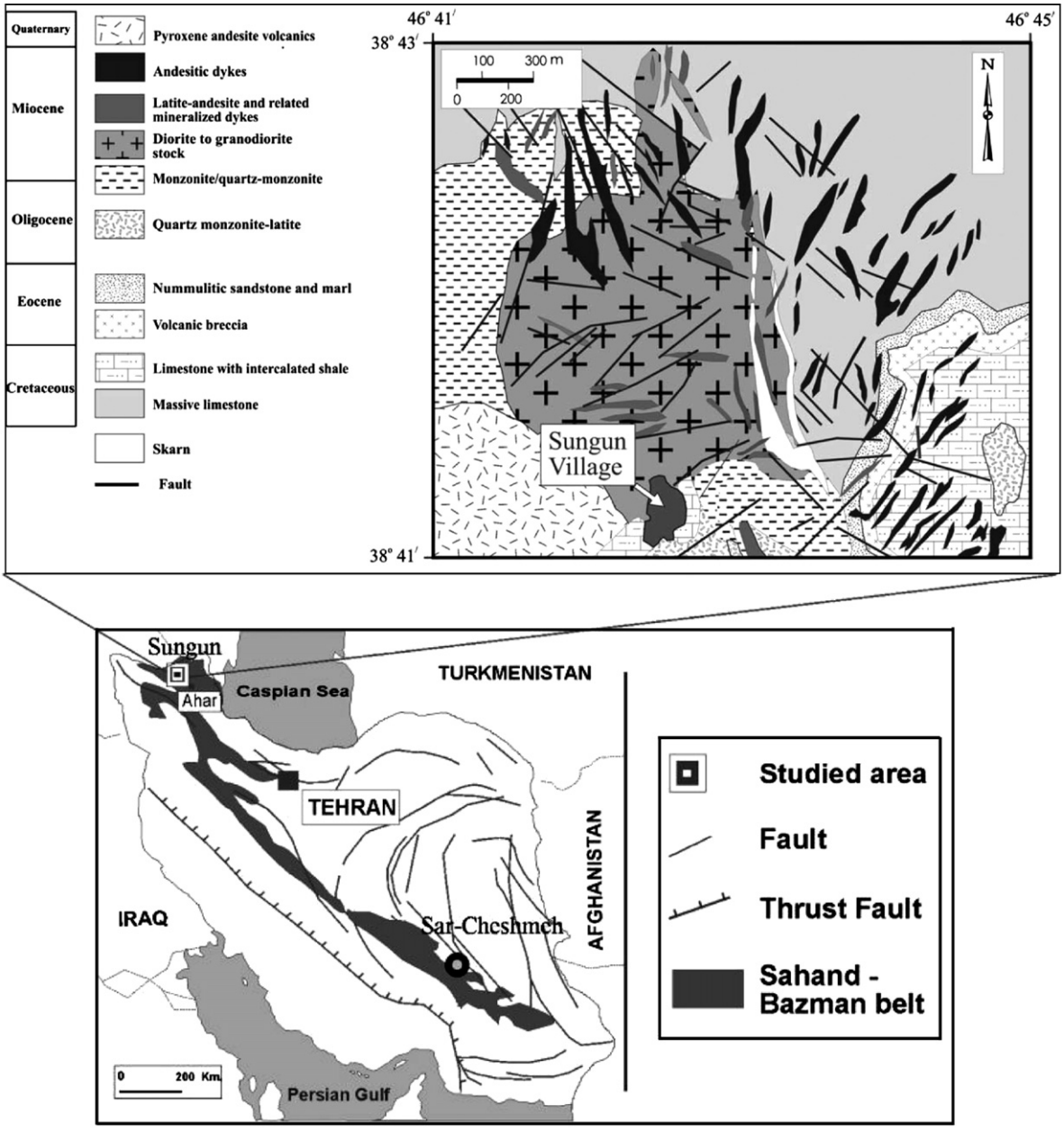

Fig. 1. Geological map of Iran (modified from: Shahabpour and Doorandish, 2008; Stocklin, 1977) showing the Urumieh-Dokhtar or Sahand-Bazman magmatic belt and the geological map of the Sungun deposit area (modified from Mehrpartou, 1993).

alteration, and the associated mineralization, occurs predominantly adjacent to the eastern margin of the stock (Fig. 1) (Hezarkhani and Williams-Jones, 1998; Mehrpartou, 1993). There are three distinct types of sulphide mineralizations including hypogene, contact-metasomatic (skarn), and supergene enrichment. Hypogene copper mineralization was probably formed during potassic alteration and to a lesser extent during phyllic alteration, and exists as disseminations and in veinlet form. During potassic alteration, the copper mineralization was deposited as chalcopyrite along with minor bornite; and the later hypogene copper mineralization was deposited mainly as chalcopyrite. Copper mineralization increments toward the margins of the central potassic zone with concentration values ranging from less than 0.20 wt.% to 0.85 wt.%. While the pyrite content is highest (3–10 vol.% of the rock) in the marginal quartz-sericite (phyllic) zone, a supergene enrichment blanket over the deposit, and a deeper clay alteration of feldspars are also present (Asghari et al., 2009).

3.2. Chah-Firuzeh porphyry Cu deposit

The Chah-Firuzeh porphyry Cu deposit is situated about 35 km north of Shahre Babak in Kerman province, SE Iran (Fig. 2). Its initial exploration was started in the 1970s by Yugoslavian geologists. The detailed exploration was carried out only in the recent years. The Chah-Firuzeh deposit is located in the main Iranian Cenozoic magmatic belt of Urumieh-Dokhtar, which is one of the subdivisions of Zagros orogenies (Alavi, 1994). It is associated with mid to late Miocene diorite/granodiorite to quartz-monzonite stocks. The Chah-Firuzeh stock is a complex igneous body consisting of two different intrusive phases: 1) diorite/granodiorite and 2) quartz monzonite/monzonite (Hezarkhani, 2009). Eocene volcanics such as andesite, dacite, tuff and lapilli tuff throughout the region were intruded by Miocene intrusive of monzonitic composition. The Chah-Firuzeh deposit is a well-zoned porphyry system composed of four K-silicate centers surrounded by quartz-sericite-carbonate-pyrite and propylitic alteration. Dioritic and latite porphyry dykes are present in this deposit (Kazemi Mehrnia et al., 2007). The Chah-Firuzeh stock is extensively and highly altered. Its hydrothermal alterations and mineralization are located in the central parts of the stock. Early hydrothermal alteration was dominantly potassic and propylitic, and was followed by later phyllic, silicic and argillic alteration. Hypogene copper mineralization appears to have been intruded during the transition from potassic to phyllic alteration and mainly during phyllic alteration. During potassic alteration, the copper mineralization consisted of chalcopyrite and minor bornite, and chalcopyrite was the only copper phase during later hypogene mineralization. Maximum concentration of Cu mineralization exists in phyllic zone and it is chalcocite in nature. In studied cores, maximum Cu grade is higher than 2% but it is rare. Supergene enrichment zone determined by geological studies indicate presence of, chalcocite, chrysocolla, azurite, malachite and digenite. (Hezarkhani, 2009).

4. Implementation of concentration–volume fractal method

An ore element distribution evaluated by estimated a block-model provides a smoothed version of the spatial distribution of that ore element. The FCV software was designed for processing of the data and for C–V calculations needed for the analysis. Since the numbers of voxels are very large and their grade variations have an extended range, the software is suitable for usage and is able to handle the huge data volume. It is notable to mention that the more voxels we make the better results in the C–V method are obtained according to optimum voxel size. Optimum voxel size is determined based on geometrical properties of deposit and grid drilling dimensions (David, 1970). The FCV software was written in VBA for Excel programming environment is easy-to-learn and user-friendly and, its capability to be accompanied on-demand usage of data worksheets, and for its programming integrated developing environment.

Grade distribution block models were generated via ordinary kriging method using the Datamine software. Ordinary kriging (OK) was developed by Matheron in the early sixties and plays a special role because it is compatible with a stationary model, only involves the variogram, and is in fact the form of kriging used most (Chile's and Delfiner, 1999). OK estimates based on moving average of the variable of interest satisfying different dispersion forms of data e.g. sparse sampling points (Goovaerts, 1997). The derived block models were used as input to the FCV software. The Sungun deposit data were obtained from 238 drill cores covering a total length of about 80,000 m. Rock samples from the drill cores were used to construct the geological models and 38,500 lithogeochemical samples from drill cores in the Sungun deposit were analyzed for Cu and the Cu data for ore element distribution estimation. In the Chah-Firuzeh deposit, the data were obtained from 26 drill cores covering a total length of 10,000 m, from which 4600 lithogeochemical samples were collected from and analyzed by ICP-MS for Cu. The experimental variograms for the Cu data in two deposits and their adjusted model are presented in Fig. 3.

4.1. C–V fractal model of the Sungun deposit

The Sungun deposit is modeled with 482,160 voxels. Each voxel has a dimension of 25 m × 25 m × 12.5 m in the X, Y and Z directions, respectively. Different $V(\rho \le \upsilon)$ and $V(\rho \ge \upsilon)$ (volumes occupied by different Cu grades of ρ) were calculated for different Cu grades (ρ) in the block model of Sungun deposit. Thresholds values were identified from log–log plot (Fig. 4), which demonstrates a power–law relationship between Cu concentrations and volumes occupied. Depicted arrows in log–log plot show threshold values (breakpoints). These are separating different straight lines segments in the log–log plot. There is a sudden change in the rate of decrease of the volume enclosed by high values of Cu (Fig. 4). Based on the log–log plot, Cu concentrations in supergene enrichment zone are considered to range between 0.7% and 1.4%. Above this range of Cu concentrations the slope of the straight line fitting the data is considered to represent high Cu value (Supergene enrichment zone) in porphyry stock. Relationships between Cu concentrations and volumes for supergene enrichment and hypogene zones in porphyry stock can be represented by the following equations of straight line segments in the log–log graph (Fig. 4):

$$V(\rho \le \upsilon) = 10^{8.8546} \quad \text{for barren host rock} \tag{5}$$

$$V(\rho \ge \upsilon) = 10^{8.5375}\rho^{-0.3795} \quad \text{for oxidation zone} \tag{6}$$

$$V(\rho \ge \upsilon) = 10^{8.2503}\rho^{-1.0645} \quad \text{for hypogene zone} \tag{7}$$

$$V(\rho \ge \upsilon) = 10^{7.3234}\rho^{-7.0526} \quad \text{for supergene enrichment zone} \tag{8}$$

$$V(\rho \ge \upsilon) = 10^{7.1132}\rho^{-3.6595} \quad \text{for skarn zone} \tag{9}$$

$$V(\rho \ge \upsilon) = 10^{11.421}\rho^{-17.598} \quad \text{for skarn enrichment zone} \tag{10}$$

These relationships show that the C–V model allows recognition of different geochemical populations in this deposit. The hypogene zone has a concentration range between 0.4% and 0.7% Cu. Supergene enrichment zone has a Cu concentration range between 0.7% and 1.4%. The first threshold from the left of the graph is about 0.1%, which is interpreted to be the threshold of background for the ore element of this deposit (barren host rocks). Between second and third thresholds from the left of the graph, Cu concentration between 0.1% and 0.4%, oxidation zone is interpreted. At Cu% equal to

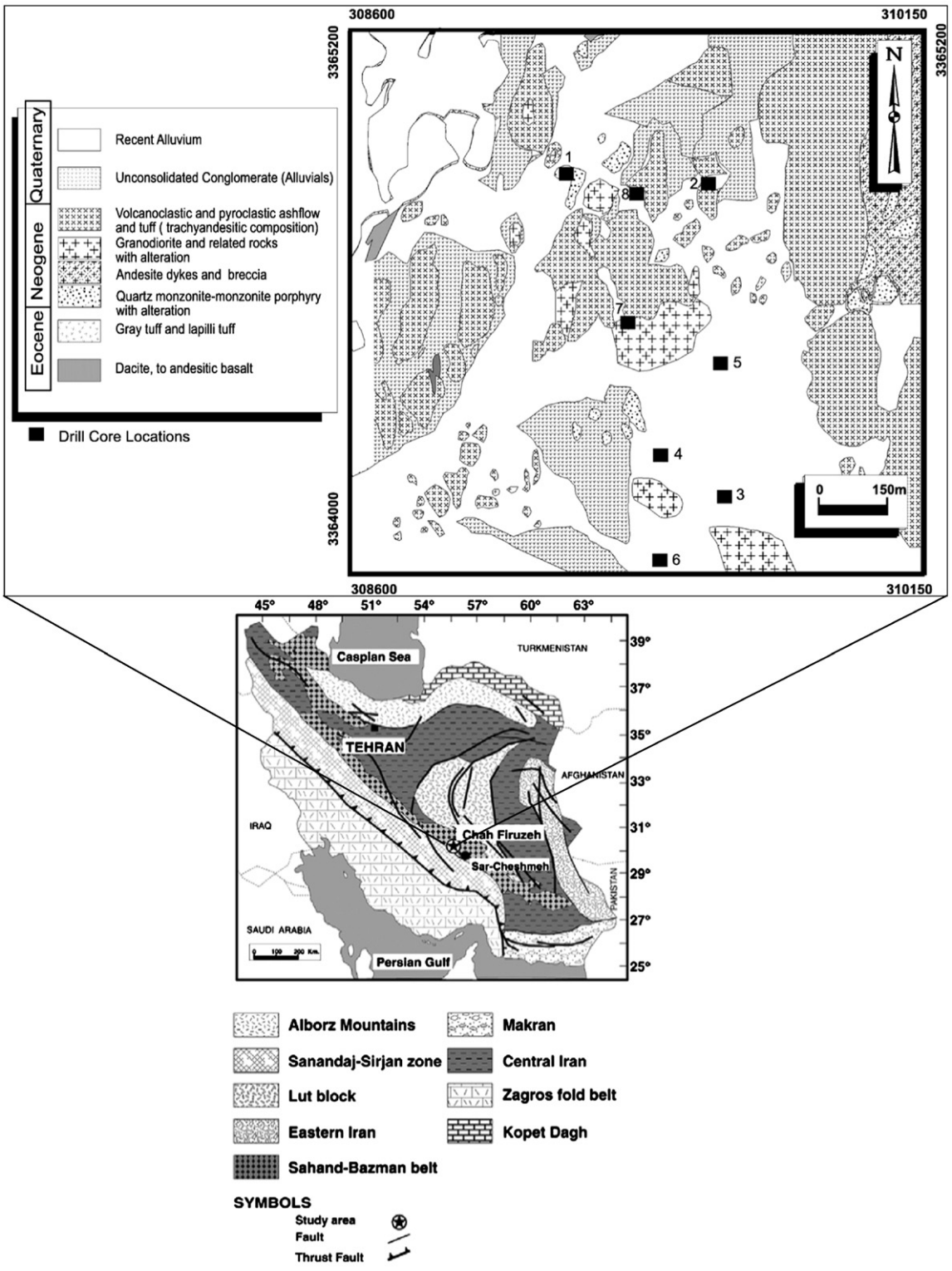

Fig. 2. Geological map of Iran showing major lithotectonic units (Hezarkhani, 2009; Stocklin, 1977) showing the Urumieh-Dokhtar or Sahand-Bazman magmatic belt and the geological map of the Chah-Firuzeh deposit area (Hezarkhani, 2009).

1.25%, an enriched copper mineralization is beginning to build up (skarn zone). It can be interpreted as a new mineralization stage separated from porphyry zones in this deposit. Threshold values defining different zones are given in Table 1. There is an apparent multifractal behavior that shows that there are two stages for copper mineralization, as depicted in Fig. 4. It can be assumed that spatial variability of ore element, i.e., Cu, which is our interest in this deposit, behaves as multifractal nature instead of mono-fractal. Skarn zone is started from Cu% equal to 1.25% but an enriched part of skarn has Cu% between 2 and 2.4 (Table 1).

Conversely, the ore volume with concentration values less than ρ, as expressed by $V(T)\text{-}V(\rho \geq \upsilon)$, where $V(T)$ is the total volume, is increased proportionally with increasing values of ρ. Cheng et al. (1994) demonstrated that element concentrations per unit area satisfy a fractal/multifractal model or that area $A(\rho)$ has a power–law relationship with ρ. As unit volume consists of several unit areas, it

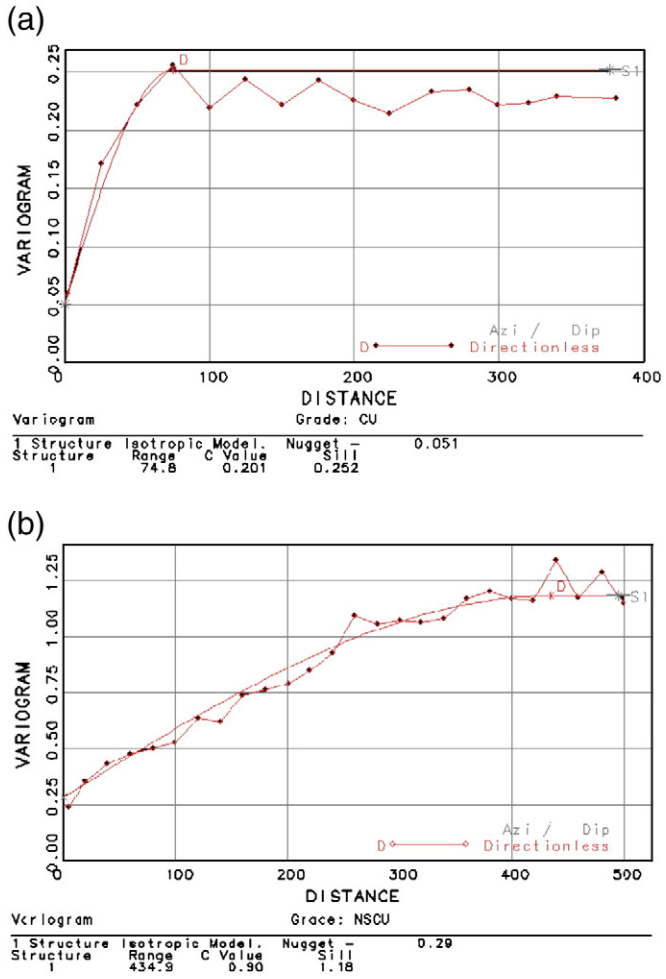

Fig. 3. Semi-variogram of Cu data in Sungun deposit (a) and Chah-Firuzeh deposit (b).

can be assumed that a power–law type relationship also exists between volume and concentration. If concentration per unit volume follows a fractal model, this power–law relationship has only one exponent. If concentration per unit volume satisfies a multifractal model, then at least one power–law relations between $V(\rho \geq \upsilon)$ and ρ can be interpreted and established. For a range of ρ, the multifractal power–law relation is:

$$V(T) - V(\rho \geq \upsilon) = C\rho^{-\beta} \tag{11}$$

where C is constant and β is an "exponent" with the maximum singularity "exponent" that represents the power law relation between

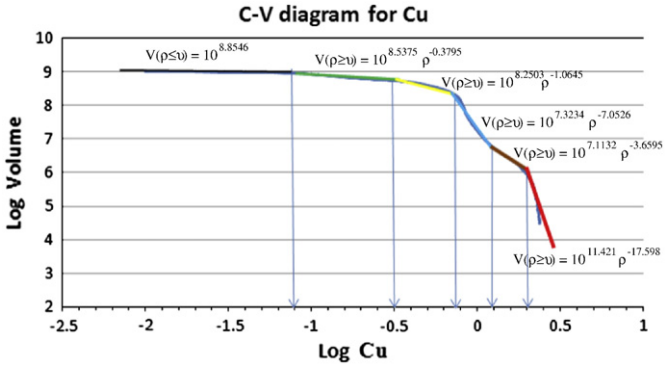

Fig. 4. Log–log graph of volume versus Cu grade in the Sungun deposit.

Table 1
Thresholds concentrations obtained concentration using C–V model based on Cu% in the Sungun deposit.

Mineralization stage	Zone	Threshold Cu%	Range Cu%
Primary	Barren host rocks		<0.1
Primary	Oxidation zone	0.1	0.1–0.4
Primary	Hypogene	0.4	0.4–0.7
Primary	Supergene enrichment	0.7	0.7–1.4
Secondary	Skarn	1.25	1.25–2
Secondary	Skarn enrichment	2	2–2.4

$V(\rho \geq \upsilon)$ and ρ. The result shown in Fig. 5 suggests that volumes with concentration less than ρ have at least two power–low relationships with ρ. Three geochemical populations are existed in log–log plot, as illustrated in Fig. 5. First population is depicted Cu% lower than 0.15% that it represents barren host rocks. Medium population shows the relation between $V(T)$-$V(\rho \geq \upsilon)$ and Cu concentration values (ρ) ranging from 0.15% to 1.2% that could represent mineralization zones in porphyry stock (Supergene enrichment and hypogene). Last population begins at Cu 1.2% that could be interpreted as skarn zone.

4.2. C–V fractal model of the Chah-Firuzeh deposit

This deposit is modeled with 285,907 voxels, each measuring 20 m × 20 m × 10 m along X, Y and Z directions, respectively. There are power–law relationships between Cu concentrations and their volumes. A threshold value representing supergene enrichment zone coincides with a sudden change in the rate of decrease of the volume enclosed by high values of Cu (Fig. 6). Depicted arrows in log–log plot show threshold values (breakpoints). These are separating different straight lines segments in log–log plot. The graph suggests that supergene enrichment zone is defined by Cu concentrations equal to or greater than 0.6%. For these Cu concentrations the slope of the straight line fit is close to 90° and the relationship between Cu concentration and volume is described by the Eq. (12) as follows:

$$V(\rho \leq \upsilon) = 10^{8.3637} \text{ for barren host rock} \tag{12}$$

$$V(\rho \geq \upsilon) = 10^{7.4005}\rho^{-0.9133} \text{ for oxidation zone} \tag{13}$$

$$V(\rho \geq \upsilon) = 10^{8.8268}\rho^{-0.094} \text{ for hypogene zone} \tag{14}$$

$$V(\rho \geq \upsilon) = 10^{8.8125}\rho^{-0.0437} \text{ for supergene enrichment zone.} \tag{15}$$

This relationship shows that the C–V is valid for representation of different geochemical populations in the Chah-Firuzeh deposit. The derived or obtained Eq. (9) is suitable to be applied to hypogene zone.

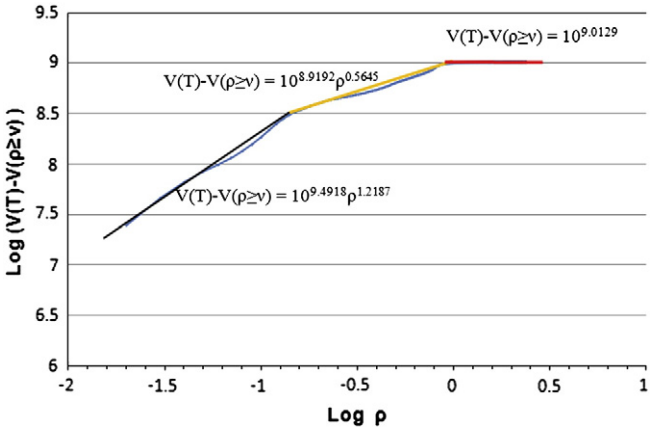

Fig. 5. Log–log graph of $V(T)$-$V(\rho \geq \upsilon)$ versus ρ [Eq. (11)] for the Sungun deposit.

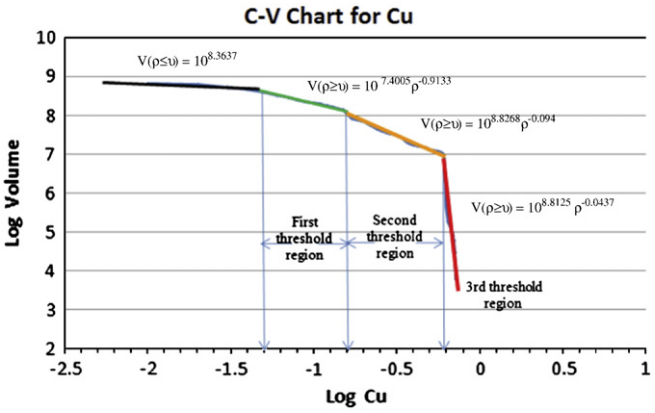

Fig. 6. Log–log graph of volume versus Cu grade in the Chah-Firuzeh deposit.

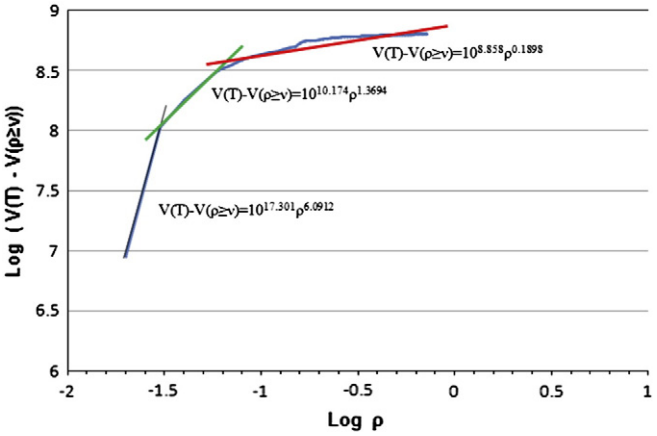

Fig. 7. Log–log graph of V(T)-V($\rho \geq \upsilon$) versus p [Eq. (11)] for the Chah-Firuzeh deposit.

This zone has Cu concentrations ranging between 0.16% and 0.6%, as depicted in Table 2. This zone has a Cu concentration higher than 0.6% which is supergene enrichment zone after the last threshold, as illustrated in Fig. 6. Oxidation zone is defined between Cu concentration of 0.05% and 0.16% (the first and the second thresholds). The second and third threshold regions are depicted in Fig. 6. The first threshold is about 0.05%, which is interpreted to be the upper limit of background for the ore element in this deposit. The zone represents barren host rocks with Cu% lower than 0.05%, as presented in Table 2. It can be assumed that spatial variability of ore element, i.e., Cu of interest in this deposit, is of multifractal nature instead of monofractal. Threshold values defining different zones in the Chah-Firuzeh deposit are given in the Table 2.

For a range of ρ the multifractal power–law relation is given in Eq. (7). Fig. 7 depicts the relationships between V(T)-V($\rho \geq \upsilon$) and ρ on log–log plot for Cu concentration. It represents the power law relation between V($\rho \geq \upsilon$) and ρ. Three geochemical populations shown on Fig.7 were identified based on the relationships between the volumes and concentrations expressed by Eq. (11). Major breakpoint is at the concentration value of 0.07% for Cu. Amounts higher than this threshold represents mineralization zones where lower values belong to barren host rocks.

5. Correlation of C–V model with geological models

Alteration models have a key role in zone delineation and also in presenting geological models, as described by Lowell and Guilbert (1970). Based on those models, phyllic and potassic alterations host major mineralization in supergene enrichment and hypogene zones. Models of Cu mineralization zones derived via the C–V method can be compared with geological data in order to validate the results of analysis in both different porphyry Cu deposits. It can be assumed that there are spatial relationships between the modeled Cu zones and geological characteristics such as mineralogical and alterations.

Geological 3D models of the deposits were constructed by using RockWorks™ v. 2006 software and subsurface data collected from boreholes. These data include collar coordinates of boreholes, azimuth and dip (orientation), lithology, alterations and mineralogy. In addition, surface data include topographical data, faults, outcrops of magmatic and other geological units and other geological features that are relevant for constructing of the 3D geological models.

Table 2
Thresholds concentrations obtained by using C–V model based on Cu% in Chah-Firuzeh deposit.

Zone	Threshold (Cu%)	Range (Cu%)
Barren host rocks		0–0.05
Oxidation	0.05	0.05–0.16
Hypogene	0.16	0.16–0.6
Supergene enrichment	0.6	0.6–1.2

5.1. Sungun deposit

The relationship between Cu distribution and alteration zones can be interpreted in Fig. 8. 3D modeling of the alterations reveals extensive phyllic and potassic alterations are associated with the deposit. Phyllic alteration is more extensive than potassic alteration. The supergene enrichment zone derived via the C–V method is located in central and eastern parts of deposit (Fig. 8), where it has a good correlation with phyllic and potassic alterations. The hypogene zone derived via the C–V method also shows good correlation with potassic and phyllic alterations and its major metal accumulation is more in the central parts of the deposit.

The C–V log–log plot (Fig. 4) suggests concentration enrichment greater than 1.25% Cu, which we consider to represent a new mineralization phase. This mineralization phase is a copper mineralization in skarn unit, especially in the SE part of the deposit (Fig. 9). This enrichment accumulation has occurred just inside the porphyry stock and is correlated with the skarn units (Fig. 9). Therefore, it can be interpreted that the skarn units are hosting Cu concentrations as mineralization zone and are located in the eastern and northern parts of this deposit. Geological investigations shows that some parts of copper mineralization occurred within the skarn units are as suggested and depicted by Fig. 9.

5.2. Chah-Firuzeh deposit

Modeling and zonation based on geological data, mineralogical and alteration 3D models were constructed in this deposit. 3D modeling of alterations reveals that extensive phyllic and potassic alterations associated with the deposit (Fig. 10). The supergene enrichment zone derived via the C–V method shows good correlation with phyllic and potassic alterations (Fig. 10). There is a good correlation between the hypogene zone modeled via the C–V method and potassic and phyllic alterations (Fig. 10).

Mineralogical distribution model is important especially for chalcocite (supergene enrichment), chalcopyrite (hypogene) and bornite (major in supergene enrichment and minor in hypogene) minerals. The supergene enrichment zone model derived via the C–V method shows good correlation with by chalcocite accumulation in the center to eastern parts of the deposit (Fig. 11). Vast extensions of this supergene enrichment zone exist in central parts of the deposit, inside and below the horizons of chalcocite and bornite accumulation (Fig. 11). In addition, there is a good correlation between the hypogene zone model and the chalcopyrite distribution (Fig. 11).

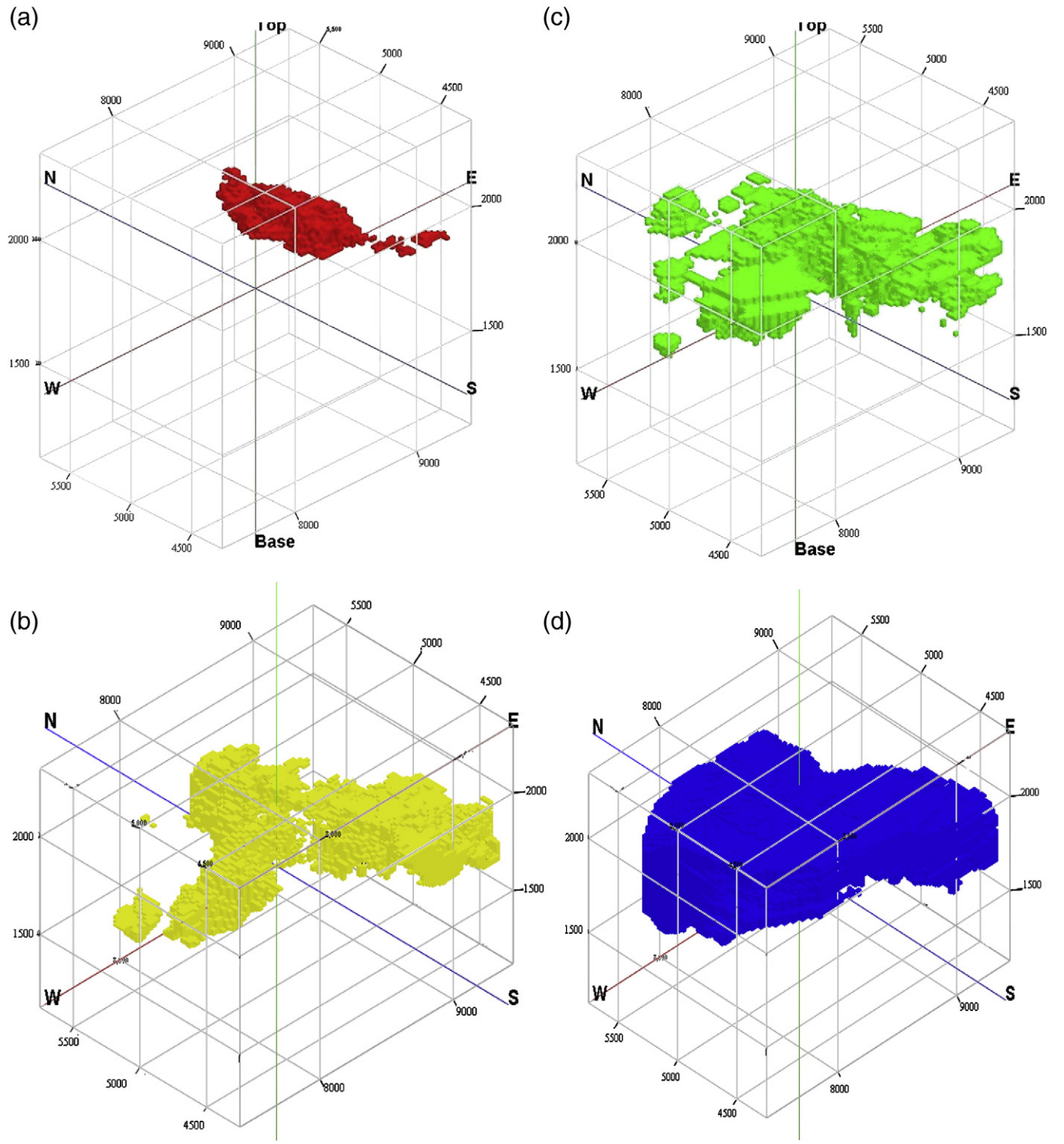

Fig. 8. The supergene (a) and hypogene (b) zones of Sungun deposit determined by C–V model and it's correlation with potassic (c) and phyllic alterations (d).

6. Conclusions

The need for handling the considerable of the voxels which are usually over 2 million and the data availability in porphyry deposits are fundamental issues should to be considered for data analysis. Therefore, FCV software which was developed by use of Excel VBA was a necessity to calculate and generate C–V log–log plots. The input data file has a very simple and well-known tab-delimited format benefitting users to handle data of huge volumes. The data classification which was carried out in this study is mainly based on user's judgment. Therefore, of course, a good data classification helps us in achieving the better results. The software presented here has also the ability to produce C–V log–log plots for detection of the threshold value for each zone.

Results from this study show that C–V method recognizes different mineralization zones in porphyry Cu deposits. Different stages of mineralization can be interpreted via the C–V fractal method. The C–V method uses relationship between the ore element concentration and enclosing volumes for example the concentration of Cu associated with

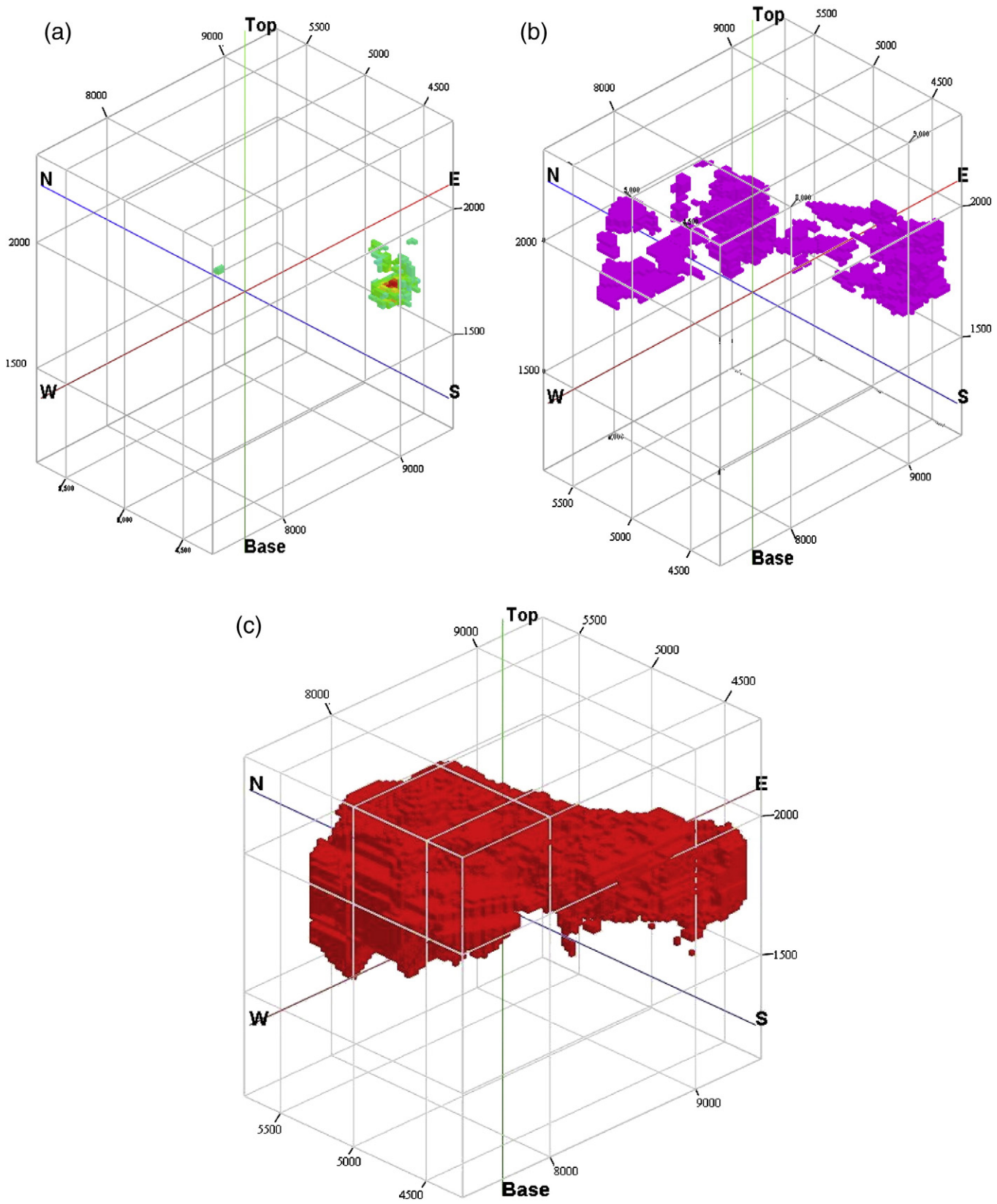

Fig. 9. Skarn parts of Sungun deposit contain high grade of Cu% (a) and it's correlation with skarns (b) and porphyry stock (c).

different zones, and satisfies power–law relationships. The proposed fractal method could be applied for delineating enrichment zones, especially supergene enrichment zone from the barren host rock, or from the background value using the concentration values of the zones in combination with characteristic features of their geometrical shapes. The proposed method is applicable to ore elements in porphyry deposits such as Cu, Mo, Au and W for which the spatial patterns of concentration values satisfy a multifractal model.

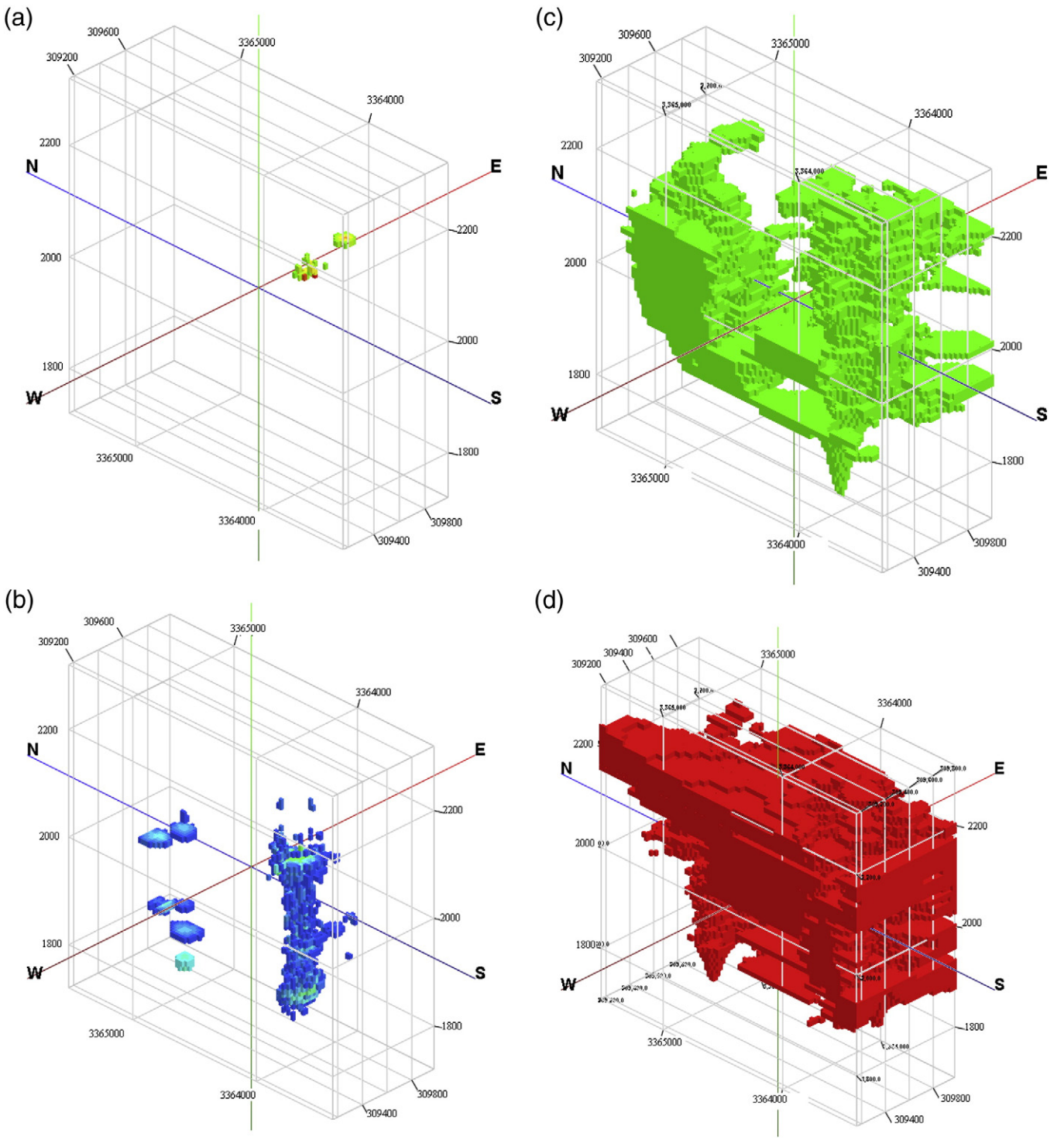

Fig. 10. The supergene (a) and hypogene (b) zones of Chah-Firuzeh deposit determined by C–V model, potassic (c) and phyllic (d) alteration models.

This C–V method has been successfully applied to model relationships between Cu concentrations and volumes in both of the Sungun and Chah-Firuzeh porphyry deposits. The supergene enrichment and hypogene zones delineated via the C–V method are correlated with alterations, mineralogical and lithological data portrayed in 3D models. The C–V log–log plot from Sungun reveals that there exists a multifractal model which fits the real situation and the two major mineralization stages can be deduced from the proposed model. Correlation between results of C–V method and geological characteristics shows that the skarns contain high grades of Cu. Supergene enrichment zone has high correlation within chalcocite and bornite accumulations in Chah-Firuzed deposit.

Acknowledgments

The authors wish to thankfully acknowledge the kind help and authorization for release of exploration data set of Sungun deposit to Dr. Omid Asghari and Dr. Farshad Rashidinejad. Also, the authors wish to acknowledge the kind agreement provided to them by Mr.

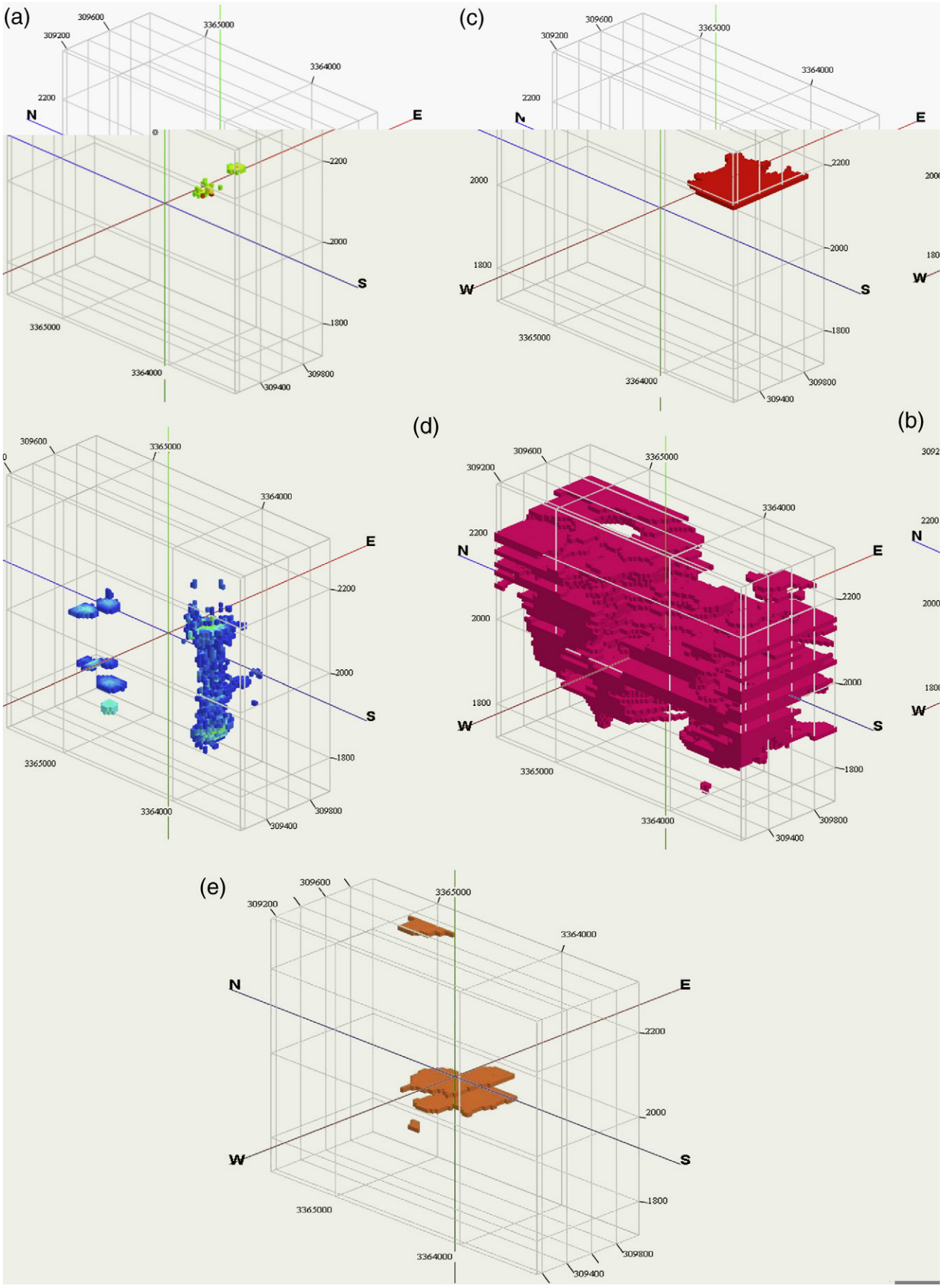

Fig. 11. The supergene (a) and hypogene (b) zones of Chah-Firuzeh deposit determined by C–V model and chalcocite distribution model (c) and chalcopyrite distribution model (d) and bornite distribution model (e).

Ebrahimi Meimand for authorizing the use of the exploration data set for Chah-Firuzeh deposit, Iranian National Copper Company.

Appendix A. The FCV software procedure

Programming in Basic language throughout Excel worksheet, considering capabilities of Excel on data processing, makes it a more useful and practical tool. A suitable interface, easy data entry, powerful data processing, clear visualization, variety of output charts in simplest from, yielding clear results were considered to design an effective software. Furthermore, extra options such as saving the results as images and printing the results as where as needed are some of other features of FCV software. This software has been designed in such a manner that the needed changes could be easily added in future. The software flowchart is illustrated in Fig. 12. The main subroutines are:

A) Reading data from a Tab-delimited text file in a simple format; four columns pointing the X, Y, Z (3D coordinate values of each voxel) and G (grade at each point in space) in sequence. Data files include center coordinates, volume and major element evaluated grades, i.e., Cu% in this case study, for different voxels which can be transferred, or come from, or be extracted from software such as Datamine, Rockworks and Surpac-Gemcom. This subroutine transfers the complete set of data into same named arrays. The delimiters in the text file between variables should be a comma or space. Generally, porphyry deposits have numerous data sets which led to overflow limitation in much software as well as in Excel. To avoid this problem a text format is used (Tab-delimited) file.

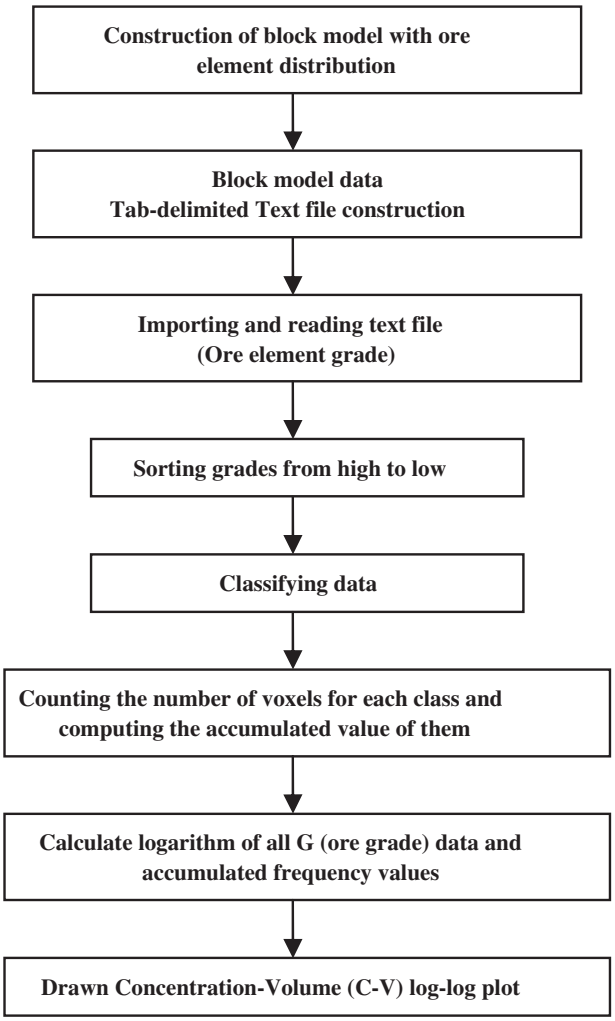

Fig. 12. The FCV software flowchart.

B) The data classified to an arbitrary number of classes which is 100 classes as considered for of the software. In next step minimum and maximum values of G (major element grade) data were calculated, then the range value for G computed and finally each class width is defined by dividing the range to the number of classes. User can determine the number of classes considering a total number of data at a requested accuracy level. Determination of class's width is designed by following equations in which Cr is the width of each class:

$$\begin{aligned} &\mathrm{minG} = \mathrm{minimum\,of\,G\,array} \\ &\mathrm{maxG} = \mathrm{maximum\,of\,G\,array} \\ &\mathrm{rangeG} = \mathrm{maxG} - \mathrm{minG} \\ &\mathrm{n} = 100 \\ &\mathrm{Cr} = \mathrm{rangeG}\,/\,\mathrm{n}. \end{aligned} \tag{16}$$

C) Counting the number of members for each class and computing their accumulated values using a loop structure which incorporates a comparison statement to find the home class for each value of G is based on the following relations:

$$\begin{aligned} &\mathrm{LBCi} = \mathrm{i} \times \mathrm{Cr} \\ &\mathrm{UBCi} = \mathrm{LBCi} + \mathrm{Cr} = (\mathrm{i}+1) \times \mathrm{Cr} \\ &\text{for all G: if } (\mathrm{LBCi} < \mathrm{Gj} \le \mathrm{UBCi}) \text{ then } \mathrm{fi} = \mathrm{fi} + 1 \end{aligned} \tag{17}$$

where, LBCi is the lower boundary of class number i, UBCi is the upper limit of class number i, Gj is grade of member j from all G data, and fj is frequency of member j. In fact, in the above approach all of considered voxels have constant volumes so they are counted as points. By multiplying the computed value of the unit volume, the calculation will be valid. Nevertheless, in fractal graph as will be discuss as below, one, does not need to carry out this conversion procedure, since the interpretation is completely based almost on trend of the data.

D) Excel internal functions were applied for calculation of logarithm of all G data and accumulated frequency volume values.

E) For drawing fractal log–log plot of concentration–volume on the basis of already computed values, Scatter plot function from Excel was used.

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