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A GIS tool to increase the visual quality of relief shading by automatically changing the light direction

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ABSTRACT

Shaded relief representations were traditionally produced manually by specifically trained cartographers. This was however a labour-intensive and time-consuming task, which gave rise to numerous attempts of automation.

Nowadays, many GIS applications implement hillshading using an oblique light source. This has standardised the method, providing a simple way to obtain reliable and consistent results. Its visual quality is however well below the standards of manual shading, where multiple light sources are employed to achieve a superior visual quality.

In this work we present a GIS tool to enhance the visual quality of hillshading. We developed a technique to illuminate the landscape from two different angles and correct the tone according to either elevation or slope. With this method the quality of the shaded relief is superior to the standard method, but its level of automation and standardisation guarantees consistent and reproducible results. This method has been embedded into an ArcGIS toolbox.

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1. Introduction

Relief shading has become the *de facto* standard for visualising high resolution terrain data, such as LIDAR. It allows practitioners to find artefacts in Digital Elevation Models (DEMs) and present their data using a form of terrain visualisation that is intuitively understood by the general public.

In the past, relief shading was done manually by specifically trained cartographers, who were able to produce detailed landscape depictions where all the most important terrain features were properly highlighted. The Swiss school of cartography, with the work of Imhof (1982), elevated this discipline to an art form. The Swiss style of shading is much more than a method to depict landscapes illuminated from an angle, it requires the cartographer to have a deep understanding of the landscape and this is translated into fine artistic maps. Manual shading implies an analysis of the landscape with the purpose of highlighting all the most important morphological features while reducing the visual impact of uninteresting details. This result is achieved by constantly changing the light azimuth and zenith in order to represent all slopes from the angle that best highlight ridges and drainage network, while decreasing the visual impact of low relief areas. It requires a subjective interpretation by the cartographer, which is

the reason why two shaded relief representations of the same area, made by two cartographers, look different (Leonowicz et al., 2010). Moreover, this was a labour-intensive and time-consuming task, which was the reason that gave rise to several attempts to automate the whole shading process in order to maintain a similar visual quality, while decreasing the processing time.

Since the 1950s several attempts has been made to automate the whole framework used in manual cartography. Researchers have used different methods to reproduce on a computer the results achieved manually by earlier cartographers. Yoeli (1959, 1965, 1966, 1967) was the first to produce computer generated shading, but it was obstructed by the technology available at that time (Horn, 1981). Nowadays, many GIS applications implement hillshading algorithms (referred to as analytical hillshading), based on the cosine law first developed by Lambert (1760) and adapted to shaded relief by Wiechel (1878). This technique provides a fully automatic, quick and easy way to produce shaded relief representations starting from DEMs. However, the visual quality of these maps is much below the standards set by Imhof (1982) and the Swiss school of Cartography; manual shaded maps are in fact still regarded as superior particularly for representing mountainous landscapes (Jenny, 2001). In contrast, because the process follows a standard and reproducible technique it allows scientists to produce consistent shaded relief representations, in a fast and easy fashion.

This is the reason why cartographers worked for substituting the cosine law as the only way of producing shaded relief images,

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with more advanced techniques able to produce results closer to manual shading. Brassel (1974) dedicated his entire Ph.D. work to recreate on a computer the results obtained by Imhof. He implemented azimuth changes, and correction for ridges and channels. His research however required an extensive work load, not just because of the technology available at the time but because in his framework the cartographer had to manually digitise all the main ridges and channels before shading. Since then terrain generalisation, the discipline that aims at removing unnecessary and distracting details from DEMs to accentuate important landforms, has produced several works where cartographers first extract important landscape features or simplify the terrain, and then proceed with the visualisation phase (Jenny, 2001; Leonowicz et al., 2010; Jenny et al., 2011; Leonowicz et al., 2012; Guilbert et al., 2014). Other techniques involved the overlay of several images, either obtained by illuminating the scene from different angles or by using DEM derivatives, to obtain shaded reliefs where ridges and channel are enhanced (Patterson, 2001a; Patterson, 2001b; Patterson and Hermann, 2004; Loisios et al., 2007; Kennelly, 2008; Leonowicz et al., 2010; Patterson, 2013). Jenny (2001) used an interactive process to locally change the light source, coupled with several other techniques to simulate all the aspects of manual shading.

These efforts elevated the visual quality of analytical hillshading to levels almost matching manual shading, but failed in creating standardised and fully automatic techniques. In fact, the only tool to automatically change the light direction in shaded relief was developed very recently by Veronesi and Hurni (2014). As a consequence, the large majority of environmental scientists are still widely using the standard analytical hillshading algorithm available in most GIS applications, even though it does not offer the same level of visual quality that can be achieved using more advanced methods.

In this work we propose a new method to produce hillshaded maps with a visual quality similar to manual shading, while maintaining the same level of automation and consistency typical of analytical hillshading, and a level of customisation comparable with advanced methods. We developed a relatively simple technique based on a sine equation to seamlessly change the image lighting between two light sources. Moreover, we developed two zenith corrections to decrease the level of details of low-relief areas, while enhancing other important features. This technique is effective in producing high-quality shaded relief and was embedded into an ArcGIS toolbox (ESRI, 2011a), so that it can be distributed to a wide audience.

2. Materials and methods

2.1. Study area

We tried this method over an area in Southern Switzerland, in Ticino. The area has an extent of approximately 440 km^2, and stretches around the city of Biasca (Fig. 1). This particular area has been selected for the presence of Pizzo Erra, which is the mountain right above Biasca. This peak is of particular interest because its ridge has a NW–SE orientation. This makes it parallel to the light direction generally chosen for analytical hillshading, 315°N. With this light source Pizzo Erra is rendered with the same grey tone on both sides, reducing its visual impact, and giving the impression to the reader that its elevation is lower than the rest of the peaks. In reality Pizzo Erra is 2400 m, which is in line with the elevation of other peaks in the area, in particular the ones towards the south (Fig. 1).

Another feature of interest is the large valley that surrounds the city of Biasca. Despite the typical alpine setting, this area is highly
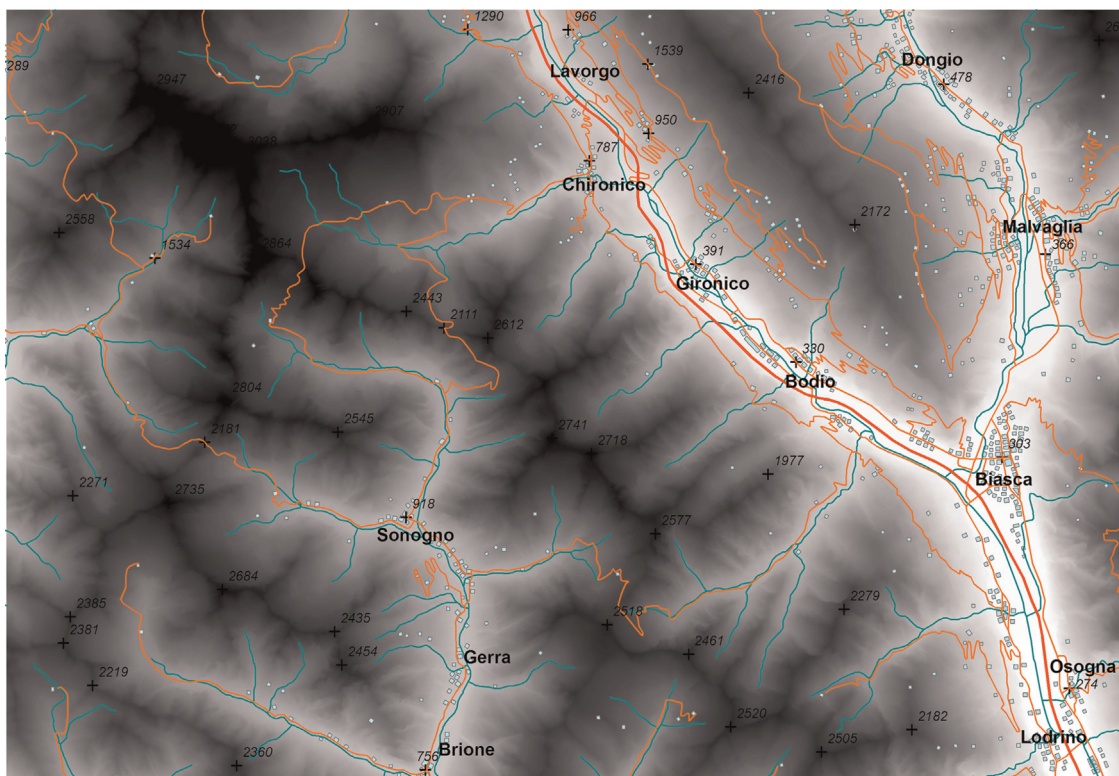

Fig. 1. General map of the study area. Here is depicted the DTM with a colour profile that stretches from white for low altitudes (the main valley in the *E* has altitude variable from 250 to 400 m), to black for mountain peaks, for which we provide the absolute altitude. The main drainage network, roads and cities are also depicted and taken from the Vector 200 topographic vector map from Swisstopo (2014). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

populated and the human intervention is still visible in the DEM, even after a phase of noise reduction made by technicians at Swisstopo (Swisstopo, 2004). This is an important characteristic of high resolution DEM, because for some application the noise in the valley presents a form of unnecessary visual distraction.

2.2. Analytical hillshading

The hillshading algorithm implemented in ArcGIS, and other applications, is a modification of the cosine method, adapted to relief shading by Wiechel (1878). With this method the grey tone of each cell of the DEM is computed as follows:

$$Shading = 255 \times \cos \theta \tag{1}$$

where θ is the angle between the light vector and the vector normal to the DEM cell.

When Wiechel proposed the method the only way to use it was calculating θ manually. The extreme difficulty of doing so made the method disappear from shading researches until almost 100 years later when Yoeli implemented it on a computer. However, in the 1950s the process was still time-consuming and remained so until the creation of DEM derivatives (Evans, 1979), which eased some of the processes involved.

Nowadays, the most common implementation of the cosine law is as follows:

$$Shading = 255 \times (\cos (Z) \cos (Sl))$$
$$+ (\sin (Z) \sin (Sl) \cos (Az - As)) \tag{2}$$

where Sl is slope, As is aspect, Az is Azimuth, which is the direction of the light vector, and Z is Zenith, which is the inclination angle above the horizon of the light vector. All these values are here expressed in radians. This equation is used in ArcGIS (ESRI, 2011a) for producing analytical hillshading maps (ESRI, 2011b).

2.3. Sine wave

In this research we propose a new method for including some of the aspects that make manual shading qualitative superior to analytical hillshading, while maintaining the same amount of automation typical of the latter.

For doing so we developed a method based on the following sine wave equation to change the light source direction continuously across the landscape:

$$Az = A \sin (\omega x + \Phi) + \Delta \tag{3}$$

where Az is the azimuth value; A is the amplitude of the wave; x is the aspect value in degrees; ω is the frequency of the wave, equal to $(2\pi)/\lambda$, where λ is the period of the wave. The parameter Φ is the horizontal offset, and it is used to shift the wave according to the direction of illumination. In this case Φ takes the value of π because we need to shift the sine wave of a period of $\lambda/2$ in order to illuminate cells with aspect between 0° and 45° with a light source coming from 280°N. Finally Δ is the vertical offset of the wave A graphical explanation of the significance of all these parameters is presented in Fig. 2.

We can clarify the meaning of each parameter by breaking up the equation. If we plot the function $\sin (x)$ we will end up with a sine wave oscillating between $+1$ and -1, and starting at 0. Of course, we cannot use this equation for our case, so we need to add parameters to adapt it to our needs. We want an oscillating function, which starts on the negative side and oscillates between 280 and 315. The amplitude (A) of the function is defined as half the distance between the maximum and minimum value, so in this case it is equal to 17.5 (315–280). The period (λ) of the function, is defined as the distance required for it to complete a full cycle; in this case it is always 180° if we keep two light sources. The frequency (ω) can therefore be easily calculated. At this point, If we do not add the two offset parameters, the function will start from 0, and oscillates between $+17.5$ and -17.5. So first we need to add a vertical offset (Δ), equal to 315–17.5, so that the function can oscillate between 315 and 280. Finally, we need to add a horizontal offset (Φ), because we need the function to reach 280 before it reaches 315. We can do this by adding a horizontal offset equal to π.

This method can be simply implemented by replacing the Az from Eq. (3) in Eq. (2).

2.4. Zenith corrections

Landforms that are generally considered important, and therefore highlighted in manual shading, are ridges and channels. Moreover, valleys are generally considered less important and represented using a flat tone. As mentioned in the introduction, one way to approach this is precede shading with some form of terrain generalisation or landform extraction, in order to interpret
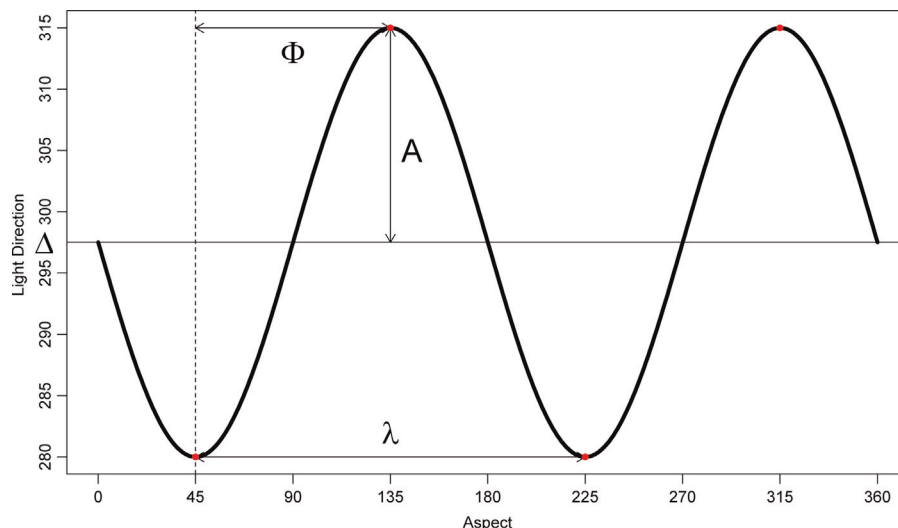

Fig. 2. Sine wave. In this image the meaning of all the parameters of the equation is graphically explained.
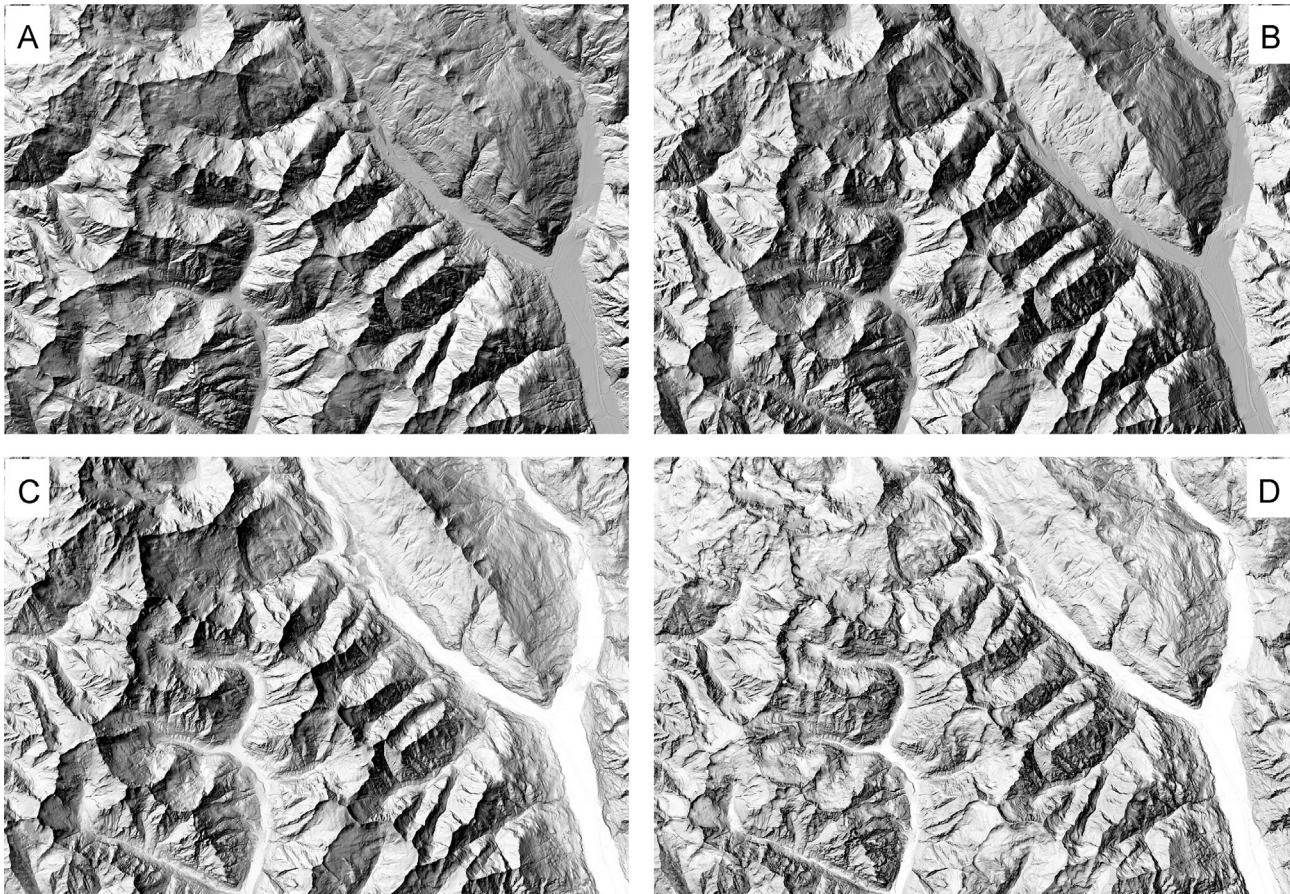
Fig. 3. Graphical comparison between (A) the standard hillshading method from ArcGIS (ESRI, 2011b), (B) hillshading with two light source but a zenith angle fixed at 45°, (C) hillshading with two light sources and zenith adjusted according to elevation, and (D) hillshading with two light sources and zenith adjusted according to slope.

the landscape and then focused the shading on areas considered important.

This may result in better shaded representations, but certainly it is not a standardised method. Numerous techniques are available in literature to extract linear features and other landforms from DEMs (few examples are: Pike, 1988; Dikau, 1989; Wood, 1996; Burrough et al., 2000; Weiss, 2001; van Asselen and Seijmonsbergen, 2006; Iwahashi and Pike, 2007; Jasiewicz and Stepinski, 2013), each with its pros and cons, and a perfect method does not seem to exist. Moreover, by introducing more intermediate steps into the shading process, it quickly becomes complex and therefore less adapted to practitioners outside cartographic disciplines.

There are ways however to increase the quality of relief representations while avoiding the need for additional procedures. In manual shading, valleys are represented with a flat tone, a similar effect can be achieved using a light source perpendicular to the ground. On the other hand, if the whole image is illuminated perpendicularly, the tone will be set according to the principle "the steeper, the darker", the scene would look flat and the only feature that is going to be highlighted is slope. For this reason we created two corrections for zenith, the first according to elevation and the second according to slope, so that the user may decide which features to be highlighted through shading. With these corrections the tridimensionality effect is still present, but practitioners can better customise the final output.

In both cases we started by transforming elevation and slope into weights, using a formula already used by Jenny (2001)

$$E_w = \frac{h_c - h_{min}}{h_{max} - h_{min}} \tag{4}$$

where E_w is the elevation weight, h_c is the elevation of each cell on the raster, while h_{min} and h_{max} are respectively the maximum elevation and the minimum elevation of the DEM. The same formula has been used for creating a slope weight (S_w), replacing the elevation values with slope values.

Subsequently, these weights are used to correct zenith values continuously across the landscape using the following two equations:

$$Z_E = (90 - Z_{MIN})E_w \tag{5}$$

where Z_E is the zenith value corrected for elevation, Z_{MIN} is the minimum value of zenith and is selected by the user. The weights always assume values equal to 1 where elevation is equal to h_{max}, and 0 where is equal to h_{min}. Thus, by multiplying the minimum value of zenith for the weight it is possible to obtain values close to Z_{MIN} on mountain tops and values of 90° above valleys. This translates into a bright white tone for valleys that hide the noise typical of high resolution DEMs, and a progressively darker tone at high altitudes, which enhance the sharpness of the ridges. The shaded relief thus created imitate the combined shading proposed by Imhof (1982).
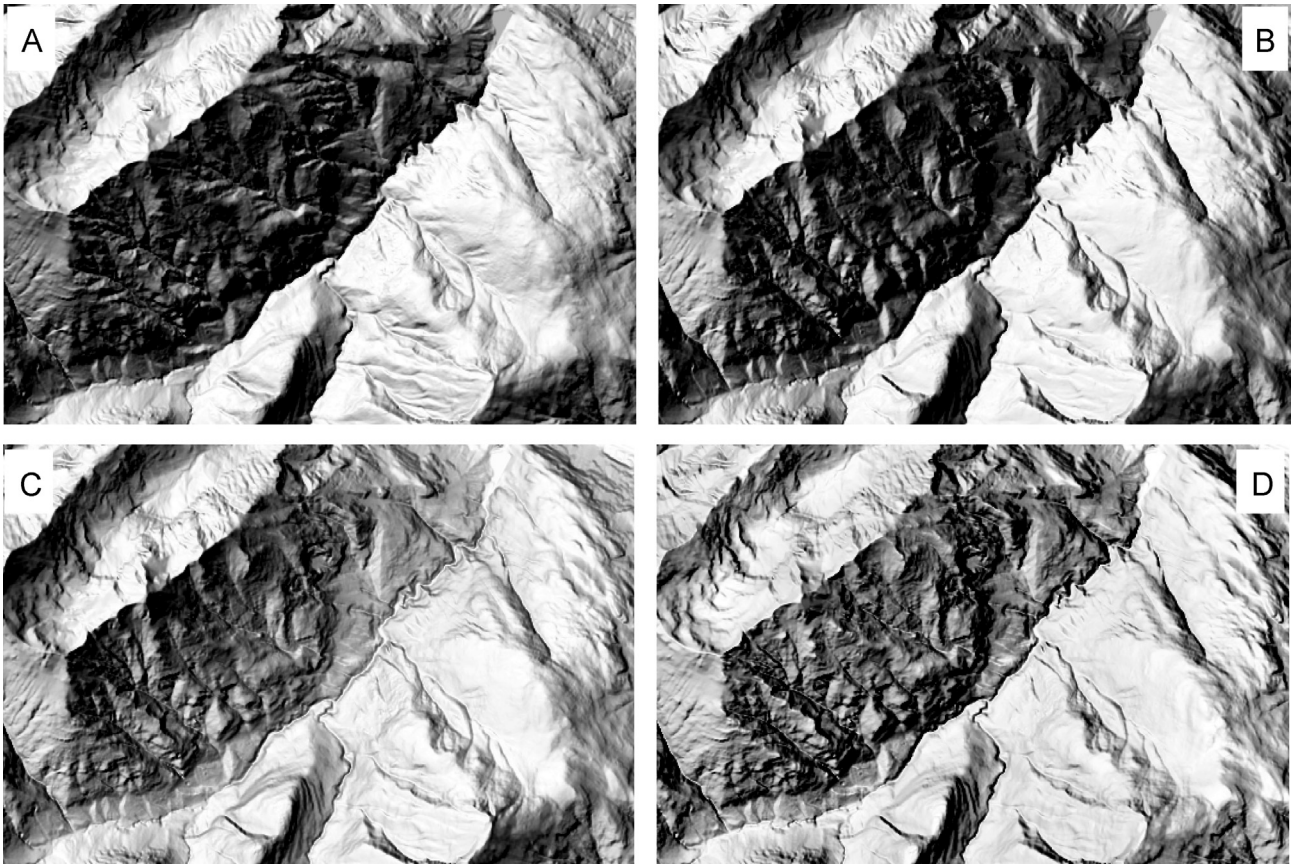
Fig. 4. Detailed comparison of the four maps in the way they represent the drainage network. Images A and B, which are not corrected for zenith, tend to hide the main river on the bottom of the valley. In contrast, correcting the zenith value for elevation or slope, even though the tridimensionality effect can be retained, it is possible to better highlight rivers. Regarding the secondary drainage on the mountain slopes, only the standard hillshaded relief (A) fails to highlight it, while the other three are capable of enhance it, even though in different ways. For example, B and C are capable of highlighting the channels perpendicular to the valley, while correcting for slope (D) can enhance also details of channels with different directions.

A similar process is employed for slope correction, using the following formula:

$$Z_S = (90 - Z_{MIN})S_W \qquad (6)$$

In this case S_W changes according to slope; it takes value 1 on vertical slopes, and value 0 on flat surfaces. The resulting shading will have a bright white tone on valleys and channels, and a dark tone for high slopes, such as rock outcrops.

2.5. ArcGIS toolboxes

The equations presented in Sections 2.3 and 2.4 were used to create a toolbox in ArcGIS for producing relief shaded representations with two light sources and three possible zenith corrections. As inputs these tools can take all the raster datasets supported by ArcGIS (although probably not raster images), in this research all raster files were in ASCII grids format.

It takes several steps to create all the data needed to solve Eq. (2). First slope and aspect are calculated, from the input DEM, with the standard tools in ArcGIS, then they are transformed from degrees into radians with "Raster Calculator", by multiplying their value for $\pi/180$.

At this point Eq. (3) is solved for each cell of the input DEM, creating a new raster with azimuth values. Zenith depends by the tool chosen by the user. There are three options: default, where the zenith values can be chosen by the user but remains constant across the entire DEM, similarly to the standard method presented in Section 2.1; elevation, where zenith is calculated solving Eq. (5); slope, where zenith is calculated solving Eq. (6). With elevation

and slope options, the user can choose the minimum zenith value, Z_{MIN}.

Finally, using once again "Raster Calculator" Eq. (2) can be solved and the shaded relief produced. Overall the time needed to complete the operation for the DEM used in this test, which has a 10 m cell size with a total of 4,725,000 cells, is less than one minute.

3. Results and discussion

The hillshading algorithm, presented in Section 2.1, has been standardised over the years and now it features in most GIS applications. It is fast, reliable, and produces consistent results, in contrast with manual shading where different interpretations produce different outputs. However, the analytical method is customisable only in the selection of the azimuth and zenith angles, it does not offer local light changes, or corrections for elevation or slope. For this reason, there is a need for an automatic method that can increase the visual quality of analytical hillshaded representations, while maintaining a level of automation that can guarantee consistent and reliable results. In this research we developed an ArcGIS toolbox to replace the standard hillshading algorithm, with one where we implemented two light sources and zenith corrections in order for the relief representation to be more informative.

The selection of the two light sources is left to the experience of the user. In general we want to illuminate each surface from an angle roughly perpendicular to the topographic surface. However,
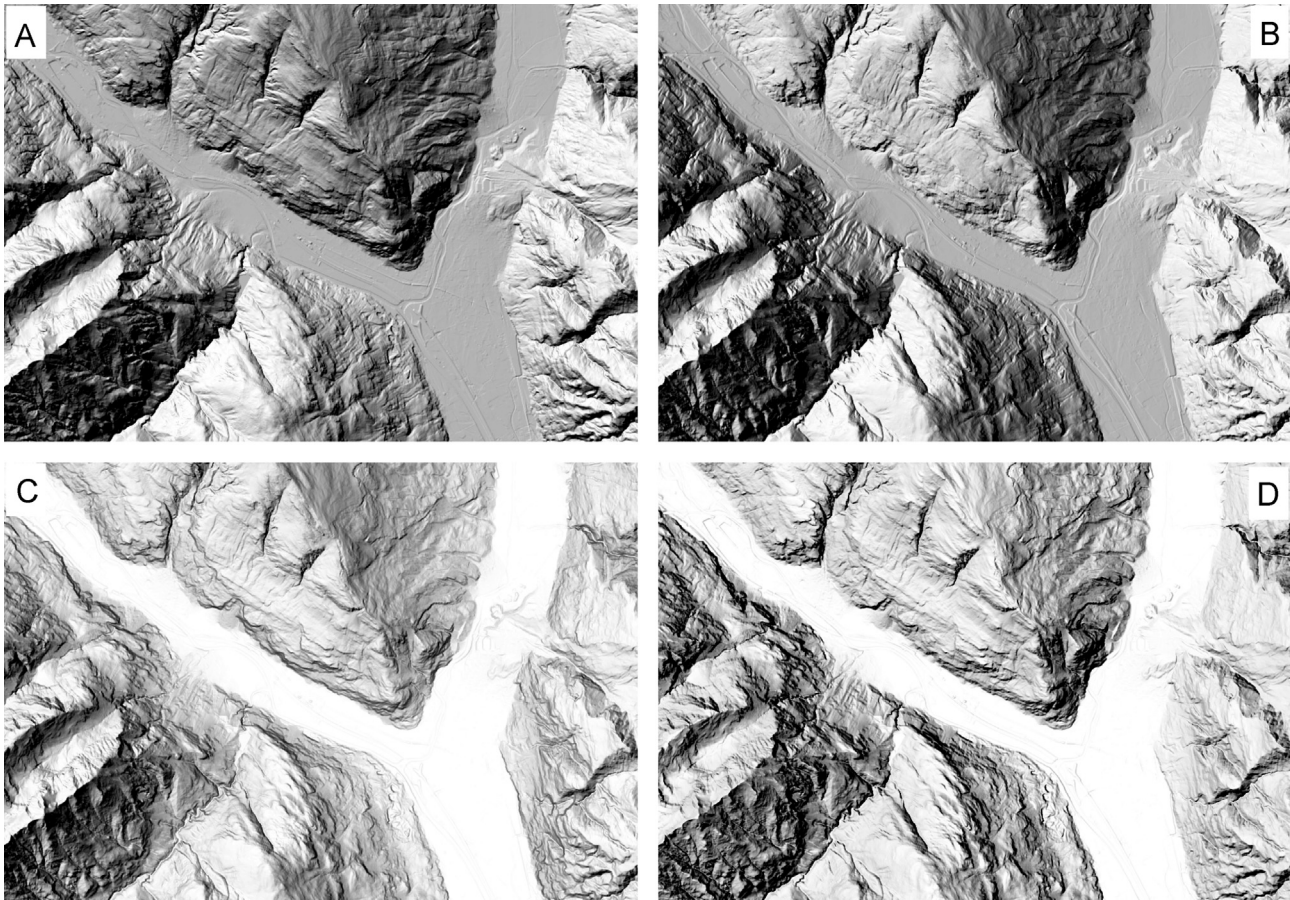
Fig. 5. Detailed comparison of the four maps in the way they depict valley and alluvial fans. The two maps of the tops, because they keep the zenith value fixed for the whole region, show all the details of the valley and some details of the alluvial fan. On the other hand, by correction zenith according to slope or elevation the uninteresting details can be excluded and the shift between valley and slopes can be highlighted.

we cannot deviate too much from the standard 315°N because otherwise there is a risk of losing tridimensionality. A shift of ± 30° should be enough for most cases (Brassel, 1974).

By implementing the equation presented in Section 2.3 we were able to seamlessly change the light azimuth across the map to illuminate each pixel from the right angle. The advantage of using this approach is the complete automation of the whole process. The sine wave changes the azimuth value based only on aspect, shifting between a maximum and a minimum value, selected by the user. This guarantees consistent results, which are presented in Fig. 3. The first (A) has been created with the standard algorithm available in ArcGIS, the second (B) implementing two light source, i.e. 315°N and 280°N. The third and fourth are created using multiple light sources and correcting zenith according to elevation (C) and slope (D).

The first correction, which results are visible in Fig. 3B, changes the light source continuously using Eq. (3), while zenith remains constant. This correction presents an enhanced tridimensionality effect because the light source changes direction according to aspect. This map probably appears more natural to practitioners used to see analytical hillshaded representations. However, the light change is a subtle but crucial element that increases the overall visual quality. In contrast, even though the tridimensionality effect is enhanced, all the unnecessary details in the valleys are still visible, and according to Imhof (1982) these should be removed to increase the readability of shaded relief representations.

For this reason we implemented two zenith corrections, which imitate the combined shading proposed by Imhof (1982).The first corrects it according to elevation, solving Eq. (5). In this case

valleys are represented with a bright white tone, which hides most of the distracting noise. For example, the main valley around Biasca is highly populated and even though the DEM has been filtered by technicians at Swisstopo (2004), most of the roads are still visible. With this simple correction all this noise disappears. Moreover, because the zenith angle changes with the increase in elevation, the image tine becomes darker toward ridges. This enhances the contrast of the ridges, which appear sharper than with the standard method (Fig. 3C). The drainage network is highlighted (Fig. 4C), and the boundary between low-relief areas and slopes is also better defined (Fig. 5C).

The second correction calculates zenith solving Eq. (6), correcting it proportionally to slope. This means that for high slope values, zenith increases and therefore the area appears darker. The results of this correction are shown in Fig. 3D. This may be useful when there is a need to highlight rock outcrops or particular geomorphological structures. This correction does not produce the same result as a light source perpendicular to the DEM. In that case the only shading principle that would be honoured is "the steeper, the darker", which enhance slope but radically decrease tridimensionality. By retaining two light sources and changing their inclination based on slope, we can produce images where the tridimensionality effect is still present, and slope in enhanced. This correction highlights not only rock outcrops but also the drainage network (Fig. 4D), even though probably less than an elevation correction. The transition from valley to slopes is certainly more emphasised that with elevation correction, and also alluvial fans are better represented (Fig. 5D).

Generally speaking, there is always a trade-off to be considered when there is a need to automate a process that has been developed manually. Two paths can be followed, on the one hand we could try to replicate as closely as possible the manual results. In this case we would need to add complexity to the technique, which could become complex and could involve a certain amount of interpretation by the cartographer, thus decreasing consistency. On the other hand a fully automatic process, even though it does not probably provide the visual quality achieved by interpreted methods, guarantees consistent results every time it is applied. In this research we developed a simple algorithm that certainly produces consistent shaded representations of superior visual quality, compared with standard algorithms. However, it does not include any form of geomorphological interpretation, the light source is changed according to aspect alone. For this reason we think it is clear that this method cannot achieve results qualitatively comparable with more complex approaches, such as Jenny (2001), Patterson (2001, 2004, 2013) or others. However, it has more chances of becoming part of the common routine of presenting spatial data, because of its level of simplicity, customisation, automation and the high level of consistency of its results.

The lack of cartographic interpretation may result in unwanted effects in rough terrains. This may be solved before shading by smoothing the DEM or by using other terrain generalisation algorithms. However, we implemented two zenith corrections that can increase the visual quality of the map by obscuring unwanted details. This certainly decreases the visual impact of DEM noise while maintaining the same level of automation and consistency.

4. Conclusions

The method presented in this paper provides a fully automatic way of seamlessly changing the light direction in shaded relief representations. The high level of automation guarantees consistent results and it also means the method can potentially replace the standard analytical hillshading algorithm. It provides a superior visual quality and a level of customisation that makes it suitable for a wide range of applications and disciplines.

The method developed in this work was embedded into an ArcGIS toolbox. This makes it available for a wide range of practitioners, both within the scientific community and outside. With this method everyone, even non-cartographer, can produce highly informative shaded relief representations, particularly in mountainous landscapes where the standard method is sometimes ineffective.

Acknowledgements

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.cageo.2014.10.015.

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