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Application of fractal models to characterization of vertical distribution of geochemical element concentration

Renguang Zuo[1,2 *], Qiuming Cheng[1,2,3], Qinglin Xia[1,2]

[1]State Key Laboratory of Geological Processes and Mineral Resources, China University of Geosciences, Wuhan, Hubei, 430074, China

[2]The Faculty of Earth Resources, China University of Geosciences, Wuhan, Hubei, 430074, China

[3]Department of Earth and Space Science and Engineering, Department of Geography, York University, Toronto,4700 Keel Street, Ontario, M3J 1P3, Canada

Abstract: Characterization of the vertical distribution of geochemical element concentration is essential for economic planning in the mining industry. 10 mineralized boreholes and 1 non-mineralized borehole from the Qulong copper deposit, Tibet, western China, were collected to identify the vertical distribution properties of Cu values using fractal models. The vertical distribution of Cu values in mineralized and non-mineralized boreholes shows a positive skewed distribution in the former and multimodal distribution in the latter. The results obtained by the box counting method show that the vertical distributions of Cu values in mineralized and non-mineralized boreholes exhibit self-similarity with box dimensions ranging from 1.28 to 1.37. The box dimensions of mineralized boreholes are greater than that of Cu values in the non-mineralized borehole, indicating that the mineralization makes the distribution of Cu values more irregular. The power-law frequency analysis reveals that Cu values in mineralized boreholes are bifractal. The two portions of the plot define a crossover point at 0.33%, for Cu values less than and greater than 0.33, fractal dimensions range from 0.1 to 0.65, in non-mineralized rocks, and range from 2.71 to 5.79, in the mineralized rocks. Hurst exponents for mineralized boreholes occur at 0.8, which are greater than 0.5,

* Corresponding author. E-mail address: zrguang@cug.edu.cn

indicating that Qulong copper deposit has a good continuity of mineralization.

Keywords: Fractal; Box dimension; Power-law frequency; Hurst exponent; Borehole; Qulong

1. Introduction

Characterization of the distribution of geochemical elements in boreholes is of significance to evaluate the quality and quantity of mineral resources in the mining industry. In the past several decades, the nature of the distributions of geochemical elements in rocks has been intensively studied in order to discover a universal geochemical law. Ahrens(1954, 1963a, 1963b, 1966) proposed that the frequency distributions of most minor and trace elements in rocks and ore deposits were positively skewed and could be satisfactorily described by the lognormal law; while the distribution of major elements is more complex. For example, the silica distribution of surface rocks where there are basic and acid maxima (Richardson, 1923) exhibits multimodal distributions (Allègre and Lewin, 1995). The distributions of geochemical elements in boreholes also exhibit power-law characteristics, which can be fitted by fractal models (Sanderson et al., 1994; Monecke et al, 2001).

A fractal is defined as a set in which the Hausdorff dimension strictly exceeds the topological dimension (Mandelbrot, 1983). It has generally the following features: (1) It has a fine structure at arbitrarily small scales: (2) It is too irregular to be easily described in traditional Euclidean geometric language; (3) It is self-similar; (4) It has a Hausdorff dimension which is greater than its topological dimension, and (5) It has a simple and recursive definition (Kenneth,1997). Fractal models have been extensively applied to physical and chemical quantities with geometrical support in the past two decades. In geology, this approach has been used for characterizing geological objects and geological features (Mandelbrot, 1983; Cheng, 1995; Wang et al., 2006; Wang et al., 2008) and for separating anomalies from background values (Cheng et al., 1994; Cheng et al., 2000), and for quantifying properties of mineralization and mineral deposits (Turcotte, 1996, 2002; Sanderson et al., 1994; Agterberg et al., 1993; Li et al., 1994; Shi and Wang, 1998). In this paper, the fractal dimensions including box dimension, power-law frequency and Hurst exponents were utilized to characterize the vertical distribution of trace elements and to evaluate the mineralized continuity based on borehole datasets. For demonstration purposes, a case from Qulong Copper Deposit will be studied. In the next section, the box dimension, power-law frequency model and Hurst exponent are briefly introduced for demonstrating how to process data.

2. Fractal Models

2.1. Box Counting Method and Box Dimensions

Box counting is one of most popular methods to estimate the fractal dimensions. It can be used for measurement of the irregularity of patterns, and can be implemented with the aid of GIS. A group of grids with box size “δ” were used to cover the concentration curve vector layers, and these grids were used for counting the corresponding number of grey boxes ($N(\delta)$) occupied by grade curves. Data pairs for box size δ and number *N* can be plotted on a log–log paper and linear regression applied to fit a straight line from which the box dimension *D* can be estimated. The box-counting method results in a power-law relation:

$$N(\delta) \propto \delta^{-D} \quad (1)$$

It can be rewritten as

$$Log[N(\delta)] = C - DLog(\delta) \quad (2)$$

where $N(\delta)$ is the number of cells containing grade curves, *C* is a constant, and *D* is the box dimension.

2.2. Power-law Frequency Model

The power-law frequency model was used to measure the frequency distribution of element concentration. This model has been demonstrated by many studies (Turcotte, 1996; Sanderson et al., 1994;

Li et al., 1994; Shi and Wang, 1998), and can be expressed as

$$N(\geq c) \propto c^{-D} \qquad (3)$$

It also can be rewritten as

$$Log[N(\geq c)] = C - DLog(c) \qquad (4)$$

where $N(\geq c)$ is the number of samples with a content greater than c, C is a constant and D is the fractal dimension.

2.3. Hurst Exponent

The Hurst Exponent proposed by Hurst (1951) is directly related to the fractal dimension of a process, which gives a measure of the process roughness. The Hurst exponent, H, is a self-similarity parameter which measures the long-range dependence in a time series, and provides a measure of long-term nonlinearity. The expected values of H lie between 0 and 1. For H = 0.5, the cumulative behavior is a random walk and the process produces uncorrelated white noise. H<0.5 represents anti-persistent behavior and H>0.5 is fractional Brownian motion with increasing persistence strength as H approaches 1.

The rescaled range statistic (R/S) analysis can be used to estimate the Hurst exponent (Mandelbrot and Wallis,1969) , which can be described as follows: The ordered data sequence is divided into d contiguous sub-series of length n, where $d\times$n = N, the total number of the samples. For each of these sub-series m, where $m = 1, \cdots, d$.

(1) Determine the mean, E_m, of each sub-series.

(2) Determine the standard deviation, S_m, of each sub-series.

(3) Normalize the data ($Z_{i,m}$) by subtracting the mean from each data point:

$$X_{i,m} = Z_{i,m} - E_m \quad m = 1, 2, \ldots, n \qquad (5)$$

(4) Use the normalized data creating a cumulative series by consecutively summing the data points:

$$Y_{i,m} = \sum_{j=1}^{i} X_{j,m} \quad i = 1, 2, \ldots, n \qquad (6)$$

(5) Use the new cumulative series to find the range by subtracting the minimum value from the maximum value:

$$R_m = \max\{Y_{1,m}, \ldots, Y_{n,m}\} - \min\{Y_{1,m}, \ldots, Y_{n,m}\} \qquad (7)$$

(6) Rescale the range, R_m / S_m by dividing the range based on the standard deviation.

(7) Calculate the mean of the rescaled range for all sub-series of length n:

$$(R/S)_n = \frac{1}{d} \sum_{i=1}^{d} R_m / S_m \qquad (8)$$

(8) The length of n must be increased to the next higher value, where $d \times n = N$, and d is an integer value. Step 1 to 7 are then repeated, these steps should be repeated until $n = N / 2$.

(9) Finally, the value of H is estimated from the slope of the regression line with log (n) versus log(R/S).

3 Geological Setting and Sampling

The study area, Qulong Copper Deposit, is a large porphyry copper deposit in the Gangdese porphyry copper belt, located approximately 110km southeast of Lhasa, the capital city of Xizang (Tibet), western China. A simplified geological map is shown in Fig.1. The host rocks are biotite-adamellite ($\beta\eta\gamma$) covering approximately 60% of the middle and east part of the district, and adamellite quartz porphyry ($\eta\lambda\pi$) located in the northwest zone. Figure 2 shows the profile of the eighth exploring line where the rocks mainly consist of quartz, biotite-adamellite granites ($\beta\eta\gamma$) and orebodies. The geological features of the Qulong deposit were studied in detail by Rui et al. (2003) and Zheng et al. (2002). The Qulong copper deposit consists of 7 orebodies, CuI, CuII, CuIII, CuIV, CuV, CuVI, and CuVII, among which CuII, the largest one, occurs in the northwest, and CuVI, the smallest one, occurs in the central part of the zone. 10

Cu mineralized datasets and 1 non-mineralized dataset, ZK1901, were obtained by continuous sampling every 2m and 7m along borehole in the zone, respectively. The properties and statistical results of the boreholes datasets are summarized in Table 1, and the line graphs of Cu values from mineralized boreholes are shown in Fig.3.

4. **Results and discussions**

4.1.Frequency distribution of Cu

The property of frequency distributions of element concentration are usually evaluated in histograms that display arbitrarily chosen, linearly scaled concentration intervals on the abscissa and the frequency of individual analyses whose results fall in a particular class interval on the ordinate. The histograms (Fig. 4) show that the distributions of Cu values in the mineralized borehole are positively skewed.

4.2. Box Dimensions

The box dimensions can be obtained from the line graphs of Cu (Fig. 3). The Cu values lines can be converted from a vector to a raster. Different appropriate cell sizes can be used to determine the count of cells. Then a log-log plot of cell size versus count made in Excel with a trend line using a power function and providing the coefficient of determination, R^2, will provide the fractal dimensions and coefficient of explanation. The log-log plots of cell size versus count are shown in Fig. 5 and Fig. 8B, and the box dimensions and R^2 are listed in Table 2. The results show that the vertical distribution of Cu in mineralized boreholes exhibits self-similarity and box dimensions range from 1.28 to 1.37, with R^2 greater than 0.99, indicating that the vertical distributions of Cu have approximate irregularity. The results also demonstrate that the box dimensions of mineralized boreholes are greater than or equal to that of the distribution of Cu in the non-mineralized borehole, indicating that the mineralization makes the vertical distribution of Cu values more irregular.

4.3. Power-law Frequency Distribution of Cu Values

The results obtained by the power-law frequency model for mineralized boreholes are shown in Fig.6 and listed in the middle of Table 2. The log-log plots of cumulative number versus Cu values show the distribution of Cu in the mineralized boreholes satisfying bifractal. The two portions of the plot define a crossover point at 0.33 (Table 2), for Cu values less than and greater than 0.33, fractal dimensions range from 0.1 to 0.65, in the non-mineralized rocks, and range from 2.71 to 5.79, in the mineralized rocks.

Larger fractal dimensions of mineralization imply more homogeneous mineralization. It can be explained that fractal dimensions are estimated from the frequency of Cu values and can reflect the proportion of Cu values. Larger fractal dimension implies less numbers of Cu values greater than a specific Cu value. It means that the Cu value change slowly along the borehole, indicating more homogeneous mineralization. The coefficient of variation (CV), which is equal to the standard deviation divided by the average, identifies the degree of change. In the case of approximate average of Cu value, a low CV implies elemental concentrations changing slightly along the borehole. In other words, it means more homogeneous mineralization. Therefore, the fractal dimensions of mineralization are inversely related to CV. Figure 7 shows the regression line existing between fractal dimensions and CV. The squared correlation coefficient R^2 is 0.84.

On the other hand, Figure 8A shows that the vertical distribution of Cu in ZK1901 exhibits a multimodal distribution, which is caused by different wall rocks containing a different Cu content. The explanation is that the distribution is the sum of several other distributions; therefore, the final density distribution can become multimodal, particularly when the parent distributions characterize well-defined source materials with different mean or modal values (Allègre and Lewin, 1995). Figure 8C illustrates three regression lines existing in a log-log plot of cumulative number versus Cu values, which could be caused by a multimodal distribution.

4.4. Hurst Exponents

The results obtained by Hurst exponents using R/S for 10 mineralized boreholes are shown in Fig.9 and listed in the last two columns of Table 2. The values of Hurst exponents range from 0.62 to 0.91, all greater than 0.5, indicating persistent phenomena and better continuity of mineralization. The Hurst exponent for the non-mineralized borehole Zk1901 is 0.87(Fig. 8D), and is also greater than 0.5 owing to the homogeneous distribution of Cu in wall rocks.

5. Conclusions

The continuity of mineralization is a key issue in potential mineral resources assessment for a given deposit. It can be recognized by characterizing the vertical distribution of geochemical concentration values in borehole datasets. In this paper, three fractal dimensions, including box dimension, power-law frequency and Hurst exponent, are used to investigate the irregularity and the continuity of Cu mineralization. The following results are obtained:

(1) The vertical distribution of Cu values in mineralized and non-mineralized boreholes show self-similarity with box dimensions ranging from 1.28 to 1.37. The box dimensions for mineralized boreholes are greater than that of Cu values in non-mineralized borehole, indicating that the mineralization process makes the distribution of Cu values more irregular.

(2) The vertical distribution of Cu values in mineralized and non-mineralized boreholes has different characteristics: the former satisfies a positive skewed distribution and exhibits bifractal; while the latter satisfies a multimodal distribution.

(3) Hurst exponents for mineralized boreholes are about 0.8, indicating that Qulong has a good continuous mineralization.

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Figure and Table Captions

Fig.1.Simplified geological map of Qulong Copper Deposit

Fig.2.The profile of the eighth exploring line

Fig.3.Plots of Cu values for 10 mineralized boreholes

Fig.4.The histograms of Cu values from 10 mineralized boreholes

Fig.5.Log-log plots of number versus box size for 10 mineralized Cu values (Logs base 10)

Fig.6.Log-log plots of cumulative number versus Cu grade 10 mineralized boreholes (Logs base e=2.732)

Fig.7. Plot of fractal dimensions for mineralization versus coefficient of variation along with regression line

Fig.8. (A) is the histogram of Zk1901 Cu grade; (B) log-log plots of number versus box size for Cu values (Logs base 10); (C) is power-law frequency for non-mineralized borehole Zk1901 and (D) is log-log plot of

R/S versus number for non-mineralized borehole (Logs base 10)

Fig.9.Log-log plots of R/S versus number for 10 mineralized boreholes (Logs base 10)

Table 1.The basic information and statistical properties for 10 mineralized boreholes and 1 non-mineralized borehole from Qulong Copper Deposit

Table 2.The fractal dimension of Cu values from Qulong copper deposit

Table 1 .The basic information and statistical properties of 10 mineralized boreholes and 1 non-mineralized borehole from Qulong Copper Deposit

NO.	X	Y	Number of Samples	Elevation(m)	Cu Max (%)	Cu Min (%)	Cu Average (%)	Std.	CV
ZK001	64.20	79.30	147	5085	1.79	0.14	0.48	0.27	0.56
ZK002	64.24	79.10	104	5210	1.21	0.02	0.34	0.15	0.44
ZK003	64.11	79.80	239	5390	2.20	0.07	0.45	0.28	0.62
ZK004	64.15	78.63	63	5215	1.01	0.03	0.27	0.16	0.59
ZK801	64.55	79.30	273	5085	1.58	0.02	0.50	0.21	0.42
ZK802	64.55	79.70	243	5080	1.06	0.20	0.51	0.11	0.22
ZK803	64.55	79.90	261	5115	1.07	0.22	0.45	0.12	0.27
ZK1601	64.95	79.30	206	5160	0.94	0.26	0.49	0.11	0.22
ZK1101	63.60	79.83	175	5375	1.29	0.10	0.37	0.16	0.43
ZK701	63.73	79.32	146	5370	1.00	0.09	0.40	0.16	0.40
Zk1901	65.50	80.5	85	5230	0.093	0.003	0.037	0.027	0.72

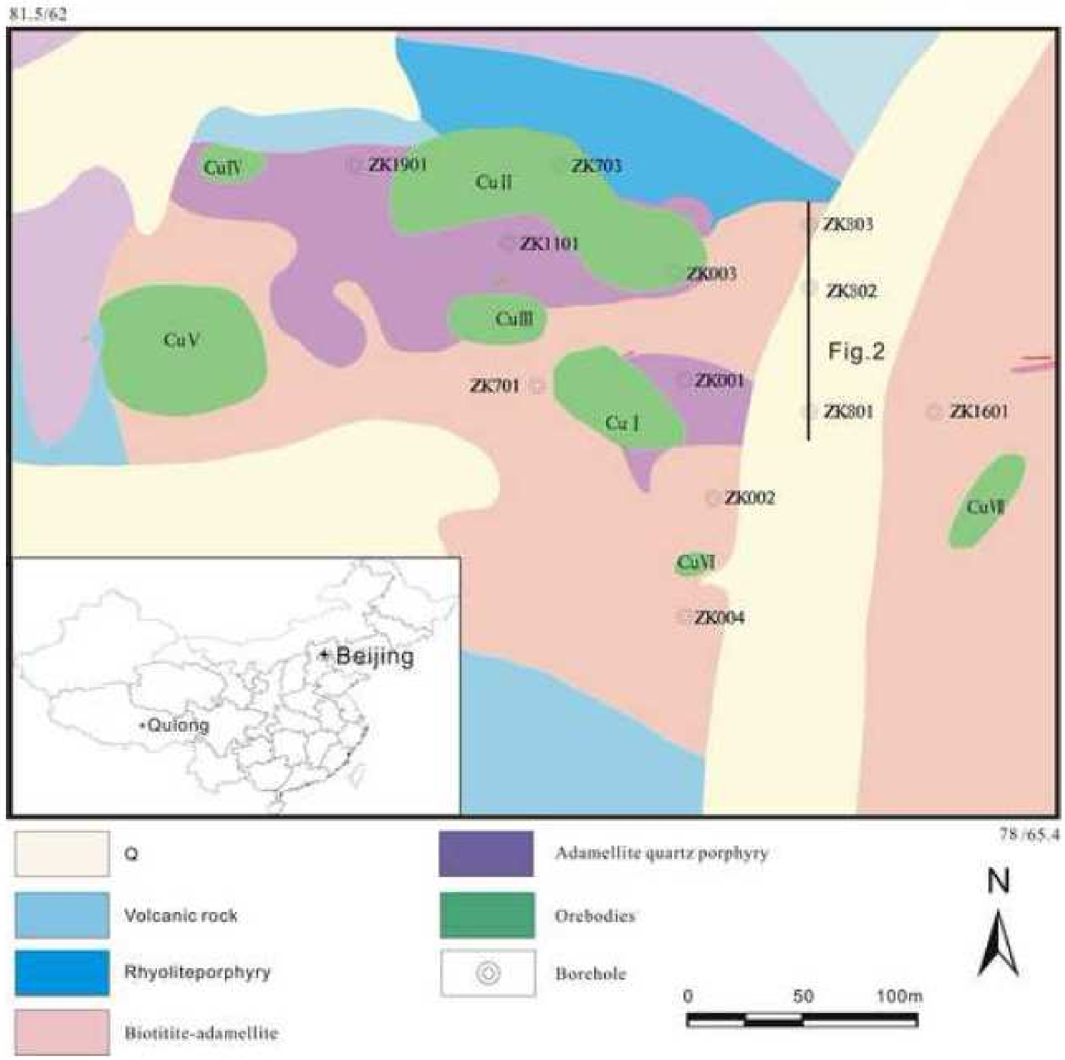
Note: Std. is the standard deviation, and CV is standard coefficient of variation

Table 2.The fractal dimensions of Cu in Qulong Copper Deposit

Borehole	**Box dimensions**		**Power-law frequency**					**Hurst exponent**	
	dimensions	**R^2**	**dimension 1**	**R^2**	**break point**	**dimension 2**	**R^2**	**Hurst**	**R^2**
ZK001	1.30	1.00	0.22	1.00	0.33	2.74	0.97	0.82	0.97
ZK002	1.28	1.00	0.1	0.64	0.33	3.75	0.97	0.75	0.91
ZK003	1.38	1.00	0.23	0.95	0.33	2.71	0.99	0.77	0.99
ZK004	1.37	1.00	0.24	0.85	0.24	2.83	0.98	0.80	0.95
ZK801	1.35	1.00	0.11	0.6	0.37	3.60	0.99	0.62	0.95
ZK802	1.37	1.00	0.3	0.64	0.45	4.25	0.99	0.91	0.98
ZK803	1.34	1.00	0.32	0.92	0.37	4.89	0.99	0.88	0.96
ZK1601	1.37	1.00	0.65	0.96	0.45	5.79	0.96	0.83	0.96
ZK1101	1.34	1.00	0.22	0.81	0.33	3.38	0.99	0.71	0.95
ZK701	1.32	1.00	0.21	0.74	0.33	3.92	0.96	0.73	0.97
ZK1901	1.28	0.99	0.2	0.97	0.22	1.67	0.96	0.87	0.96

Note：Dimension 1 denotes the negative slope of the upper regression line in power-law frequency, and dimension 2 denotes the negative slope of the lower regression line in power-law frequency.

Figure 1


81.5/62
CuⅣ
ZK1901
CuⅡ
ZK703
ZK803
ZK1101
ZK003
ZK802
CuⅢ
CuⅤ
Fig.2
ZK701
ZK001
CuⅠ
ZK801
ZK1601
ZK002
CuⅦ
CuⅥ
ZK004
Beijing
Qulong
78/65.4
Q
Volcanic rock
Rhyoliteporphyry
Biotitite-adamellite
Adamellite quartz porphyry
Orebodies
Borehole
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0
50
100m

Figure 2

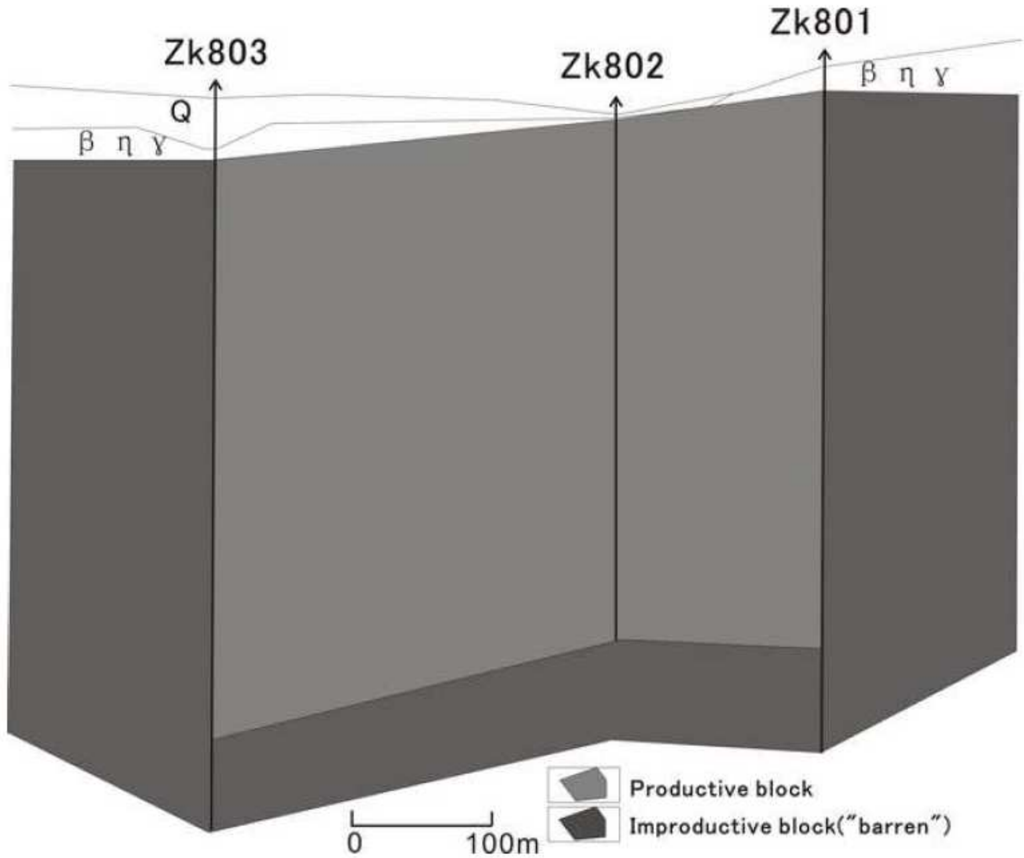
Zk803
Zk802
Zk801
Q
β η γ
β η γ
0
100m
Productive block
Improductive block("barren")

Figure 3

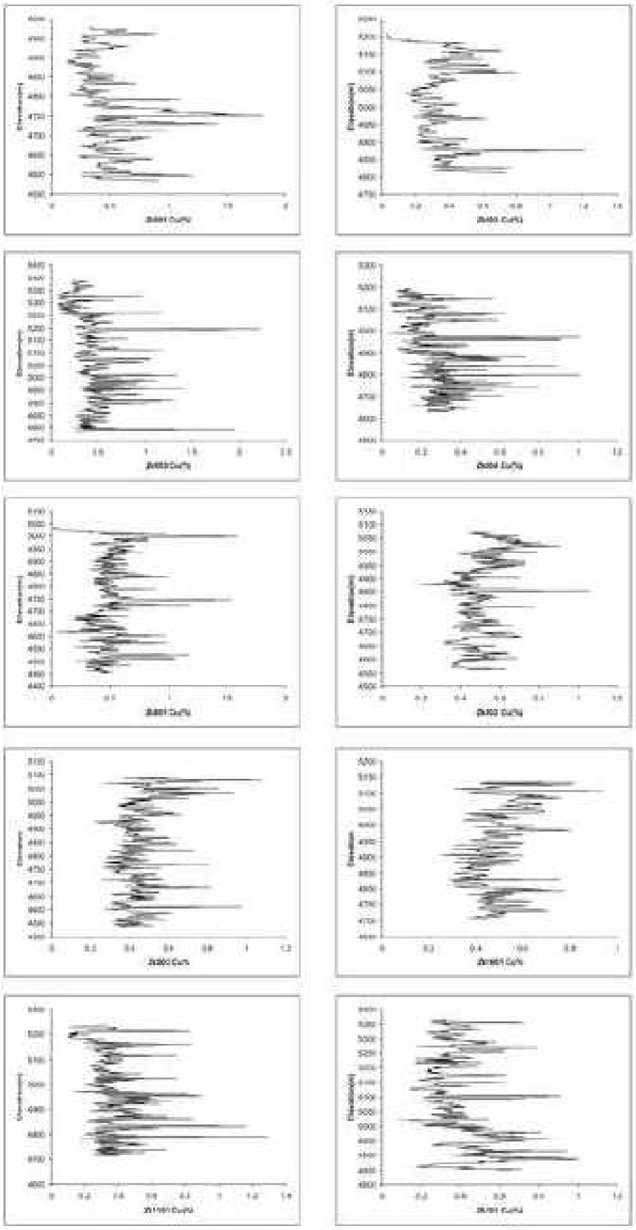

Figure 4

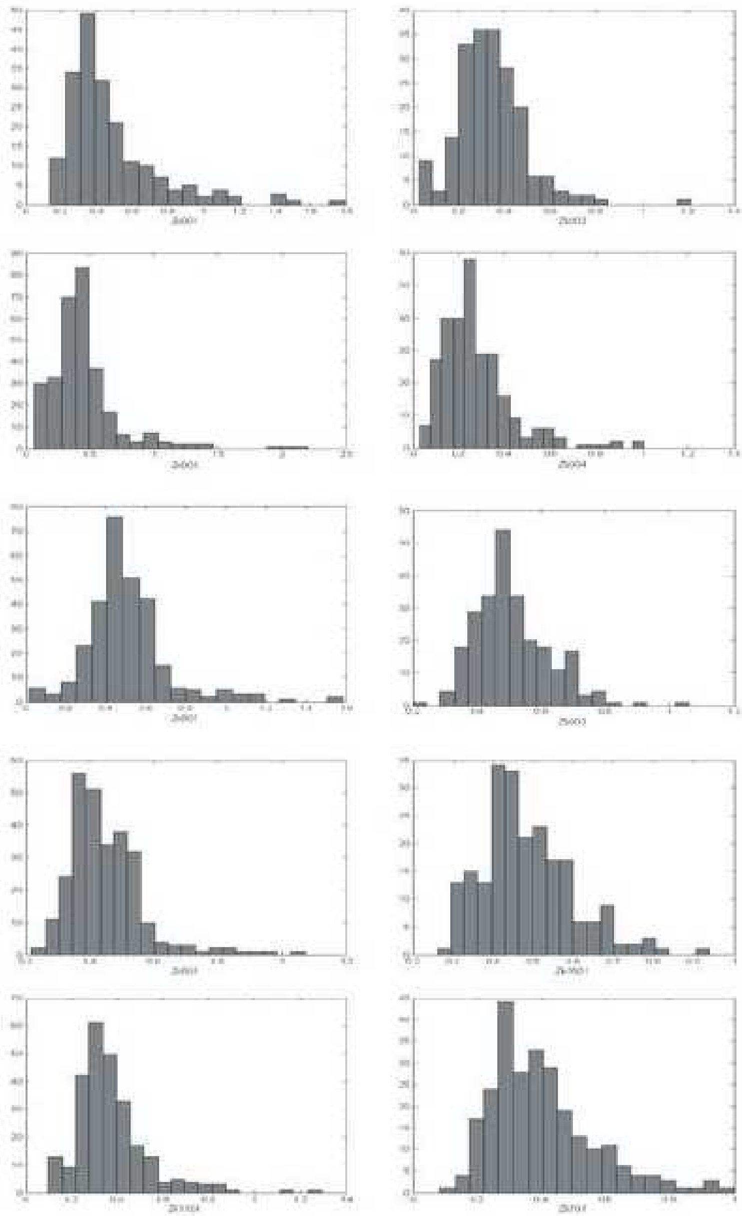

Figure 5

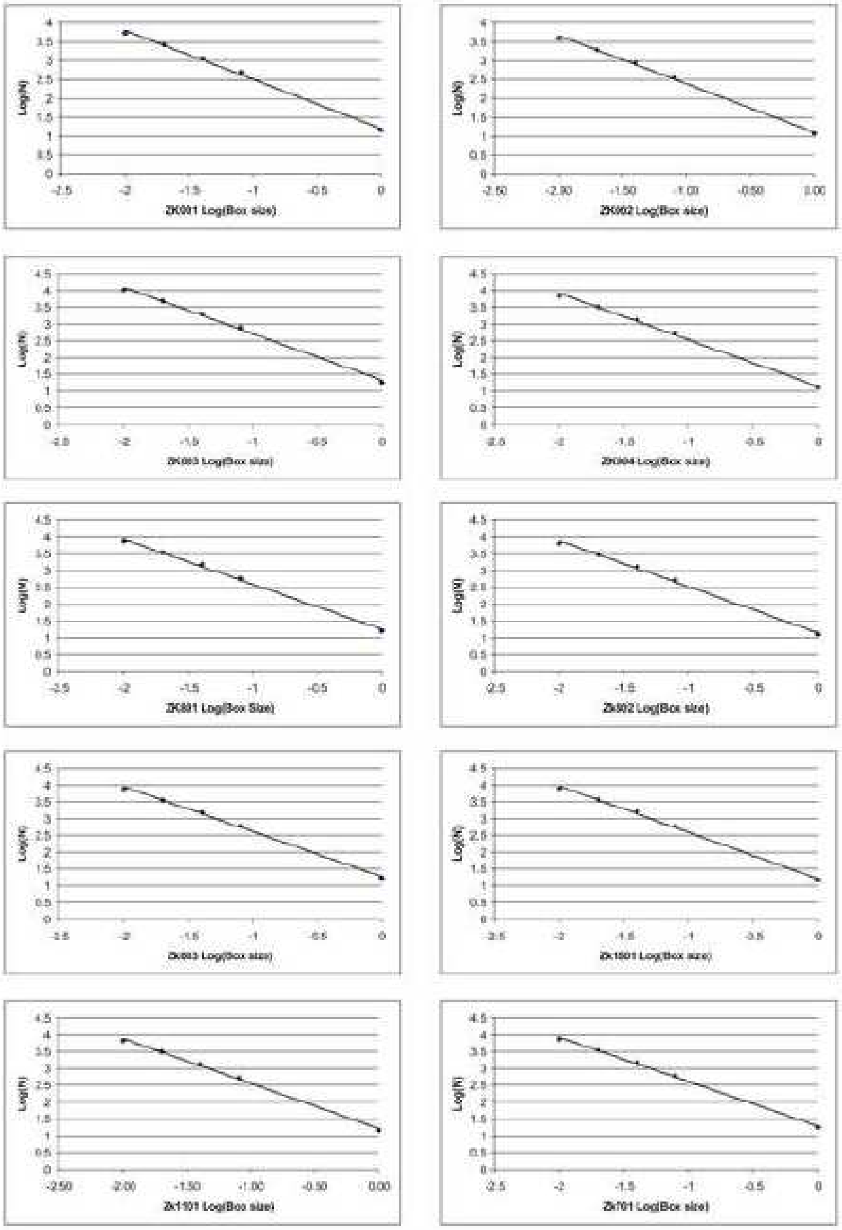
Log(N)
ZK001 Log(Box size)
ZK002 Log(Box size)
ZK004 Log(Box size)
Zk1101 Log(Box size)
Zk701 Log(Box size)

Figure 6

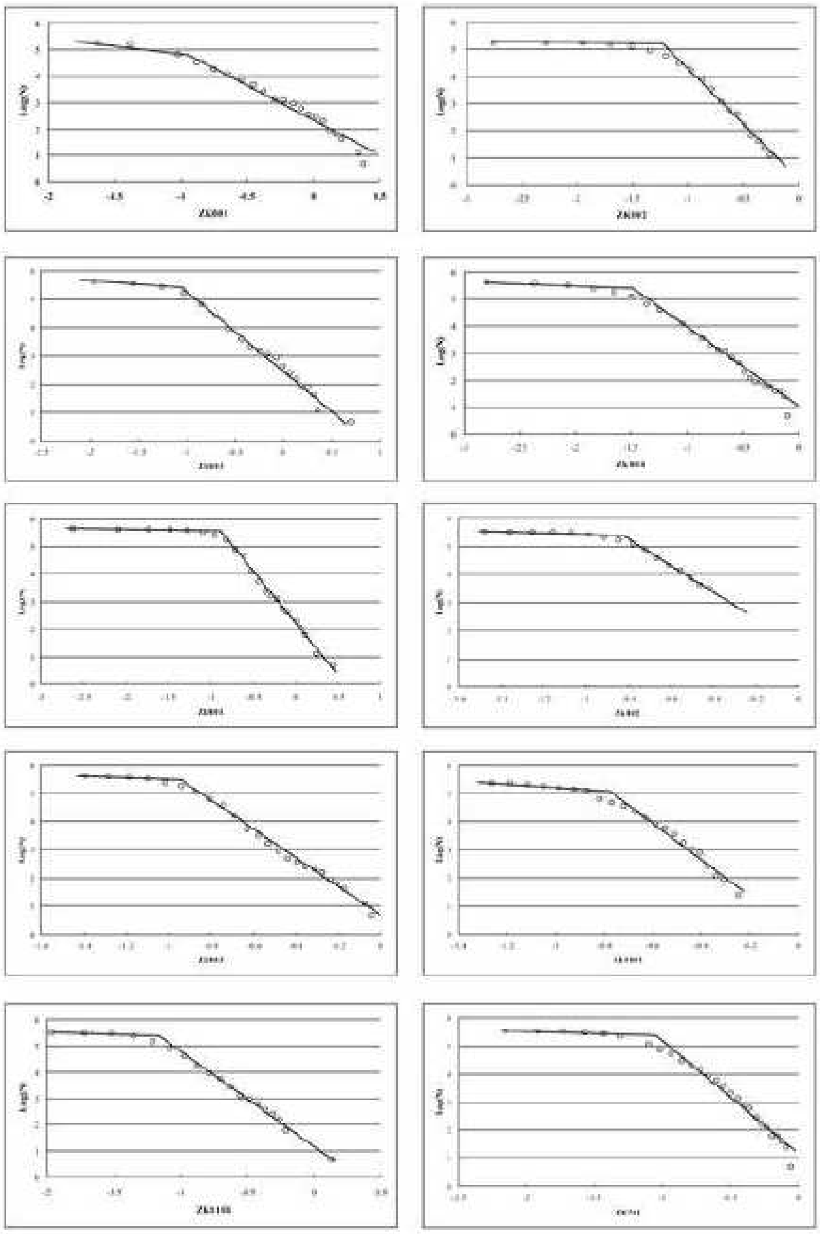

Figure 7

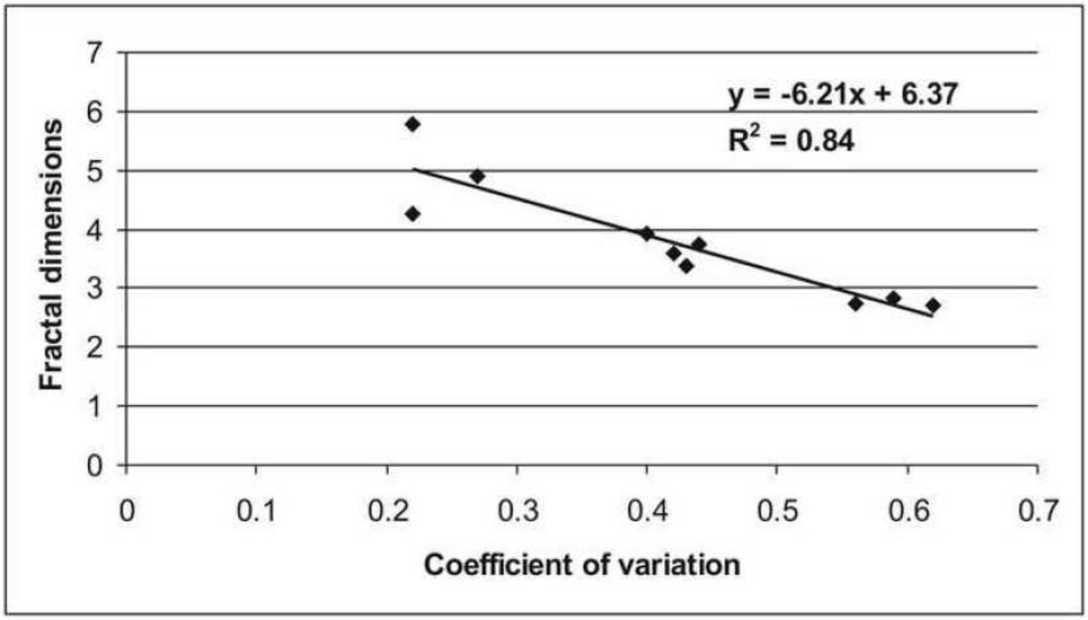
y = -6.21x + 6.37
R^2 = 0.84
Fractal dimensions
Coefficient of variation
0
1
2
3
4
5
6
7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Figure 8

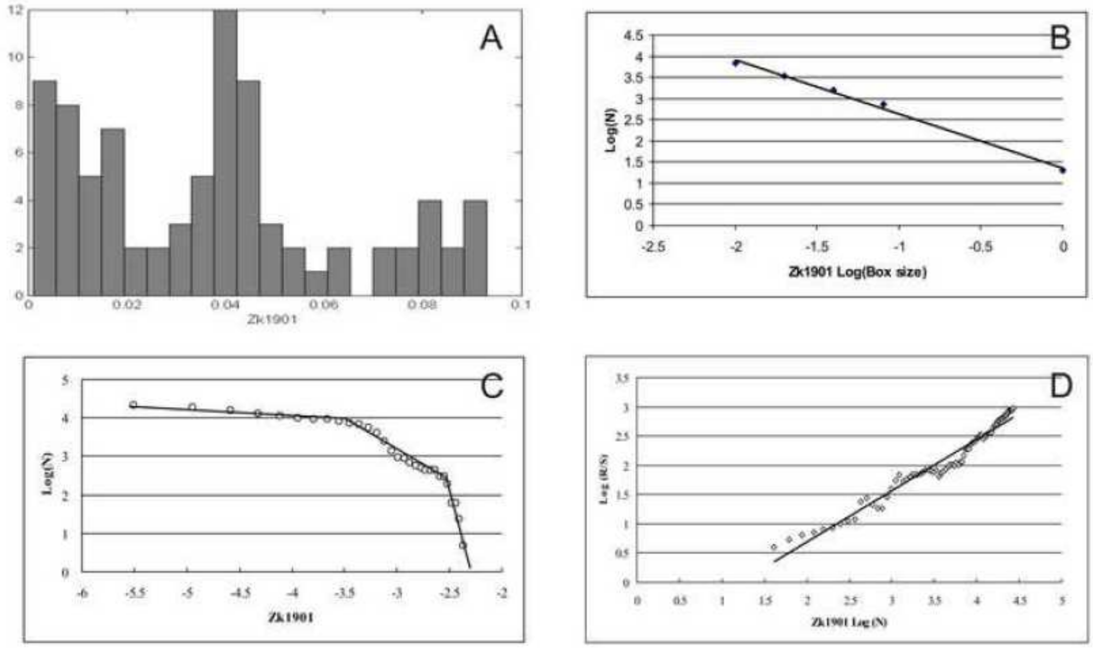
A
Zk1901
B
Log(N)
Zk1901 Log(Box size)
C
Log(N)
Zk1901
D
Log (R/S)
Zk1901 Log (N)

Figure 9