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Application of power-spectrum–volume fractal method for detecting hypogene, supergene enrichment, leached and barren zones in Kahang Cu porphyry deposit, Central Iran

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ABSTRACT

The aim of this study was to identify various mineralization zones especially supergene enrichment and hypogene zones in Kahang Cu porphyry deposit by higher than 60 million tonnes of sulfide ore with an average grade of 0.6% Cu and 70 ppm Mo, which is situated in central Iran, based on analyzing the subsurface data using a proposed power spectrum–volume (P–V) fractal method. P–V method is used in frequency domain provided by application of Fourier series transformation on assay data. Straight lines fitted through log–log plots, showing P–V relations for Cu, were employed to separate supergene enrichment and hypogene zones from leached zone and barren host rock in the deposit. In the proposed P–V fractal method, the identification of mineralization zones is based on power–law relationship within power spectrum field (S) and the rock volume hosting Cu mineralization at different grades by applying a $V(\geq S) \propto S^{-2/\beta}$ multifractal model. The subsurface data from deposit was analyzed by P–V fractal method and the results have been compared with geological models which included alteration and mineralogical models. The comparison shows that the interpreted zones based on the P–V fractal method have noticeable consistency to the geological models. The proposed P–V method is its either new approach to define zones in a mineral deposit and there was no professional software available to perform the relevant calculations; therefore, Fractal Power Spectrum–Volume (FPSV) software was programmed by the authors to achieve this goal.

1. Introduction

One of the main aims in the exploration of porphyry deposits is the recognition of supergene enrichment and hypogene zones from leached zone and barren host rocks. Customized geological methods for separating hypogene and supergene zones from those of leached and barren zones in porphyry deposits are usually based on mineralographical, petrographical and alteration assemblages of minerals, specifically in potassic and phyllic alteration zones (e.g., Beane, 1982; Berger et al., 2008; Schwartz, 1947; Sillitoe, 1997). A conceptual model of the lateral and vertical relationships with the alteration zones was first proposed by Lowell (1968), and Lowell and Guilbert (1970). Based on this model, similar models were developed by Sillitoe and Gappe (1984), Cox and Singer (1986) and Melfos et al. (2002). Additionally, the fluid inclusions studies in porphyry deposits, e.g., Roedder (1971), Nash (1976), Ulrich et al. (2001) and Asghari and Hezarkhani (2008) and investigation of ^{34}S isotope value variations in different zones in porphyry deposits by Wilson et al. (2007) are other methods for detection of mineralization zones. Ore grade variation of principal ore however was not considered in the exploration of porphyry copper deposits using all of the mentioned methods. It is a fact that porphyry deposits exhibit zonation based on grade variations in ore elements. The geological borehole log data are usually important for obtaining properties of different zones in porphyry copper deposits. Additionally, borehole log records of ore grade vary with respect to changes in the geological properties of the different zones such as mineralogy and alterations. Different geological interpretations could be presented for definition of boundaries between different zones in porphyry deposits which may also lead to different results if the ore element grade distribution is not taken into consideration.

Fractal theory as an important branch of nonlinear sciences was proposed by Mandelbrot (1983), and then has been widely and effectively applied in geosciences so far, (e.g. Afzal et al., 2010; Agterberg et al., 1993; Ali et al.;, 2007; Bai et al., 2010; Cheng et al., 1994; Goncalves

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et al., 2001; Li et al., 2002; Li et al., 2003; Li et al., 2004; Ortega et al., 2006; Mao et al., 2004; Sim et al., 1999; Turcotte, 1986; Shen and Zhao, 2002). Cheng et al. (1994) and Cheng (1999) studies showed that the distribution of chemical elements in any part of the earth's crust is the result of the repeated, multi-episodic geological processes to which that particular part of the earth's crust was subjected over protracted periods of time. These processes lead to elemental enrichments and geochemical distribution patterns that often have fractal or multifractal characteristics, and may represent mineralized zones. In addition, the geological and geochemical environmental control and effects of the separation of geochemical populations are of essential importance in interpretations by fractal methods (Carranza, 2009; Carranza and Sadeghi, 2010; Cheng, 1999; Goncalves et al., 2001; Li et al., 2003; Sim et al., 1999).

Non-linear characteristics in concentration of ore-elements in rocks and related surface environments such as water, soil, stream sediment, till, humus, and vegetation can be exhibited in hydrothermal mineral deposits, in particular in porphyry deposits (Cheng, 2007; Cheng, 2008; Cheng and Agterberg, 2009). In addition, Cheng et al. (1994) and Zuo et al. (2009) applied fractal models to characterize the horizontal and vertical distribution of geochemical element concentration in porphyry copper deposits. Afzal et al. (2011) proposed concentration–volume (C–V) fractal method for identification of various mineralization zones, especially supergene enrichment and hypogene, in Cu porphyry deposits.

The aim of the study was to apply power spectrum–volume (P–V) fractal method to distinguish supergene enrichment and hypogene zones, which are assumed to be related to the main mineralization event, from the oxidation and barren host rocks in the Kahang copper porphyry deposit. “FPSV” software was implemented in MATLAB coding environment to apply the P–V fractal model on our data set. Subsequently, a discussion is presented whereby zonation threshold values are correlated with relevant geological data specifically alteration data. In addition, we attempted to utilize the main results of the P–V fractal analysis and interpretations derived in this study in 3D analysis of geochemical data.

2. Power spectrum–volume fractal method

Geochemical determinations of total or bulk concentration of elements in soils, humus, tills and rocks are commonly used in geological studies to investigate influences of particular geological processes such as mineralization (Afzal et al., 2010; Agterberg et al., 1996; Goncalves et al., 2001; Li et al., 2003). Decomposing bulk values into component patterns to reflect the specific geological features is often an essential part of prediction of mineral resources and exploration (Cheng, 2006; Cheng and Li, 2002). In computer and communication sciences, signals are decomposed into components to distinguish and reduce the ratio of noise to signal. The procedure is also well known in pattern recognition. A number of parameters have been used in the past for pattern recognition. These include the magnitude of patterns, frequency distribution, geometry and texture. More recently, scaling properties have been recognized and incorporated in the pattern recognition (Ali et al., 2007; Cheng, 2006; Cheng et al., 1994; Cheng et al., 1999; Panahi et al., 2004). Scaling refers to the property that the change of measuring unit does not alter the type of function between measure and measuring unit.

Fractal or multifractal scaling of a measurement usually involves both the range of scale (ε) over which the measurement holds the scale invariant property and the actual scaling governing power-law itself, $\mu(\varepsilon) \propto \varepsilon^{\alpha}$. The scaling range may be limited due to either the nature of the relevant physical process or the resolution and quality of the observed data (Cheng, 2006). Therefore, while it is important to utilize the power–law exponent to characterize the scaling properties of the measurement, the scaling range itself plays an important role for differentiating between superimposed fractals/multifractal measurements with different scaling properties. There have been many examples of fractal quantities showing bi-fractal properties (texture and structure dimensions). Most of the bi-fractal examples documented in the literature are defined in the space domain with a geometric measuring scale. A new power–law model was developed from a multifractal point of view by Cheng (2006) to characterize the scaling property of the power-spectrum in the frequency domain. Power–law relationship between the power spectrum values and the occupied volume can be expressed as the following general form:

$$V(\geq PS) \propto PS^{-2/\beta}. \tag{1}$$

It involves power–law relationships, Eq. (1), between the power spectrum values ($PS = -||F(Wx, Wy, Wz)||$) and the “volume” of the set with power spectrum values above PS, {Wx, Wy, Wz: ≥PS}, where F denotes fast Fourier transformation (FFT) of the measurement $\mu(x, y, z)$; Wx, Wy and Wz respectively represent wave numbers (angular frequencies) in X, Y and Z axes directions on a 3D model.

The original borehole data of ore element concentrations were interpolated by using geostatistical estimation method. The estimated block model is used for this purpose. The range of the exponent is $0<\beta\leq 2$ or $1\leq 2/\beta$ with the special case of $\beta=2$ or $2/\beta=1$ corresponding to non-fractal or monofractal measure μ and $1<2/\beta$ to multifractals (details in Cheng, 2006). This model holds true for both isotropic measures and others with generalized scale invariance. Most patterns extracted from geochemical data can be considered as mixtures of multiple components according to several processes. It may be anticipated that the power–law relationship $V(\geq PS) \propto PS^{-2/\beta}$ shows a multi-scaling property over multiple scale ranges. The scaling breaks bounding the multiple ranges of power spectra can be identified on log–log plots of $V(\geq PS)$ vs. PS. Each such range of scaling then can be used to build define a frequency filter. For example, taking two filters and assuming two ranges of power spectrum can be identified by fitting two different power–law relationships with exponents $\beta1$ and $\beta2$, respectively, then the threshold PS_0 obtained from those two power–law relations can be used to form the two sets $\{W_x, W_y, W_z: PS\leq PS_0, \beta_1\}$ and $\{W_x, W_y, W_z: PS_0\leq PS, \beta_2\}$, which can be further used to define two filters $G_1(W_x, W_y, W_z)=1$ if $W_x, W_y, W_z \in \{W_x, W_y, W_z: PS\leq PS_0, \beta_1\}$ and otherwise $G_1(W_x, W_y, W_z)=0$. The other filter can be $G_2(W_x, W_y, W_z)=1-G_1(W_x, W_y, W_z)$. Inverse fast Fourier transformation (IFFT) then was applied with these filters to move the decomposed components back to the space domain: $\mu_1(x, y, z)=(F\,G_1)^{-1}$ and $\mu_2(x, y, z)=(F\,G_2)^{-1}$. $\mu_1(x, y, z)$ and $\mu_2(x, y, z)$ are the decomposed patterns of $\mu(x, y, z)$. In this special case where $G_1+G_2\,\mu1$, then, $\mu1(x, y, z)+\mu_2(x, y, z)=\mu(x, y, z)$. In the more general case that a small range of power spectrum corresponding to a noise component is removed during the definition of the filters, the sum of the decomposed components will be slightly different from the original patterns. The decomposed components, $\mu1(x, y, z)$ and $\mu_2(x, y, z)$, can be non-fractal, fractal, or multifractal quantities with less variability in comparison with the bulk measure $\mu(x, y, z)$.

3. Geological setting of Kahang porphyry deposit

The Kahang Cu porphyry deposit (with 5 km^2 surface area) is located in 73 km NE of Isfahan, Central Iran. This deposit contains more than 60 million tonnes of sulfide ore with an average grade of 0.6% Cu and 70 ppm Mo. This deposit occurred within the Cenozoic Urumieh-Dokhtar magmatic belt, one of the subdivision of Zagros orogenies (Alavi, 1994; Berberian and King, 1981; Stocklin, 1977). This belt extends for some 2000 km from NW to SE Iran. All of the Iranian large porphyry copper deposits and systems such as Sarcheshmeh, Sungun and Meiduk are located in this belt (Shahabpour, 1994). Geological, geophysical, geochemical, alteration patterns as well as drilling data show that there could be a large Cu porphyry system as Kahang (Afzal et al., 2010; Tabatabaei and Asadi Haroni, 2006).

The porphyry system is mainly composed of Eocene volcano-pyroclastic rocks, intruded by Oligo–Miocene quartz monzonite, monzodiorite to dioritic intrusions (Fig. 1). The extrusive rocks, including tuffs, breccias and lavas, are dacitic to andesitic in composition. Magmatic events in Kahang area can be defined as the following (Afzal et al., 2010):

1— Explosive eruptions and ejection of pyroclastics such as tuff and tuff breccia.
2— Flows of andesitic to dacitic lavas with porphyry texture from the volcano edifice. It is probable that eruptions of pyroclastic rocks and lavas were repeated periodically.
3— Emplacement of subvolcanics and intrusives rocks with compositions of dacitic, andesitic, monzonitic and dioritic nature, respectively. In western part of the deposit, silisic breccias covered high level parts of subvolcanic bodies.

The main structural features are two fault systems trending NE–SW and NW–SE locally, their feather fractures and joints are intense. The main alteration zones (potassic, phyllic, argillic and propylitic) were accompanied by the vein to veinlets of quartz, quartz-magnetite and Fe-hydroxides fillings. Mineralization has mostly occurred within the brecciated and intrusive rocks. The ore minerals, including chalcocite, covelite, chalcopyrite, pyrite, malachite, magnetite, hematite, jarosite, goethite, chalcantite, are mostly associated with intense zones of quartz stockwork, hydrothermal-tourmaline breccia, quartz-sericite and locally potassic alteration.

4. Implementation of power spectrum–volume fractal method in Kahang deposit

An ore element distribution, evaluated by a block-model, provides a smoothed version of the spatial distribution of that ore element. The FPSV software was designed for data processing and for P–V calculations; needed for the analysis. Since the numbers of voxels are very large and their grade variations have an extended range, this software is suitable for usage and is able to handle the huge data volume. It is notable to mention that the more voxels we make the better results in the P–V method are obtained.

Grade distribution block models were generated via ordinary Kriging method using the Datamine software. The derived block models were used as input to the FPSV software. The Kahang deposit data were obtained from 11 drill cores covering a total length of about 2800 m, as presented in Fig. 2. The rock samples from the drill cores were used to construct the geological models and 1240 lithogeochemical samples from drill cores in the Kahang deposit were analyzed for Cu and several

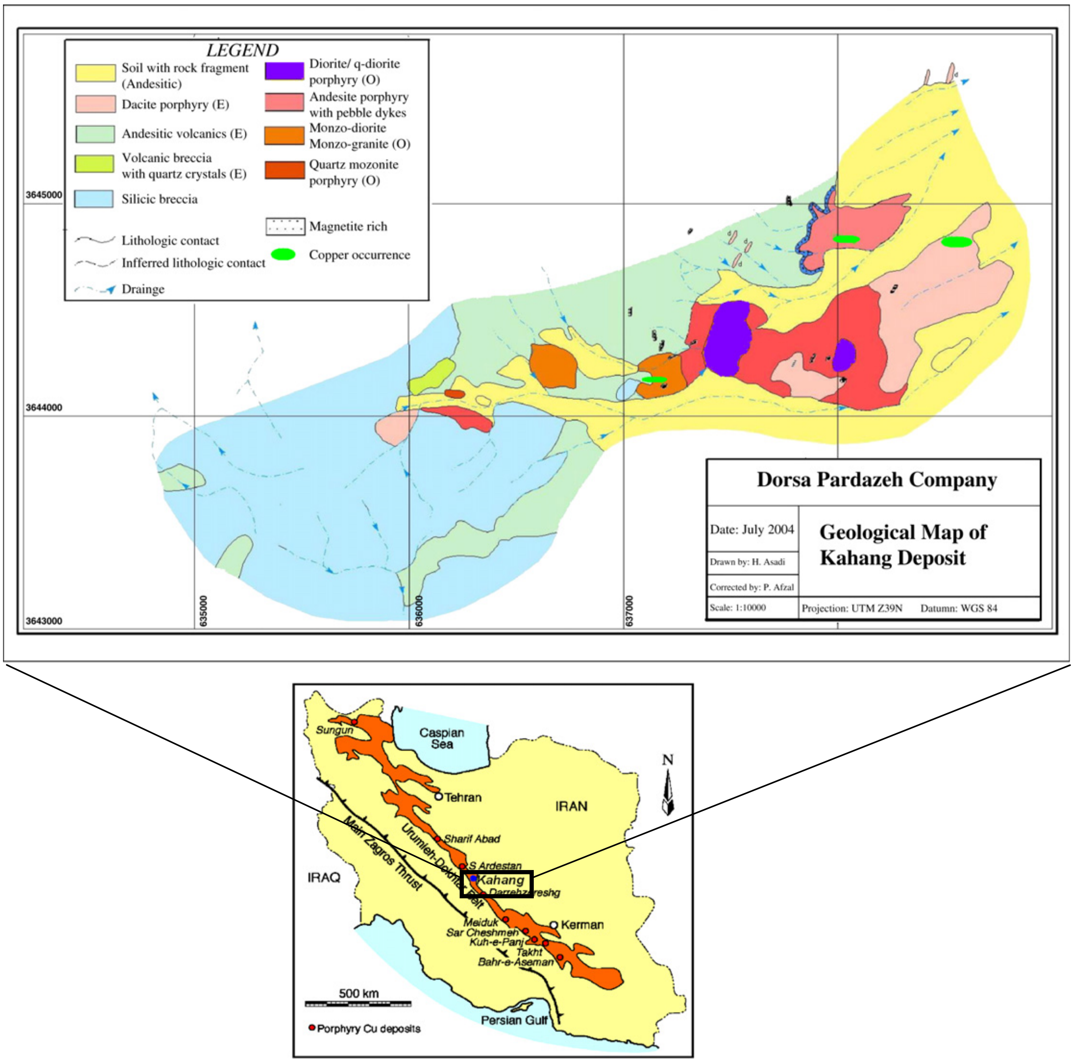


Fig. 1. Geology map of Kahang deposit (E and O are Eocene and Oligo-Miocene), showing the Urumieh-Dokhtar magmatic belt and known porphyry deposits there in the belt. (Modified based on Shahabpour, 1994; Afzal et al., 2010).

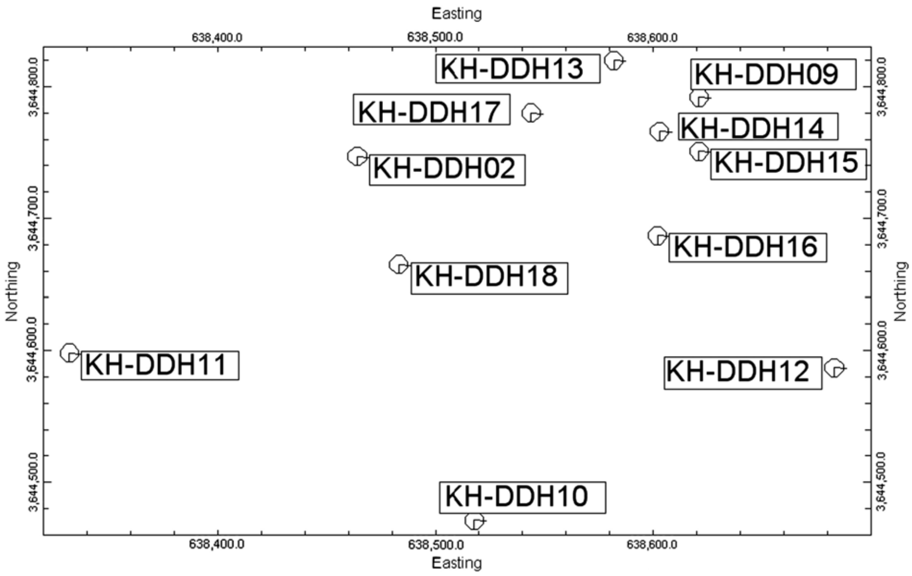
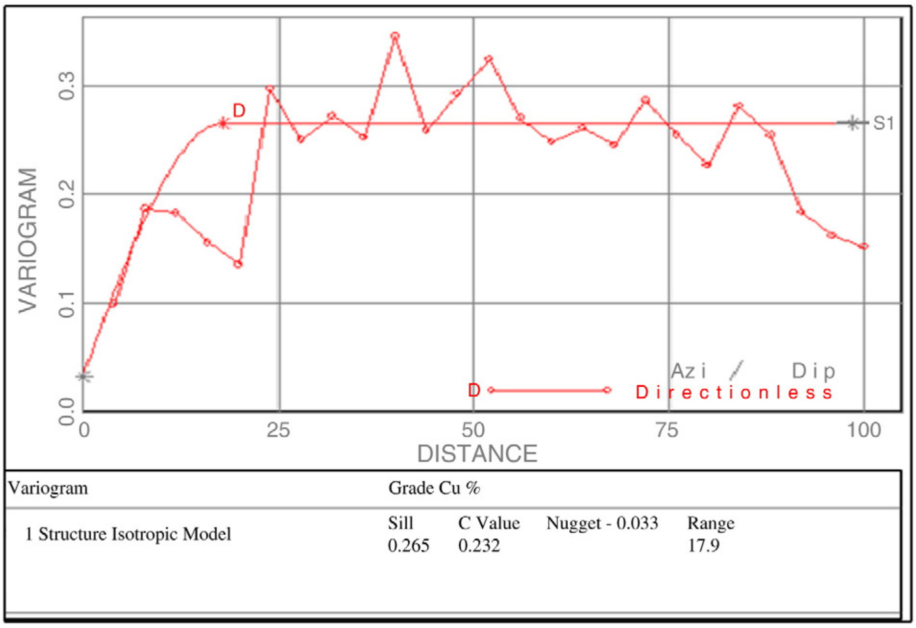


Fig. 2. Drilling core location map in Kahang deposit.

other elements. Collected samples from hypogene, supergene enrichment, leached and barren zones were 1032, 32, 74 and 102 samples, respectively. The Cu data was used for ore element distribution estimation. The experimental variogram for the Cu data in the deposit and the fitted model are presented in Fig. 3. Cu mean and median are equal to 0.22% and 0.1%, as illustrated in Table 1. In addition, the mean values of Cu are 0.04%, 0.35% and 0.9% for leached, hypogene and supergene enrichment zones, respectively (Table 2).

The Cu grade distribution in the Kahang deposit was evaluated via ordinary Kriging for 51,660 voxels. Each voxel has a dimension of 10 m in the X, Y and Z directions. The power spectra (S) were calculated for the 3D elemental distributions using 3-D Fourier transformation by FPSV (see Appendix A for methodology). Logarithmic values of PS and V were plotted against each other as shown in Fig. 4. Straight line segments, depicted in Fig. 4, were fitted to the values representing different power–law relationships.

Results have consistently shown that the power spectra of the measures, Cu, possess three different power–law relationships as observed from the log–log plot. The multifractal properties may imply that multiple geological processes lead to the observation of the patterns of Cu. The components with relatively higher frequencies (lower values of PS) give larger slopes ($2/\beta > 1$). The observed multifractal properties may imply that multiple geological processes have formed the patterns of Cu. The thresholds $PS_0 \sim 1174897$ (log PS = 6.07) and $PS_0 \sim 3981071$ (log PS = 6.60) were determined from log–log P–V plot, depicted in Fig. 4. PS for different mineralization zone reveal in Table 3. There is a sudden change in the rate of decrease of the volume enclosed by high values of power spectrum (Fig. 4). Based on these thresholds values, filters were designed for separation of mineralization zones. Inverse fast Fourier transformation was applied with these filters to convert the decomposed components back to the space domain by FPSV (Appendix A.). Based on this method, Cu concentrations in the hypogene zone



Variogram	Grade Cu %			
	Sill	C Value	Nugget - 0.033	Range
1 Structure Isotropic Model	0.265	0.232		17.9

Fig. 3. Index variogram in Kahang deposit.

Table 1
Statistical parameters of Cu in Kahang deposit.

Parameters	Cu %
Mean	0.22
Median	0.10
Maximum	5.1
Minimum	0
Range	5.1

are considered to range between 0.1% and 0.73%. Above 0.73% Cu concentrations the slope of the straight line fitting the data is considered to represent high Cu values (supergene enrichment zone). First population, less than 0.1% Cu, in log–log plot is illustrated leached zone and barren host rock.

5. Correlation results of P–V model with geological models

Alteration models have a key role in zone delineation and also in presenting geological models, as described by Lowell and Guilbert (1970). Based on these models, phyllic alterations host major mineralization in supergene enrichment and hypogene zones. Relationship between Cu mineralization zones, derived via the P–V method, can be compared with geological data in order to validate the results of analysis in the Kahang porphyry copper deposits. It can be assumed that there are spatial relationships between the modeled Cu zones and geological characteristics such as mineralogy and alterations.

Subsurface data collected from borehole include alterations and mineralogy used for validation model. The supergene enrichment zone, derived via the P–V method, is located in central part of the deposit (Fig. 5), where it has a good correlation with phyllic alteration. Also, this zone model, derived via the P–V method, shows good correlation with chalcocite accumulation in the central to eastern parts of the deposit (Fig. 5).

The relationship between hypogene zone, resulted from P–V method and phyllic alteration zone can be interpreted in Fig. 6. The hypogene zone, derived via the P–V method, also shows good correlation with phyllic alteration and there is a good correlation between the hypogene zone model and the chalcopyrite distribution (Fig. 6). In addition, Cu concentration ranges between 0.1% and 0.73% was considered from P–V log–log plot for hypogene zone entered into primary hypogene zone model that is constructed on the basis of geological data, as depicted in Fig. 6. Also, the Cu concentration range is entered into chalcopyrite distribution model (Fig. 7). The resulting models have good correlation with hypogene zone, derived via the P–V method, as depicted in Fig. 7.

6. Conclusions

Results from this study are showing that P–V method could be effectively applicable for separating different mineralization zones in porphyry copper deposits, especially in supergene enrichment and hypogene zones. Also, different stages of mineralization can be interpreted via the P–V fractal method. The P–V method is a proper fractal modeling procedure which uses relationship between volumes containing and power spectrum values that encloses the volumes for power spectrum values of Cu associated with different zones, and

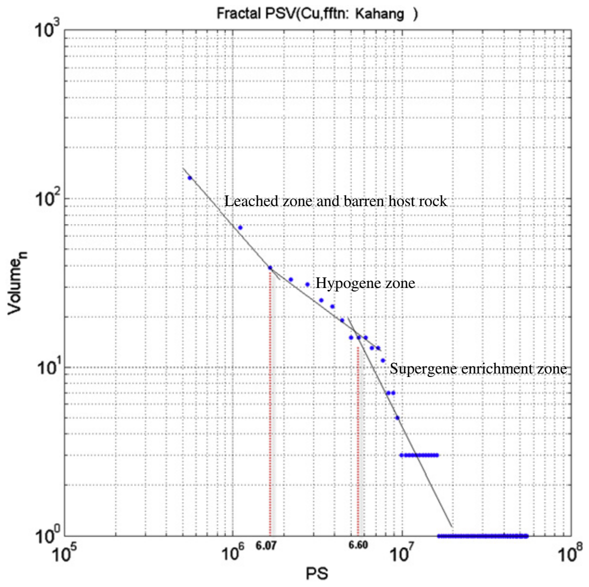


Fig. 4. A log–log plot showing relationships between power spectrum values calculated for Cu and volume with power spectrum above thresholds. Straight-lines were fitted to the values by means of least squares.

satisfies power–law relationships. The proposed fractal method, as introduced in this work, could be applied for delineating mineralized hypogene and supergene enrichment zones from the barren host rock or from the background value using the concentration values of the zones in combination with characteristic features of their geometrical shapes. The proposed method is applicable to power spectrum of ore element in porphyry deposits, such as Cu, Mo, Au and W, for which the spatial patterns of concentration values satisfy a multifractal model.

This P–V method has been successfully applied to separate Cu mineralized zones in Kahang porphyry deposit. Breaking points in log–log plot show that enrichment in supergene enrichment zone has threshold values of 0.73% and the hypogene threshold values are 0.1% Cu. Less than 0.1% Cu concentration in Kahang deposit is considered as weakly mineralized zones, background or barren host rock which is correlated with Cu mean in leached zone and barren host rock. The resulting zones from P–V method are correlated well with brecciated and intrusive rocks, quartz-sericite alterations and mineralogical data as well as supergene enrichment and hypogene zones. Supergene enrichment zone has high correlation within chalcocite accumulations in near surface mineralized zone. Also, hypogene zone is correlated with phyllic alteration and chalcopyrite accumulation models.

The need for handling over 2 millions of voxels and large amount of data in porphyry deposits, powerful software was needed for calculation of power spectrum and drawing the P–V log–log plots. Accordingly, the FPSV software was presented and developed by using MATLAB for this purpose. The input data file has a very simple and well-known tab delimited format benefitting the user in data handling of huge volumes. Data classification, as intended in this study, is based mainly on user's judgment and opinion, and a good data classification helps in giving better results. The software presented here

Table 2
Statistical parameters of Cu in different mineralization zones of Kahang deposit.

Zone	Sample no.	Mean Cu %	Median Cu %
Leached zone	74	0.04	0.02
Hypogene	1134	0.35	0.1
Supergene enrichment	32	0.9	0.97

Table 3
Ranges of PS for different mineralization zone in Kahang Cu porphyry deposit.

Zone	PS threshold	Range of PS
Barren host rock and leached zone	–	<6.07
Hypogene	6.07	6.07–6.60
Supergene enrichment	6.60	>6.60

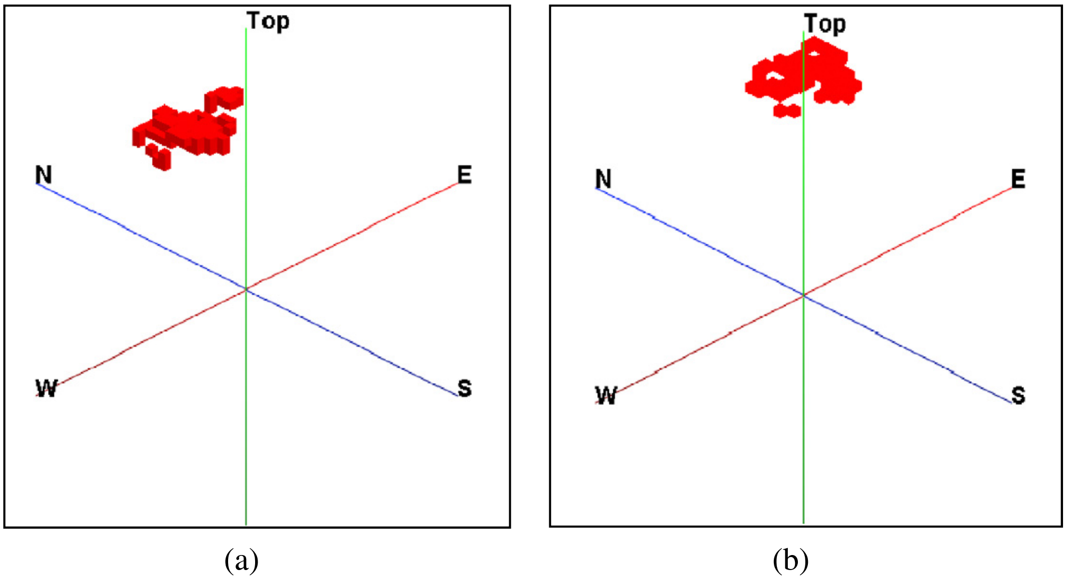


Fig. 5. The supergene enrichment zone (a) of Kahang deposit determined by P–V method and chalcocite distribution model (b).

has also the ability for construction of P–V log–log plot for zone's threshold values detection.

Appendix A. The FPSV software procedure

Programming in MATLAB, considering capabilities of data processing, makes it a very useful and practical tool. A suitable interface, easy data entry, powerful data processing, clear visualization, variety of output charts in simplest from, yielding clear results were considered to design an effective software. FPSV inherits these advantageous moreover, extra options such as saving the results as images and printing the results as where as needed are some of its friendly features. This software has been designed in such a manner that the needed changes could be easily added in the future. The software flowchart is illustrated in Fig. 8. The main subroutines are:

A) Reading data from a Tab-delimited text file in a simple format into workspace; four columns pointing the X, Y, Z (3D coordinate values of each voxel) and G (grade at each point in space) in sequence. Data files include center coordinates, volume and major element evaluated grades, i.e. Cu concentration in this case study, for different voxels which can be transferred, or come from, or be extracted from software such as Datamine, Rockworks and Surpac-Gemcom. This subroutine transfers the complete set of data into same named arrays. The delimiters in the text file between variables should be a comma or space. Generally, porphyry deposits have large data sets which cause overflow issues in much software as well as in Excel. To avoid this problem a text format is used (Tab-delimited file).

B) 3D fast Fourier transformation of the Cu concentrations and calculation of their power spectrum.

C) The data classified to an arbitrary number of classes which is 100 classes as considered for of the software. In the next step, minimum and maximum values of PS (power spectrum of major element grade) data were calculated, then the range value for PS computed and finally each class width was defined by dividing the range to the number of classes. User can determine the number of classes considering a total number of data at a requested accuracy level.

D) Counting the number of members for each class and computing their accumulated values using a loop structure which incorporates a comparison statement to find the home class for each value of PS.
In fact, all of the considered voxels have constant volumes so they are counted as points. By multiplying the computed value of the unit volume, the calculation will be valid. Nevertheless, in fractal

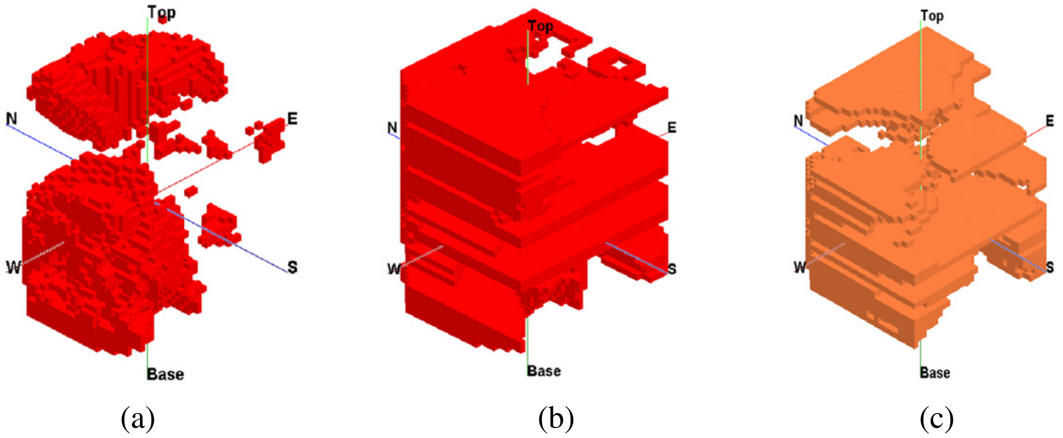
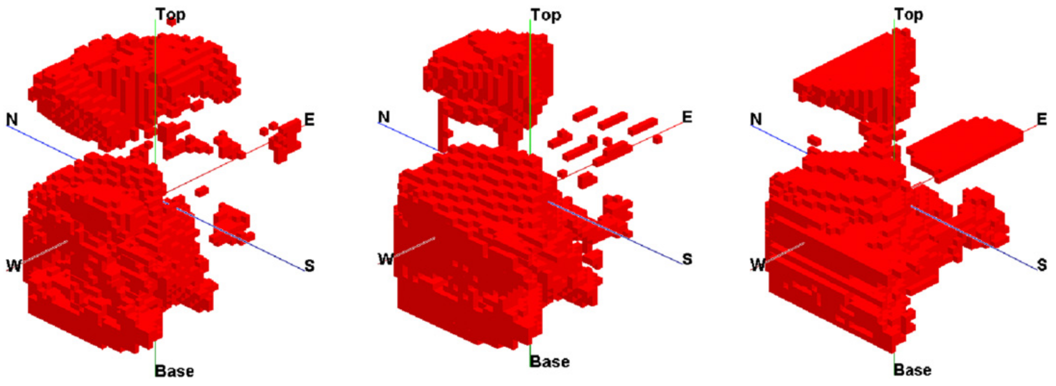


Fig. 6. The hypogene zone of Kahang deposit determined by P–V method (a) and phyllic alteration model (b) and chalcopyrite distribution model (c).



(a) (b) (c)

Fig. 7. The hypogene zone of Kahang deposit determined by P–V method (a) and primary geological hypogene zone model with Cu concentration range between 0.1% and 0.73% (b) and chalcopyrite distribution model with the Cu concentration range (c).

graph as will be discussed below, one, does not need to carry out this conversion procedure, since the interpretation is completely based almost on trend of the data.

E) Calculation of logarithm of all PS data and accumulated frequency volume values and drawing fractal log–log plot of power spectrum–volume based on previously computed values.

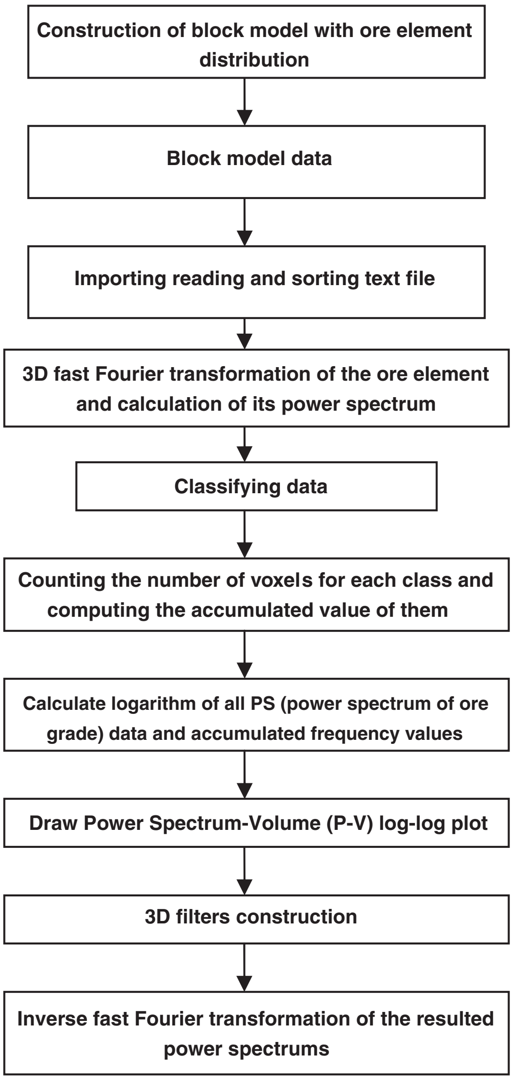


Fig. 8. The FPSV software simplified flowchart.

F) 3D filters construction based on threshold values resulting from P–V log–log plot and using on the data.

G) Inverse fast Fourier transformation of the power spectrum values were screened by filters.

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