Ore Geology Reviews 78 (2016) 336–345

Contents lists available at ScienceDirect

Ore Geology Reviews

journal homepage: www.elsevier.com/locate/oregeorev

A practical methodology to optimise marginal mineral deposits using switching real options

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ARTICLE INFO

Article history:
Received 18 January 2016
Accepted 20 April 2016
Available online 25 April 2016

ABSTRACT

Currently depressed commodity prices have rendered many mining projects marginal irrespective of their geological merit. Tight capital markets discourage investment in their development because of their unappealing deterministic NPVs, which in the majority of cases reflect conceptual designs focused on achieving primarily economies of scale often at the expenses of operating flexibility. Given that project profits and cash flows are highly sensitive to movements in volatile commodity prices, circumstances now call for a re-direction of emphasis towards creating managerial flexibility to facilitate and minimize the cost of temporarily placing projects in care and maintenance and re-opening them in response to increases in prices. This flexibility, that is to say the option to alternatively switch the project between an open and closed state, can be created through an appropriate combination of mine design, commercial procurement arrangements and mode of operations that enables managers to anticipate and take advantage of future hikes in prices, while minimizing the negative effect of downturns. This paper presents a practical example of how to estimate the real option value (ROV) of this type of switching option, which is generally not captured by the deterministic DCF/NPV of projects. To facilitate the numerical presentation, initially the binomial lattice method is applied only to the first 2 years of a realistic DCF model of a gold mine, with an expected life of 5 years and a negative deterministic NPV. The model is limited to assessing the ROV created by introducing switching flexibility as a result of the volatility of the gold price in isolation. A consistent ROV is then obtained using as an alternative the unrelated decision tree methodology. This result is considered important as using decision trees for this type of analyses in cases where more than one source of uncertainty is involved (e.g. that of grades, costs, and exchange rates) does not require, as in the case of binomial lattices, estimating the volatility of a project cash flow. This process, which may create computational ambiguity and possible bias, can be avoided in decision trees as each source of uncertainty is represented by an individual event node. Finally the ROV of the project, including the switching option, is calculated over its whole 5-year life to provide some indication of the amount that could justifiably be invested up-front to create the necessary switching flexibility.

1. Introduction

The price of most mineral commodities has been falling in recent years to levels not seen for over a decade. As a result, many mining projects have essentially become financially marginal in spite of their inherent geological merit. Many of these projects had either been developed during recent boom years, when commodity prices were high and development capital was easily available, or alternatively remained undeveloped. These circumstances have created great opportunities for discerning investors to apply innovative selection criteria for identifying and acquiring potentially valuable mineral projects.

During recent periods of boom, mine design has been primarily focused on achieving economies of scale generally on the expectation of sustained commodity prices in the future, resulting in significant capital investments and in many instances relatively inflexible mining designs and practices. Although progressive falls in prices have stimulated significant improvements in productivity, these have, in some cases, escalated into extreme and sometimes counter-productive forms of cost-cutting. Inevitably, after operating at a loss, some mining projects had to close down at significant cost and confront the prospect of having to incur major re-opening costs when commodity prices hopefully improve in the future.

Yet, with the wisdom of hindsight, a number of flexibility measures could have been adopted at relatively low cost at the time of development to anticipate and soften the impact, and indeed, take advantage of the volatility in mineral prices. One of these approaches would have been for management to create project flexibility through both mine design and related commercial arrangements to facilitate alternative

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[1] CET is an unincorporated joint venture between the University of Western Australia, Curtin University and the mining industry, established with seed funding provided by the State of Western Australia under their Centre of Excellence Program.

http://dx.doi.org/10.1016/j.oregeorev.2016.04.016
0169-1368/

switching of the project from the 'operating' or 'open' state to the 'close' state, where operations are temporarily suspended and placed under 'care and maintenance,' and back in response to movements in the price of the mineral commodity being produced (Fig. 1).

In the context of this paper the term 'close' implies temporary suspension of operations and is fundamentally different from a permanent closure or abandonment of the mine. In the first instance, at a minimum, some personnel must be retained to keep the mine dry and to service and maintain plant and equipment in readiness for possible re-opening at short notice. Temporary closure of operations raise important questions as to what degree a company would be justified in maintaining a critical core of technical competence while facing negative cash flows to ensure continuity and facilitate possible resumption of operations. In the second, essentially all commitments to staff and other parties are severed, plant and equipment salvaged, the mine openings are made safe or filled in and the site is rehabilitated to a standard required by law before the land can be returned to government.

The change from the 'open' to the 'close' state involves two types of costs:

- A once-off 'closing' cost, and
- Annual 'care and maintenance' costs while the mine is not operating.

The change from the 'close' to the 'open' state involves:

- A once-off 'opening cost' and
- Normal annual operating expenses while the mine is in production.

The capacity to switch with relative ease between alternative open/closed states, or between inputs and outputs of projects creates 'switching option' value. This is generally not captured by the NPV obtained through a deterministic DCF model of the project. In effect the difference between the Expanded NPV (ENPV) of the project including the value of flexibility (Munn, 2002) and its static NPV represents the Real Option Value (ROV) of the switching option.

As it will be discussed in detail below, in the case of a marginal mining project with a finite life of say 5 years, the ENPV represents the value of exploiting the reserves not necessarily over the 5 years implicit in the static DCF model of the project, but over a potentially longer period. During this period the 5 individual years of production may either be continuous or separated by years during which the project is not operating and is placed in care and maintenance in response to low commodity prices.

In addition to the above designed flexibility, the geological characteristics of some mineral deposits may display inherent or natural flexibility. An example of this type of flexibility can be found where high-grade reserves are surrounded by halos of adjacent, or at any rate reachable, progressively lower-grade resources, providing potential trade-offs between tonnages and grades of reserves exploitable, given an appropriate mine design, at different commodity prices. This type of 'chooser option', to expand from Run of Mine (ROM) production grades and volumes to lower grades and higher volumes in cases of persistent commodity price rises, or lower tonnages of higher-grade ore, or even close, in response to severe falls, is discussed in Guj (2013). Samis (2001) provides a quantitative model of an open cut gold mine designed to take advantage of natural flexibility by facilitating a possible pit wall cut-back and/or a portal to reach gold resources adjacent to and/or below the planned pit that were considered to be sub-economic at the time of the initial feasibility study.

An exhaustive review of literature relating to the theory and application of real options to mining in general can be found in Shafies et al. (2009). Potential sources of project flexibility are discussed, among others, in Samis and Davis (2004), while Kazakidis and Mayer (2010) focus specifically and in detail on the creation of flexibility in underground mines.

Depending on the circumstances the effect of designed and inherent flexibility may compound into more complex real options as illustrated in Fig. 2. Solving these types of switching options requires the formulation of a more sophisticated DCF model of the mining project capable of accommodating changes in capital and recurrent expenses, both in the form of potential increases and savings, which would be brought about by possible changes in production levels. For the sake of simplicity, however, this paper will focus exclusively on the type of open/close switching option illustrated in Fig. 1.

The potential significance of the ROV of the switching option of alternatively opening or closing a mine producing a mineral commodity with a volatile price has been academically recognized for around four decades and has generated a large body of relevant literature, including fundamental contributions by Brennan and Schwartz (1985), Slade (2001) and Moel and Tufano (2002).

In spite of its conceptual power, practical industrial application of real option modeling in the area of mining investment has proven relatively slow. This is primarily because of a persistent degree of confusion and lack of understanding of the ROV processes in general and of a, in many ways justifiable, perception that the related mathematical calculations can prove impractically complex. As it will be seen, such complexity can be largely overcome through appropriate use of user-friendly computer spreadsheets and off-the-shelf decision tree software

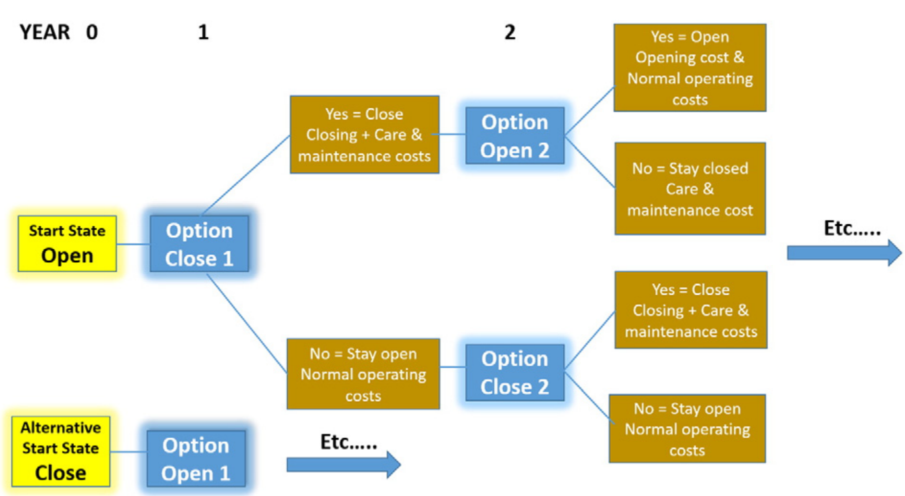

Fig. 1. Diagram illustrating the structure of a typical close/open switching option and related cost consequences. Please note that the analysis could have started from a 'Close' state in which case the first option would have been 'Open 1'.

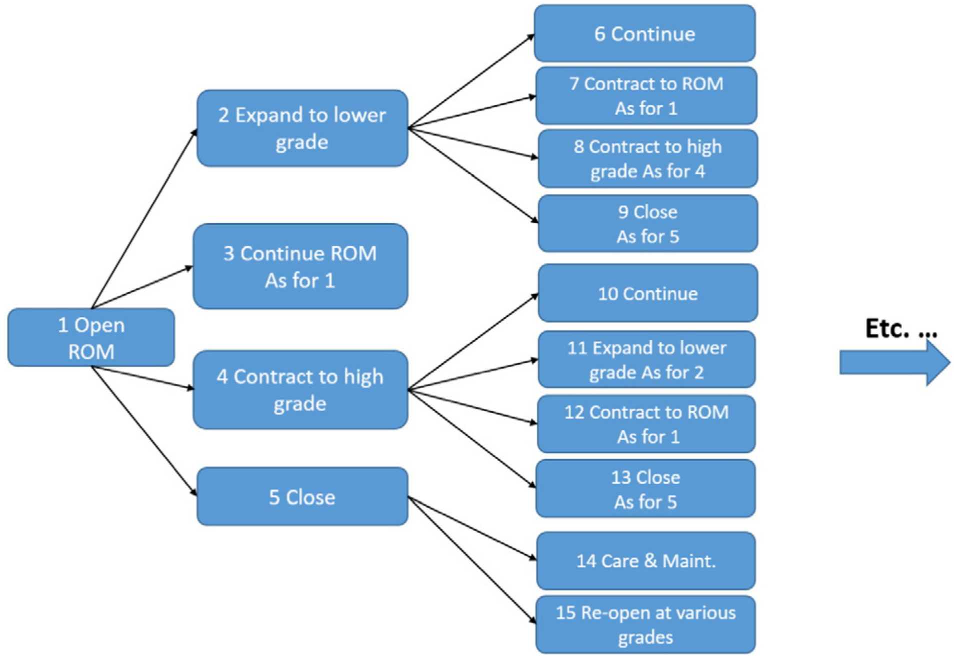

Fig. 2. Real option structure compounding designed and inherent flexibility. Please note that the analysis could have started from either a 'Close', 'Contracted' or 'Expanded' state.

with dynamic programming capacity, as shown by the quantitative example provided in this paper based on a simplified but realistic gold mining project.

2. Real option value (ROV): background

Central to the creation of ROV is the presence of uncertain value drivers for the project, such as volatile commodity prices, and the capacity of management to progressively adjust its course of action in a manner that allows it to take advantage of upswings and avoid, or at least minimize the deleterious effects of downswings. This capacity may be enhanced if management has anticipated potential future scenarios and has built into its mine plans and *modus operandi* the necessary flexibility to more easily adjust to emerging circumstances as uncertainty is progressively resolved, that is to say with the wisdom of hindsight. Such an approach, while entailing some up-front and on-going costs, may result in an overall financial performance for the project that can be largely superior to that of persisting with the original mining plans or of having to react with unanticipated and costly changes to an essentially inflexible project plan.

The value of an option is a function of a number of parameters including the annual volatility of the returns on holding the underlying asset (σ), its spot value (S_0) and exercise price (X), both of which, in the case of derivatives based on financial assets or commodity prices, are regularly quoted in the market, the risk-free rate of interest (Rf) and the time to expiry of the option (T). Option values are also adjusted for any dividend paid to the owners of the underlying assets up to the time of expiry.

Calculating the annual volatility (σ) of the price of a continuously traded asset, such as that of a mineral commodity, is a relatively straight forward process. It implies first finding the standard deviation of the daily logarithmic returns on holding the asset ($s = SD(\ln(S_{t+1}/S_t))$) from published market price series (where S_t is the price of the asset at time t) and then annualizing it by multiplying it by the square root of the number of trading days in the year. The latter generally ranges between 252 and 254.

The exercise price, X, is in effect an up-front capital investment that buys the present value of all future net after-tax operating cash flows that are expected to be generated by the underlying asset, as reflected in its current spot price (S_0), that is to say in its market capitalization. The same logic can be transferred to options on real assets that are infrequently traded. In mining projects, for instance, the present value of the initial investment in the mine, mill and related infrastructure, as well as of subsequent sustaining capital investments during the life of the mine, is a proxy for X as it secures the present value of all future expected net after-tax operating cash flows from mine production, a proxy for S_0. This so called 'Market Asset Disclaimer' or MAD approach is discussed in considerable detail by Copeland and Antikarov (2003), who recommend deriving S_0 and X from the DCF model of the project under consideration after application of an appropriate risk and time adjusted discount rate (RADR).

They also suggest a methodology to estimate the aggregated volatility of the cash flows (σ) for projects where more than one source of uncertainty, besides that of the commodity price, is present. They define the project volatility as the standard deviation (SD) of the logarithmic returns from the project, obtained using a Monte Carlo simulation of the formula:

$$\sigma = SD\left(\ln\left(\Sigma_{\text{from } t=1} PV_{\text{at } t=1} \text{ of NATOCF} / \Sigma_{\text{from } t=0} PV_{\text{at } t=0} \text{ of NATOCF}\right)\right)$$

where NATOCF = the Net After-Tax Operating Cash Flow expected to be generated by the project. Their method implies keeping the denominator of the above fraction constant during the simulation, with only the numerator being affected.

The accuracy of their method, however, has been questioned by Brandão et al. (2012), who claim that it over-estimates the volatility at low probability levels and propose modifications to reduce its bias. In addition, contrary to financial assets that always have a positive or at worst a zero value, Monte Carlo simulation of the DCF model of a marginal mining project is likely to generate, over a large number of iterations, instances where the sum of the present values of all future cash flows at time $t = 1$, that is to say the numerator of the above formula, is negative. As there is no natural logarithm for negative numbers, these negative values must be discarded, introducing further potential inaccuracy. As it will be discussed, this issue does not arise if real options are solved using decision trees as there is no need to use an aggregated σ of the values of the underlying asset as an input. Instead, decision trees can handle each source of uncertainty as an individual event node.

It is generally accepted that the price of continuously traded assets, such as mineral commodities, follows a stochastic process, typically a Geometric Brownian Motion (GBM). Furthermore, the DCF/NPV of a mining project, which is a function of a stochastic process, as for instance the price of the commodity produced, is, according to the Ito's Lemma, itself a stochastic process. This implies that common option valuation methodologies based on the GBM may be applied to value options in which the underlying asset is a real asset such as a mining project.

There are a number of methods to calculate the ROV including primarily:

- Close form equations, mostly derived from the classical Black and Scholes' formula. While ideal for valuing financial derivatives, these formulae have limited application in complex real option models;
- Binomial lattices developed as a function of the volatility of the value of the underlying asset, following the methodology established by Cox et al. (1979) where risk is hedged through the use of 'replicating portfolio' techniques, including the associated 'state prices' and 'risk-neutral probabilities' and discounting at the risk-free rate of interest (Benninga, 2008);
- Decision trees mapping all possible project scenarios and facilitating calculation of ROV using dynamic programming (Borison, 2003; Smith and McCardle, 1999); and
- Monte Carlo simulation using the DCF model of the project modified to include the necessary option maximization rules.

Among these methodologies, binomial lattices and decision trees are in the author's opinion the most suitable to model the complex structure of options often encountered in realistic models of mining projects and will, therefore, be utilized in this paper. The mechanics of these option valuation techniques are discussed in some detail in the context of the following numerical example.

3. Calculating the ROV of the open/close switching option of a marginal gold mining project

3.1. Description and DCF model of the mining project

The example involves the evaluation of an operating gold mine with a residual life of 5 years, which could be acquired in full for $20 million. All mine assets have been fully depreciated for tax purposes. The $20 million purchase price can be fully depreciated for tax purposes over the 5-year life of the mine using the straight line method. The mine produces 50,000 oz of gold per annum at a variable unit cash cost of $900 per ounce with a fixed annual cost of $6 million. Costs are estimated at year 0 values and are expected to escalate in nominal terms at a rate of 3% per annum, of which about 2% is attributable to general inflation. The nominal mean price of gold, by contrast, is expected to escalate on trend at around 1% per annum from $1184 per ounce in year 0, that is to say at ½ the rate of general inflation, thus reducing slightly in real value over the period of the analysis. The starting price of $1184/oz adopted for year 0 is the mean for the year preceding the evaluation date, i.e. 1st October 2015. Mineral royalties are levied at a rate of 2.5% of gross revenue and the rate of corporate income tax is 30%. The nominal cost of capital and the risk-free rate of interest (Rf) are assumed to be 11 and 5% respectively. Possible un-recouped losses carried forward beyond the closure of the mine are disregarded.

As displayed in Table 1, on these assumptions, the NPV of the mine over its residual life of 5 years and including the $20 million acquisition price is negative at −$6.192 million, hence strictly on a DCF/NPV basis the investment should be rejected.

3.2. Introducing open/close switching flexibility

The mine is currently operating, i.e. in the 'open' state. Under current mine design and operating arrangements, temporarily placing the mine under care and maintenance, i.e. moving to a temporarily 'closed' (but not finally abandoned) state, in the event of falling gold prices generating negative cash flows would entail prohibitive and above all unanticipated expenses. As a consequence, given their current limited flexibility the owners would reluctantly continue to produce gold at a loss and hope for improvements in the gold price. There would obviously be limits to the degree to which a mine could realistically continue to incur losses and eventually, assuming continuing deterioration in the commodity price, a point will be reached where the owners of a mine will decide to close irrespective of the consequential costs.

While carrying out due diligence, however, the potential buyers have identified opportunities for a range of modifications to the current mining design and plan, while the takeover would also provide the opportunity to re-negotiate and re-structure a range of procurement,

Table 1
DCF model of gold mine. The grey area represents the extent of the analysis carried out in Sections 3.2 to 3.4.

	Year					
	0	1	2	3	4	5
Au Price $/oz		1195.8	1207.8	1219.9	1232.1	1244.4
Revenue	All figures below $´000	59,792	60,390	60,994	61,604	62,220
Royalty		−1495	−1510	−1525	−1540	−1555
OPEX		−52,530	−54,106	−55,729	−57401	−59,123
Depreciation		−4000	−4000	−4000	−4000	−4000
Income before tax		1767	774	−260	−1337	−2459
Losses carried forward		0	0	−260	−1597	−4056
Taxable income		1767	774	0	0	0
Tax		−530	−232	0	0	0
NCF	−20,000	5237.0	4542.0	3739.9	2662.7	1541.3
PVCF	−20,000	4718	3686	2735	1754	915
NPV	−6192.3		NPV2	=PV of CFs Years 1 + 2 =	8404.4	

ASSUMPTIONS			
Sigma	0.16	Deltat	1
Gold price at Y0	1184	Up factor	1.1600
Rf	0.05	Down factor	0.8621
T	2	Risk-neutral p	0.6308
Steps	2	Discount factor	0.9524
CloseCost	2500		
OpenCost	4000		
Care and maintenance	3500		
Cost escalation	0.03		
Gold price growth	0.01		

YEAR	**0**	**1**	**2**	
Possible states of nature	Start	up	up-up	
		down	up-down, down-up	
			down-down	
		=1184*1.01*1.16		
Possible gold prices	1184.0	1387.2	1625.2	=1387*1.01*1.16
		1030.9	1207.8	=1030.9*1.01*0.8621
		=1184*1.01*0.862	897.6	=1030.9*1.01*0.8621

Fig. 3. Binomial lattice displaying possible gold prices corresponding to a volatility of 16% and a mean growth rate of 1%.

labour and other commercial commitments with the potential to significantly reduce the cost involved in closing and re-opening the mine as well as limiting costs while on care and maintenance, which are now estimated at $2.5 million, $4.0 million and $3.5 million at year 0 values respectively.

In this light, as shown in Fig. 1, the mine manager has the option to continue operating or temporarily close the mine depending on whether the expected net after-tax operating cash flow to be realized in remaining open during year 1 exceed the cost of closure and of care and maintenance during the year (i.e. $2.5 million plus $3.5 million respectively escalated by 1 year) or not.

In the following year the manager would have two different options depending on the decision made in the previous year. If the mine had remained open he/she would once again have a closing option. If on the other hand, he/she had opted to close the mine in the previous year, he/she would now have the option of re-opening it as long as the expected net after-tax operating cash flow less the cost of re-opening the mine (i.e. $4.0 million) exceed the cost of keeping the mine on care and maintenance (i.e. $3.5 million), both escalated by 2 years. The same options would be present in each subsequent year until the mine reserves are eventually completely exhausted.

Although in the model that follows it has been assumed that the mine is in the 'open' state at the start, a similar model could have been built on the assumption that the mine begins in the 'closed' state, that is to say under care and maintenance.

It is interesting to note that a binomial lattice covering the full life of the mine would have 32 nodes, and a corresponding decision tree (assuming price event nodes with three branches) in excess of 7776

YEAR		**0**	**1**	**2**	**3**	
Possible PVCF		Open	11767.2	18785.8	= Max(18785.8,-2652.3-3713.2)	
	Max(-2273.6,-2575-3605) =		-2273.6	4542	= Max(4542.0,-2652.3-3713.2)	
				-6365.5	= Max(-10347.9,-2652.3-3713.2)	
Optimal optional decisions		Open	Stay open	Stay open	Stay open	
			Stay open	Stay open-Re-open	Stay open	Etc.....
				Close	Re-open	
					Stay closed	
		=(18785.8*0.6308 +4542.0*(1-0.6308))*0.9524				
Possible ENPV			12882.7	18785.8		
			490.3	4542.0		
				-6365.5		
	=(24649.9*0.6308 -1783.3*(1-0.6308))*0.9524					
ENPV		**14181.3**	24649.9	=11767.2+12882.7		
			-1783.3			
ROV		**5776.9**	=14181.3-8404.4=ENPV-NPV2			

Fig. 4. Binomial lattice displaying in the top part the optimal annual cash flows corresponding to the possible gold prices of Fig. 2, including exercise of options where justified and rolling back of optimal values to their cumulative present value in year 0, i.e. to the project ENPV.

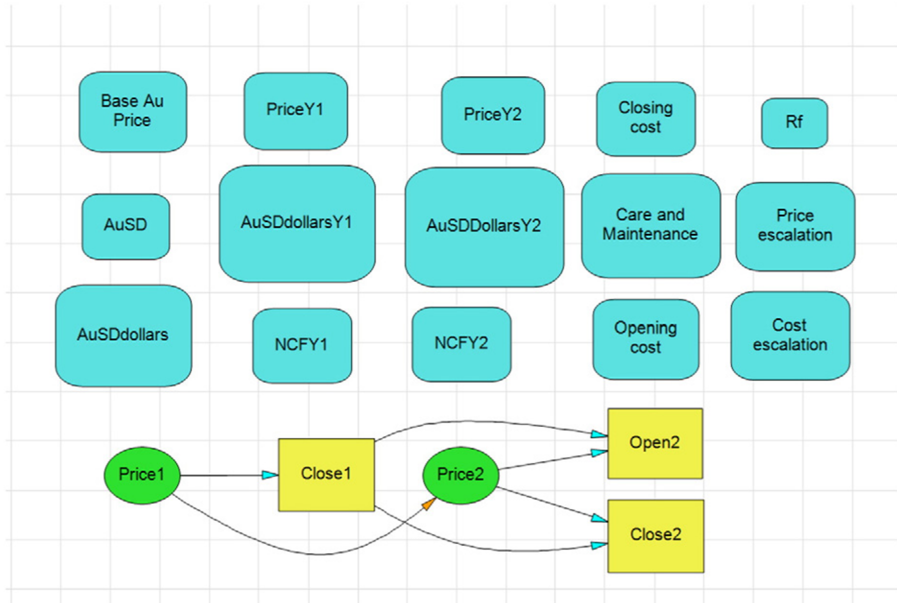

Fig. 5. Influence diagram showing main inputs as rounded rectangles, event nodes as ellipses and decision nodes as rectangles. The presence of an arrow denotes conditioning between nodes.

branches. Hence, to facilitate detailed quantitative discussion, the following ROV analysis of this realistic gold mine will be carried out in three parts:

- First, the analysis will be limited to the cash flows generated during the first 2 years of the mine life (highlighted in grey in Table 1) using the binomial lattice methodology, then
- The same cash flows will be analyzed using a decision tree, which will demonstrate that consistent ROV results can be obtained irrespective of the methodology used, and finally
- The ROV for the project over its whole life will be calculated using a decision tree with dynamic programming capacity, which simplifies the calculations, providing an upper limit to the amount of investment that a mine manager could theoretically devote to creating the postulated switching flexibility.

3.3. Calculating the ENPV and ROV of the open/close switch option using a binomial lattice

As already mentioned, in the DCF model it was assumed that the mean of future gold prices would be equivalent to the mean price in the year preceding the analysis (i.e. \$1184 per ounce) escalated by 1% per annum. As gold prices distribute lognormally, the mean and standard deviation of the prices between 1st October 2014 and 30th September 2015 were calculated using the formulae:

- μ = Mean of the price = $\exp(\alpha + 0.5*\beta\hat{}2)$ = \$1184/oz and
- SD = Standard Deviation of the price = $\mu\hat{}2*(\exp(\beta\hat{}2)-1)$ = \$46.2/oz, equivalent to 3.9% of μ

where α = Mean (ln(price)) and β = SD(ln(price)).

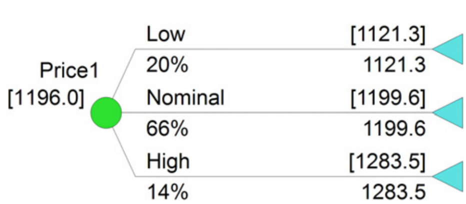

Fig. 6. Year 1 gold price discretized from its continuous lognormal distribution using the moment matching method.

In addition, the annualized volatility of the logarithmic returns on holding gold was calculated using the formula:

- $\sigma = SD(\ln(S_{t+1}/S_t))*SQRT(252) = 0.16$, (where S_t is the price of the asset at time t)

Option valuation using the binomial lattice method involves the use of the following four main parameters, which were calculated in their discrete form, given the discrete nature of the DCF model of the project based on yearly time intervals, using $\sigma = 0.16$, a Rf = 0.05 and a time interval (Δt) of 1 year:

- Up factor = 1 + σ = 1.1600
- Down factor, D = 1/U = 0.8621
- Compounding and discounting factors, at the risk-free rate of interest Rf, i.e. $R = 1 + \mathrm{Rf} = 1.05$ and $1/R = 0.9524$
- Risk-neutral probability, p = (R − D)/(U − D) = 0.6308

Cox et al. (1979, p.232) point out that the risk-free rate of interest (Rf) needs to be lower than the up rate (u) and greater than the down rate (d). In addition analysts must ensure that Rf should be consistent with the risk and time adjusted cost of equity (Re)[2] used as a discount rate (RADR) in generating the values for S_0 and X following Copeland and Antikarov's MAD methodology.

On the basis of the above parameters the binomial lattice of possible gold prices shown in Fig. 3 and subsequent option valuation of Fig. 4 were generated.

Applying each of the possible prices in turn to the DCF model of Table 1 the optimal annual cash flows for each possible state of nature, whether price up or down, were computed for each year as displayed in the top of Fig. 4. The optimal figures were obtained applying the appropriate maximization rule. For example for year 1 the rule was CF (in \$′000) = MAX(between the possible cash flow from the project if in operation or the cost of closure and care and maintenance escalated by 1 year). Under both the up and down state of nature, it was optimal to stay open, as even in the down state, that produces a negative CF of

[2] According to the Capital Asset Pricing Model (CAPM), i.e. Re = Rf + β ∗ (Rm − Rf), where Rm is the return on the market portfolio.

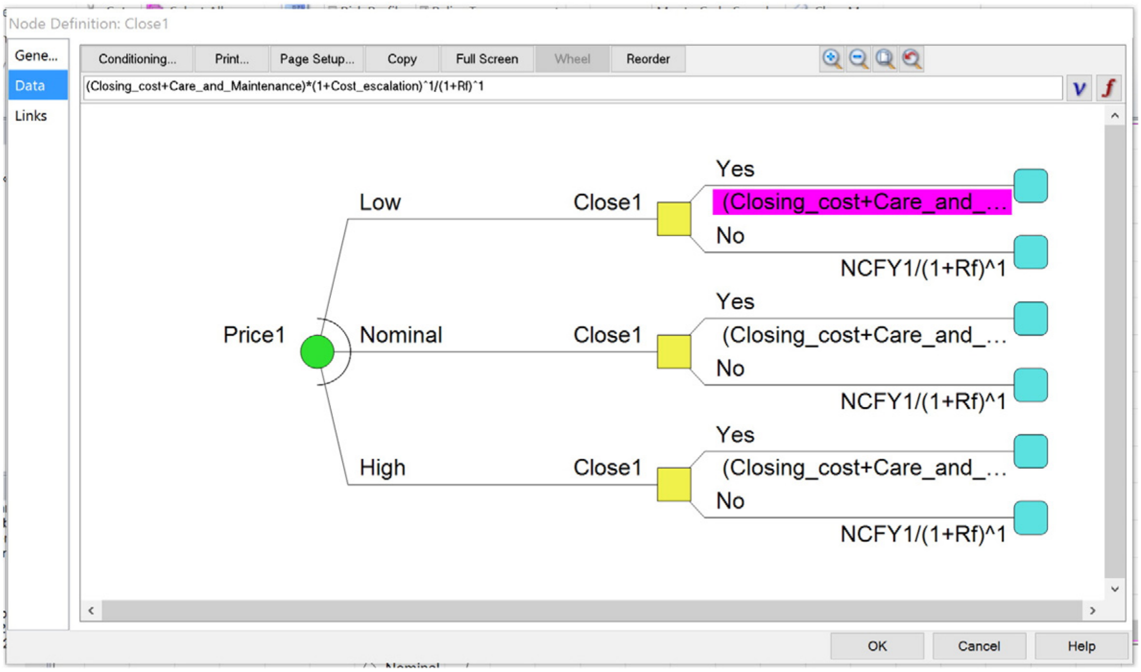

Fig. 7. Definition of the Close1 decision node showing the potential optional payoffs relating to each branch. Please note that the analysis could have started from a 'Close' state in which case the first decision would have been 'Open 1'.

around −$2.274 million, this is less than the cost of closure of (−$2.5 million–$3.5 million) ∗ (1 + 0.03)^1 = −$6.18 million. Hence the closing option is not exercised.

By contrast, if a down-down state occurs in year 2, the potential cash flow would be -$9.226 million, which is well in excess of the closing cost escalated by 2 years, i.e.(−$2.5 million–$3.5 million) ∗ 1.03^2 = −$6365.4 million. As a consequence the closing option is exercised and the mine is placed into care and maintenance.

Having generated the binomial lattice of possible optimal cash flow values for the various states of nature, they are then rolled back to their present value in Year 0, that is to say to the ENPV of the project including its ROV. This process entails two steps. First, the certainty equivalent for each possible cash flow value must be obtained by neutralizing its risk, in the example using the appropriate discrete risk-neutral probability (Munn, 2002) (p = ((1 + Rf) − D)/(U − D) = (1.05–0.8621)/(1.16–0.8621) = 0.6308) for up state values and (1 − p) for the down state values and then discounting their sum by the discrete discount factor 1/(1 + Rf) = 1/1.05 = 0.9524, as shown in the second part of Fig. 4. The rolled back possible cash flows from year 2 are cumulated with those of year 1 in the third part of Fig. 4 and then rolled back to their present value in year 0. The year 0 value of $14.181 million represents the ENPV of the cash flows of the first 2 years of the project, including the value of the switching option. By subtracting from this the cumulative present value of the cash flows obtained in the deterministic DCF model for years 1 and 2 (NPV2 in Table 1), that is to say $8.404 million, the corresponding first 2 year ROV of $5.777 million is obtained as illustrated in the fourth part of Fig. 4.

3.4. Confirming the binomial lattice ROV using a decision tree

In this section the same real option problem of Section 3.3 will be solved using an off-the-shelf decision tree program, in this case Dynamic Programming Language 8 (DPL8), marketed by Syncopation. The uncertainty surrounding the price of gold in each year is brought to play in the form of two event nodes, Price1 and Price2, shown as green ellipses in the influence diagram of Fig. 5. The price event node contain the parameters of their lognormal distribution, for instance in year one μ = $1184*1.01 = $1196.0 per ounce and s = $1196.0*0.039 = $55.8 per ounce. However, to construct the tree price distributions need to be discretized into tree-branch event nodes using the moment matching technique (Smith, 1993), as illustrated for year 1 in Fig. 6.

It needs to be pointed out that, the standard deviation of the gold price and the volatility of the cash flows of the gold project, while functionally related, are vastly different measures and that, as a consequence, a down state in the binomial lattice does not correspond to the outcome of a low gold price in the decision tree. For instance the former may lead to temporary closure of the mine, while the latter may not.

The arrow connecting Price1 to Price2 in Fig. 5 denotes that the latter is conditioned by the former. Similarly, the various decision nodes, symbolized as rectangles, i.e. Close1, Open2 and Close2, are conditioned by the relevant prices and Open2 and Close2 by Close1.

The decision tree is then derived from the influence diagram of Fig. 5 by attributing to each branch of each decision node algorithms consistent with maximization of the relevant option payoffs, as shown in Fig. 7.

The corresponding values needed to populate the decision tree of Fig. 8 are obtained by linking its various nodes to the Excel DCF model of the mine provided earlier in Table 1, which imports inputs from the tree and exports calculated model outputs for the various scenarios of the tree. The values of the 32 branches of the tree are then rolled back to the origin resulting in an ENPV of $13.795 million. This represents the present value of the net after-tax operating cash flows for the first 2 years including the value of the relevant options. When the static DCF present value for the first 2 years, amounting to $8.404 million, is deducted from the above ENPV an ROV of $5.390 million is obtained. This represents a difference of only $0.387 million or 6.7% from the ROV of $5.777 million obtained by valuing the same option using a discrete binomial lattice with the risk-neutral probability, as discussed in Section 3.3. This margin of error is remarkably low considering that the two ROVs were obtained by completely different techniques each involving their own approximations. The difference is also well within the normal margin of error typical of the DCF model of the mine on which the ROV calculations are based.

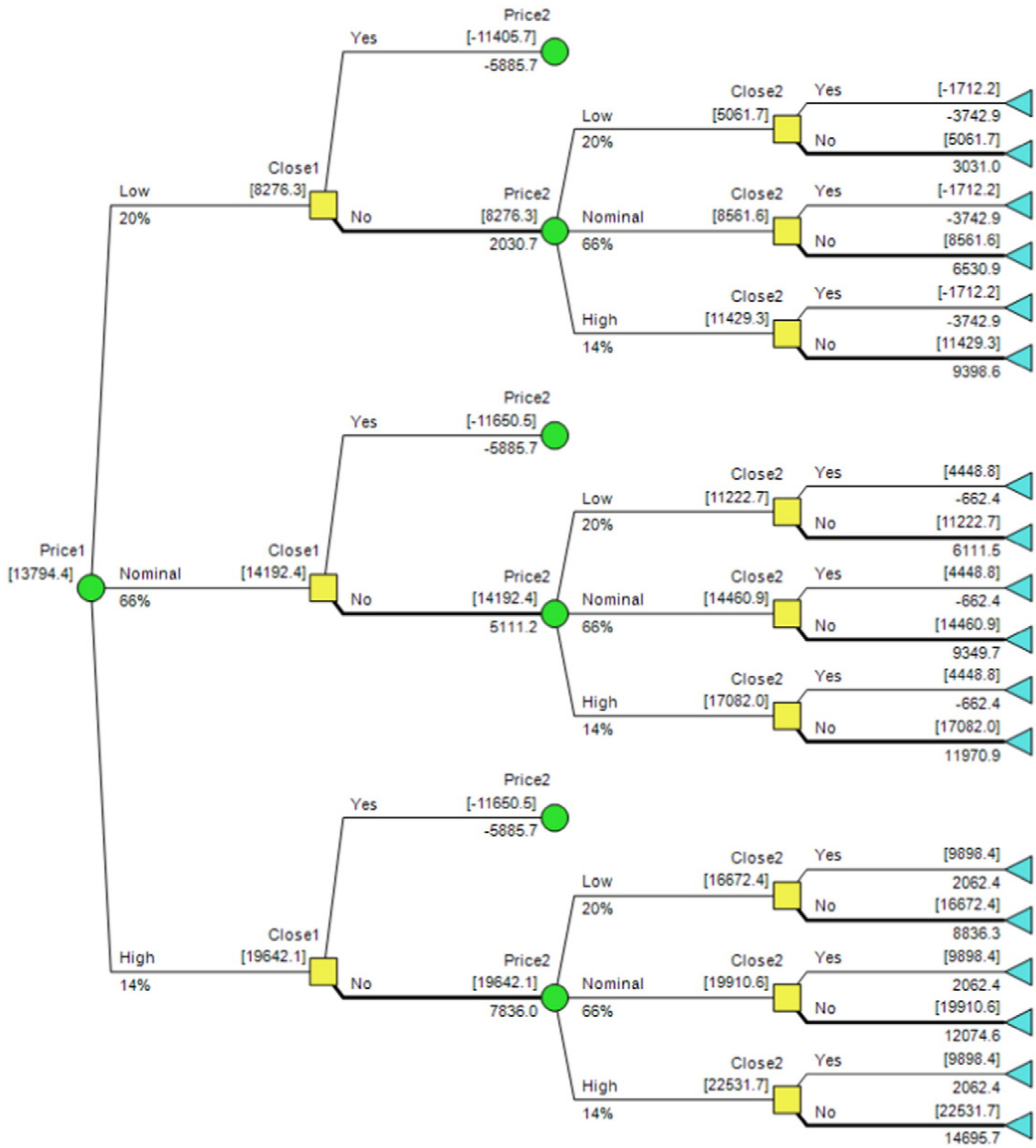

Fig. 8. Fully computed decision tree displaying the ENPV = $13.795 million for the first 2 years at the origin. Note that to fit the page all negative scenario nodes are not fully displayed.

The above results are consistent with those of previous, as-yet unpublished, modeling of a copper mining project, conducted by the author in co-operation with Messiers Atul Chandra and Ricardo Garzon,[3] using three separate methodologies, i.e. a binomial lattice using the risk-neutral probability, a hybrid decision tree combining a risk-neutral probability for the market risk with other uncertainties handled as individual event nodes and a decision tree where all the sources of uncertainty were disaggregated. These three methods displayed close but progressively lower ROVs, which is explained by the decreasing influence of the risk-neutral probability calculated using the volatility of the cash flows of the underlying project using the Copeland and Antikarov's method, which as already discussed may introduce a positive bias. The current results are considered important in that they indicate that use of a decision tree may be a valid alternative to the binomial lattice in ROV calculations. Besides not requiring node by node calculations, decision trees have the additional distinct advantage of accommodating multiple uncertainties in the form of individual event nodes, thus not requiring the calculation of the volatility of the cash flows of a project, an area that, as already discussed, involves a degree of ambiguity, potential bias and controversy.

3.5. Extending the model to the full 5-year mine life

As already discussed, while possible, it would be impractical to calculate the ROV over the full 5-year life of the mine using a binomial lattice, as this would entail calculating values for 32 individual lattice nodes. Extending the decision tree to the full mine life, however, may prove easier, even though the tree will grow into 7776 individual branches, because skillful use of the dynamic programming language embodied in the software enables analysts to carry out repetitive calculations through single or limited lines of programming.

Because of space limitations, only the top half of the summary tree, emanating from a possible decision to close the mine in year 1, is displayed in Fig. 9. The pay-offs for the two branches of each decision node, including any real option value, are conditioned by the relevant

[3] PhD candidates carrying out research on the application of real option valuation to the resources industry under the author's supervision at the University of Western Australia and Curtin University respectively.

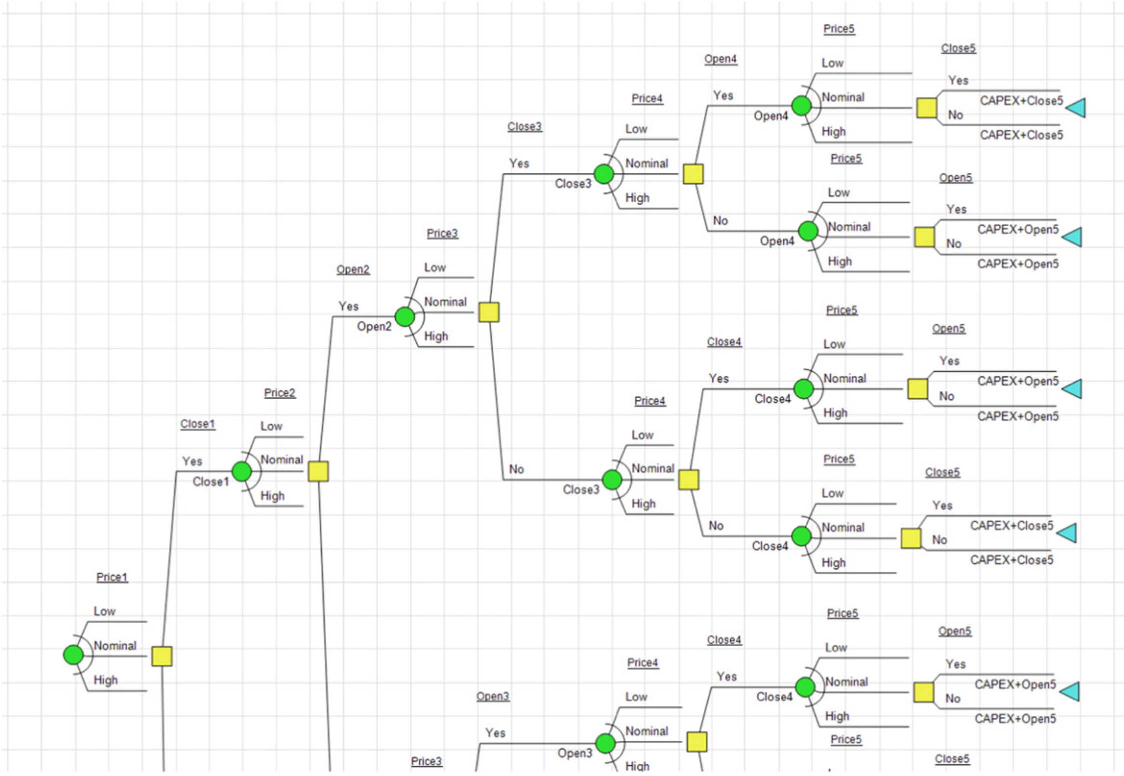

Fig. 9. Summary diagram displaying the structure of upper half of the decision tree for the whole 5-year life of the mine.

price outcome and cumulated over the years into their total in year 5, represented by the sum of the values of Close5 and Open5. Solving the tree, and subtracting from Close5 + Open5 the initial $20 million capital investment (CAPEX), the project ENPV including the value of all the options, amounting to $19.083 million, is obtained. The ROV of the project, amounting to $25.276 million, is in turn derived by subtracting from the above ENPV the static DCF/NPV of the project (i.e. −$6.192 million).

3.6. Improving the realism of the ROV model

The ROV obtained in Section 3.5 above is calculated against the alternative of keeping the mine open no matter what the emerging circumstances are. In reality, totally irreversible investments are rare and inflexibly designed mines will still close under extreme duress and then re-open when circumstances improve, albeit at very high cost.

The ROV analysis can, therefore, be improved if this is recognized and the cost of unanticipated closure, care and maintenance and re-opening are estimated. These estimates can then be used in the decision tree model developed in 3.5 above to calculate the ENPV of the project with low unplanned flexibility relative to a totally inflexible situation. The difference between the ENPV of the mine model including a high level of planned flexibility to facilitate and minimize the cost of temporary closure and reopening of the mine and that of the unplanned limited flexibility one provides a measure of the maximum level of investment that would be justifiable to create the additional high level of flexibility. As an example Table 2 provides a comparison based on the assumption that enough flexibility can be created to half the potential unplanned costs of closing, placing the mine on care and maintenance and re-opening operations.

Table 2
ENPV of the 'Low unplanned flexibility' scenario relative to that of the 'High planned flexibility' one designed to half the cost of closure, re-opening and care and maintenance.

Project characteristics scenario	Cost of flexibility as multiple of base case	Enpv relative to totally inflexible scenario $ m
High planned flexibility	Closing = $2.5 M Re-opening = $4.0 M Care and maintenance = $3.5 M	25.276
Low unplanned flexibility	2 × High flexibility Closing = $5.0 M Re-opening = $8.0 M Care and maintenance = $7.0 M	18.958
Difference representing justifiable investment in project flexibility		6.318

If for instance, one were to assume that the unanticipated cost of closing, re-opening and care and maintenance were double those that could be achieved by building flexibility into the mine plans and operations, that is to say $5.000 million, $8.000 million and $7.000 million respectively (Low unplanned flexibility case in Table 2), then the ENPV of the project with low flexibility relative to a totally inflexible project would be $18.958 million. The difference between the ENPV of the 'High planned flexibility' project (i.e. $25.276 million) and that of the 'Low unplanned flexibility' project (i.e. $18.958), amounting to $6.318 million, represents the ROV of introducing a higher level of close/open flexibility. This represents the significant maximum amount that would be justified being invested in the creation of additional close/open flexibility.

Carrying out this type of sensitivity analysis is very easy once the DCF and decision tree models are established and linked as it only involves changing the values of the relevant inputs in the list of assumptions.

4. Conclusions

The ROV analysis illustrated in this paper supports a number of important conclusions:

1. As under current and foreseeable commodity price conditions many mining projects will be financially marginal, there is a

need for a paradigm shift in investment criteria from economies of scale, which may involve potentially significant capital investments and relatively inflexible modes of operation, to building operational flexibility to make it easier to temporarily close and re-open marginal mines in response to price volatility.

2. This type of switching option can have high ROV because the cost of closing and putting the mine on care and maintenance and then re-opening it, if unanticipated, is invariably very high, justifying upfront investment in creating the necessary flexibility at the development stage.
3. The ROV can be calculated using either a binomial lattice, hedging risk using risk-neutral probabilities, or by building a decision tree where price uncertainty is introduced as an event node, with decision nodes containing algorithms for the maximization of optional payoffs.
4. Use of decision trees with dynamic programming capacity is considered a better approach because, even though they may develop a large number of branches over the life of a project, they can be solved with a relatively limited number of algorithms that enable a large number of repetitive calculations.
5. Decision trees also have the advantage of incorporating various sources of uncertainty as individual event nodes, thus avoiding the need to estimate the volatility of the cash flows of a project, with its related potential complexity, inaccuracy and positive bias.
   a. It is critical that mine planners assess the potential cost of having to close and re-open their operations for various technically feasible mine designs and modes of operations, and determine avenues for and the potential ROV benefits of investing, at the acquisition and/or development stages, in flexibility measures to facilitate these processes and reduce their cost.

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