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Letter to the Editor

Comments on “computation of the gravity field and its gradient”

1. Introduction

In the paper of Dubey and Tiwari (2016), the method of jointly calculating gravity and gravity gradient anomalies is presented. Several regular geological bodies are taken into consideration and also the anomaly of irregular 3D geometries is calculated with vertical rectangular prisms or infinite cells. And at last a comparison is drawn between the forward calculation results of the described technique and open geophysical software. It is demonstrated that the proposed method in this paper can provide reasonable modelling data. Moreover, it enables us to detect the edges and boundaries of irregular bodies with smaller element size and higher resolution. However, after looking through the paper, including the forward calculation formulas in the appendix (Dubey and Tiwari, 2016) and MATLAB codes in the attached file, it is noteworthy that errors exist in formulas and codes and some descriptions about data processing and interpretation are not exact. We give a correct form of some formulas and show the results with figures and attached codes. Comments are also made on some viewpoints of the authors.

2. Analysis of the theory

In Dubey and Tiwari's paper, the forward formulas of prism satisfies Laplace's equation, but the results of sphere and vertical cylinder do not. After looking through equations and codes, errors exist in the codes for sphere, and the outputs of codes of prism and slab are empty. For the vertical cylinder, the forward formulas of its gravity and gravity gradient are quite complicated in other literatures as elliptic integrals are involved. But in their paper, formulas are simple and it is easy to prove that Laplace's equation is not supported theoretically. In the appendix (Dubey and Tiwari, 2016), the authors provide the forward formulas of vertical cylinder referenced from Zhang's et al. (2000) paper. But formulas of sphere instead of cylinder are found after looking through the Zhang's paper. There is no analytical formula readily available to calculate the gravity anomaly vector and gravity tensor gradients of the cylindrical mass model. In order to make the problem unambiguous, steps below are done.

2.1. Verify the formulas of vertical cylinder

First of all, the units of the calculation results should be in agreement with gravity and spatial derivative of gravity. Taking gravity anomaly and a component of gravity gradient tensor for example, the formulas are given (Dubey and Tiwari, 2016) as

$$Gz = \frac{4\pi G\Delta\rho R^3}{3}\frac{z}{r^2}, \tag{1}$$

and

$$Tzz = -\frac{4\pi G\Delta\rho R^3}{3}\frac{z^2-x^2-y^2}{r^4}, \tag{2}$$

As stated in their paper, the unit of R, x, y, z and r is m, $\frac{4}{3}\pi\Delta\rho R^3$ equals to the mass of the body, whose unit is kg, and the unit of gravitational constant G is $m^3/(kg\cdot s^2)$, so the unit of the final result of Eq. (1) is $\frac{m^3}{kg\cdot s^2}\cdot kg\cdot\frac{m}{m^2} = m^2/s^2$, for Eq. (2) it is $\frac{m^3}{kg\cdot s^2}\cdot kg\cdot\frac{m^2}{m^4} = m/s^2$. It's the same case for the other tensor components and we can easily find out that the above units are not correct for gravity and gravity gradient.

The second thing is that the diagonal components of the tensor should satisfy Laplace's equation, which states

$$Txx + Tyy + Tzz = 0, \tag{3}$$

Summing the equations of the three components, we have

$$\begin{aligned} Txx + Tyy + Tzz &= \frac{4\pi G\Delta\rho R^3}{3}\frac{y^2+z^2-x^2+x^2+z^2-y^2+x^2+y^2-z^2}{r^4} \\ &= \frac{4\pi G\Delta\rho R^3}{3}\frac{x^2+y^2+z^2}{r^4} = \frac{4\pi G\Delta\rho R^3}{3}\frac{1}{r^2} \neq 0, \end{aligned}$$

Laplace's equation is disconfirmed here. The above two points prove that errors exist in the related formulas.

2.2. Run the codes to calculate modelling response

In the attached files of Dubey and Tiwari's paper, we find that formulas of Txx and Tyy of the sphere and all seven formulas of the vertical cylinder

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are not in agreement with those in Appendix A (Dubey and Tiwari, 2016). In formulas of the cylinder, the key parameter, *i.e.* radius (or we might say mass) of the cylinder is missing. And the '*for*' loop is meaningless for k = 1 to 2, because results of the second loop will cover those of the first one. Still, the diagonal components are added and the result is not equal to zero, *i.e.* Laplace's equation is not supported.

2.3. Give the analytical form of forward formulas of vertical cylinder

We refer to a couple of papers about gravity forward calculation of a vertical cylinder, but no one gives a full set of formulas of gravity anomaly and the six tensor components. A common point for the formulas is that the form is more complicated than those in the paper of Dubey and Tiwari (2016) and complete elliptic integrals are utilized. In the paper of Zhang et al. (2000) referenced by Dubey and Tiwari in Appendix A, the main content is about Euler deconvolution of gravity gradient data and forward formulas of prism instead of vertical cylinder are described. Nabighian (1962) gave the analytical expressions to calculate gravity and horizontal derivatives of gravity of a vertical circular cylinder. Singh (1976) provided an approach to calculate gravity anomaly of a vertical right circular cylinder using Lipschitz-Hankel type integral. In the papers of Veryaskin and McRae (2008) and Chen et al. (2016), we get the formulas of *Txz*, *Tyz* and g_z, *Tzz*, respectively. Then we unify the notations of parameters and reorganize the formulas. Assuming that h is depth to the top of the cylinder, a is the radius, d is the length and the depth of any point of the cylinder Z locates in the range of [h+z, h+d+z]. The coordinate of observation point is (x, y, z), z = 0 means measuring on the ground, (x_0, y_0) is the coordinate of its horizontal center, and so the horizontal distance from observation point to the center is $r=\sqrt{(x-x_0)^2+(y-y_0)^2}$. So we have

$$g_z = 2G\rho\left[\begin{array}{l}\frac{a^2-r^2}{((r+a)^2+Z^2)^{1/2}}K(k)+((r+a)^2+Z^2)^{1/2}E(k)+\\ \frac{\pi}{2}Z\Psi(\phi,k)-\pi Z\end{array}\right]_{h+d+z}^{h+z} \tag{4}$$

$$T_{xz} = G\rho\frac{x-x_0}{r^2}\left[\sqrt{Z^2+(r+a)^2}\,((2-k)E(k)-2K(k))\right]_{h+d+z}^{h+z} \tag{5}$$

$$T_{yz} = G\rho\frac{y-y_0}{r^2}\left[\sqrt{Z^2+(r+a)^2}\,[(2-k)E(k)-2K(k)]\right]_{h+d+z}^{h+z} \tag{6}$$

$$T_{zz} = -2G\rho a(h+z)\left[\frac{(r^2+Z^2-a^2)E(k)-((r-a)^2+Z^2)K(k)}{((r-a)^2+Z^2)((r+a)^2+Z^2)}\right]_{h+d+z}^{h+z} \tag{7}$$

In Eqs. (4)–(7), $\Psi(\phi,k)$ is *Heuman-Lambda* function, ϕ is the *Heuman-Lambda* angle,$K(k)$, $E(k)$ are respectively the first kind and the second kind complete elliptic integral, and k is the modulus. We have their calculation formulas as follows

$$\begin{cases}\Psi(\phi,k)=E(k)F(\phi,k)+K(k)E(k)-K(k)F(\phi,k)\\ F(\phi,k)=\int_0^{\phi}\frac{1}{\sqrt{1-k^2\sin^2\theta}}d\theta\\ \phi=\frac{\pi}{2}+\arctan(\frac{r-a}{Z})\\ K(k)=\int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}\\ E(k)=\int_0^{\pi/2}\sqrt{1-k^2\sin^2\theta}\,d\theta\end{cases} \tag{8}$$

where $k^2=4ar/((r+a)^2+Z^2)$. Using the above equations, we could calculate the gravity anomaly and three gravity gradient anomalies. The other tensor components would be obtained through the transforming equations in frequency domain (Nelson, 1988; Blakely, 1995)

$$\begin{cases}\mathcal{F}[T_{xx}]=-k_x^2/|k|^2\mathcal{F}[T_{zz}]\\ \mathcal{F}[T_{yy}]=-k_y^2/|k|^2\mathcal{F}[T_{zz}]\\ \mathcal{F}[T_{xy}]=-k_xk_y/|k|^2\mathcal{F}[T_{zz}]\\ \mathcal{F}[T_{xz}]=ik_x/|k|\mathcal{F}[T_{zz}]\\ \mathcal{F}[T_{yz}]=ik_y/|k|\mathcal{F}[T_{zz}]\end{cases} \tag{9}$$

where k_x, k_y and k are wavenumber in x, y and radial direction, respectively.

3. Model test

To verify the proposed formulas (4)–(8), we design a model of a vertical cylinder and rewrite MATLAB codes. At the same time, anomalies of two prisms which have similar size to the cylinder are also calculated, one is inscribed in the cylinder and the other one is circumscribed to it. The

Table 1
Parameters of the models.

Model	Radius/Side (m)	Thickness (m)	Depth to top (m)	Center (m)
Cylinder	10	30	20	(50, 50)
Prism A	$10\sqrt{2}$	30	20	(50, 50)
Prism B	20	30	20	(50, 50)

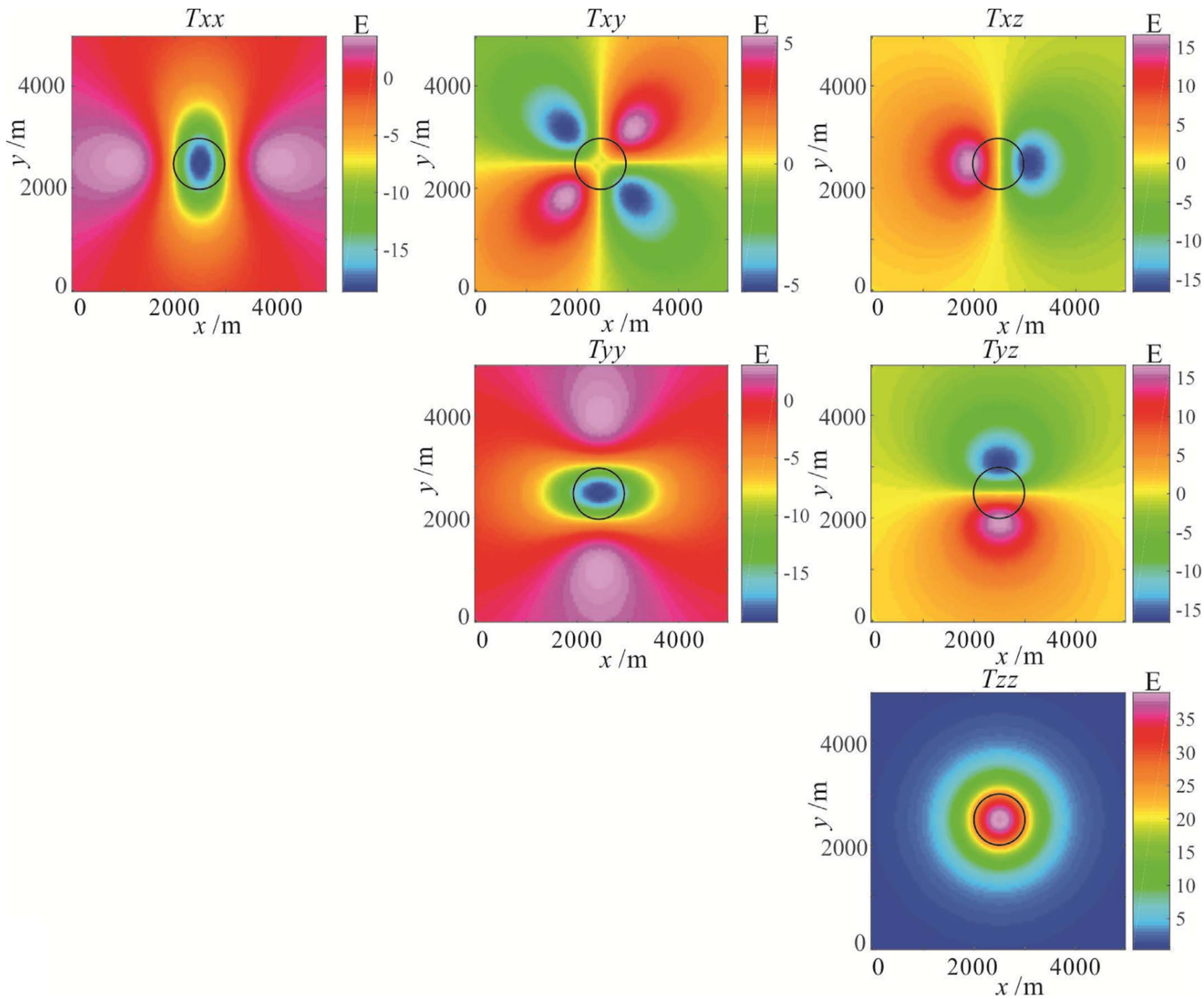

Fig. 1. Calculated gravity gradient tensor components of the vertical cylinder with sampling interval of 50 m in x and y direction.

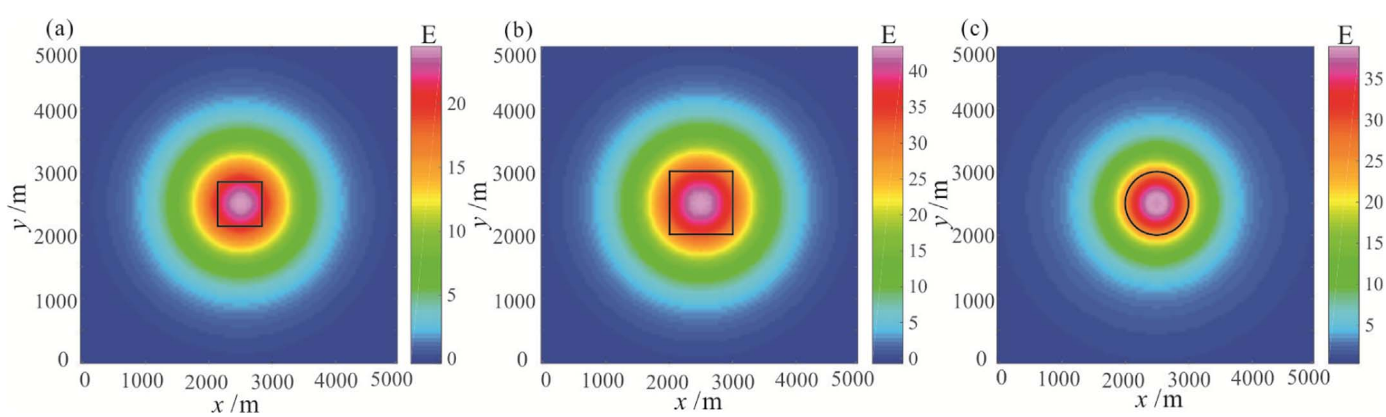

Fig. 2. The component of Tzz of a vertical cylinder and two prisms, (a) the smaller prism, (b) the larger prism, (c) the cylinder.

parameters are given in Table 1, and the residual density is 1 g/cm^3. As we can know, gravity anomaly or gravity gradient response of the cylinder should be within that of the two prisms. After running the codes, we find that the results of (5) and (6) do not meet the criteria stated above (calculation of the two components is given in the attached code), but formula (7) does. So we would use (7) as a foundation and calculate other components with Eq. (9). Calculation results of the six tensor components are shown in Fig. 1, and the comparison between Tzz of the vertical cylinder and two prisms is demonstrated in Fig. 2.

In Fig. 1, we also plot the horizontal location of the cylinder with a black circle which can be used to evaluate the shape of calculated anomalies. The tensor have some shallow information of the geological body, including geometric parameters, *e.g.* the components Txx, Tyy identify the north-south and east-west edges of the dome, Txz and Tyz can be used to identify the central axes of the dome, and Tzz locates the dome. As we can see the relative location between anomalies and the black circle, the forward results provide right horizontal position of the cylinder. So the shape of the anomalies is appropriate. When it comes to the amplitude of anomalies, in Fig. 2, sub-maps (a), (b) and (c) correspond to anomalies of the smaller prism, the larger prism and the vertical cylinder, respectively. Also, the location and size of the model is drawn with black square and circle to illustrate the relative position. From the color-bar on the right of each map, we can find that the amplitude of the cylinder is right between that of the two prisms and which is reasonable. Additionally, the coverage area of the cylinder's anomaly is smaller than both prisms, *i.e.* the anomaly of the cylinder is more focused. Different from prism which has right angle and sharp edges, the anomaly of vertical cylinder would be narrowed down and concentrated more to the center, which is clear in Fig. 2(c).

Besides shape and amplitude, there are also characteristics which would reflect approximate error in calculation. The first one is that the trough value in Fig. 2(c) is not below zero as shown in (a) and (b). Secondly, the value of four points in the middle of Fig. 2(c) is smaller than the surrounding points, which should be larger as they are projected to the center of the cylinder. The two points could be attributed to the approximate calculation and the rounded shape of this vertical cylinder. In formulas (7) and (8), approximation is involved in the derivation of formulas and numerical calculation, so errors exist in the result of Tzz.

4. Discussions and conclusions

In Dubey and Tiwari's paper, a comparison is drawn between the forward results of formulas and the software IGMAS. However, this calculation is only about a single prism and it is quite simple. So there will not be obvious difference between results with different methods. In the case of 3D calculation, we need to divide the irregular geological body into cubes with approximate homogeneous density and forward calculation here is more complicated and more time-consuming. So looking for an appropriate way to improve calculation efficiency seems to be more meaningful. Applying the basic formulas of gravitation and its differentiation into different ways to do forward calculation will not help to promote accuracy obviously.

In their paper, forward response of interpretation results of real data is calculated and then compared to observed data to illustrate that the forward method is effective. It is less convincing as the accuracy of inversion is the key point in evaluating the fitting between calculation and observation. Actually, forward and inversion are related tightly and forward is the foundation of inversion.

Dubey and Tiwari have done a good job in calculating gravity and gravity gradient anomalies of regular and irregular bodies, which provides a better understanding of the properties of gravity gradient tensor. However, minor errors still exist in some forward formulas and attached codes. We try to shed light on the ambiguities and modify the formulas and codes of the vertical cylinder. Analytical formulas are given in our paper, and we implement them with MATLAB codes. The results are reasonable and the approximation may cause error, as shown in the figures. MATLAB function *VerticalCylinder_T.m* is written and attached with this paper.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cageo.2017.06.017.

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