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Case study

Integrating geological uncertainty in long-term open pit mine production planning by ant colony optimization

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ABSTRACT

Meeting production targets in terms of ore quantity and quality is critical for a successful mining operation. In-situ grade uncertainty causes both deviations from production targets and general financial deficits. A new stochastic optimization algorithm based on ant colony optimization (ACO) approach is developed herein to integrate geological uncertainty described through a series of the simulated ore bodies. Two different strategies were developed based on a single predefined probability value (*Prob*) and multiple probability values ($Prob_n^t$), respectively in order to improve the initial solutions that created by deterministic ACO procedure. Application at the Sungun copper mine in the northwest of Iran demonstrate the abilities of the stochastic approach to create a single schedule and control the risk of deviating from production targets over time and also increase the project value. A comparison between two strategies and traditional approach illustrates that the multiple probability strategy is able to produce better schedules, however, the single predefined probability is more practical in projects requiring of high flexibility degree.

1. Introduction

The long-term open pit mine production planning is a large combinatorial optimization problem that involves specifying the blocks extraction sequence and their destination during mine life. Mathematical formulation is aimed to maximize the net present value (NPV) of the mining operation subject to a series of operational constraints such as reserve, slope, mining capacity, and milling rate. The operational research techniques, which have been developed to solve long-term production planning since 1960s, could be categorized in two major classes of deterministic and stochastic-based approaches.

All inputs are assumed as fixed value in the deterministic approaches. In early investigations, Dagdelen and Johnson (1986) suggested an approach based on Lagrangian relaxation. Later, a branch-and-cut algorithm was developed by Caccetta and Hill (2003). The major drawback of these methods was their disability in applying on real scale deposits where, typically, include hundreds of thousands to millions of blocks. Several attempts have been spent on reducing the problem size such as Fundamental Trees methodology of Ramazan (2007). Moreover, the other class of researches focused on the heuristic methods (Gershon, 1987), combination of dynamic programming and heuristics (Tolwtnski and Underwood, 1996), and meta-heuristic approach such as genetic algorithm (Denby and Schofield, 1994), particle swarm algorithm (Ferland et al., 2007), and ant colony algorithm (Sattarvand, 2009). A detailed review of the solution approaches could be found in Osanloo et al. (2008).

Ignoring any kind of uncertainty is the common weakness of all deterministic algorithms, which leads to create un-realistic plans in terms of operational requirements. Dimitrakopoulos classifies the uncertainties of mining projects into three major sources as geological, technical, and economical uncertainties (Dimitrakopoulos, 1998).

Grade uncertainty is the major source of deviations from production targets and general financial deficits. Vallee (2000) reported that the average production rate of 60% observed mines in the early years of the mining is 70% less than predicted rates, mainly due to grade uncertainty. Uncertainty-based open pit optimization approaches could be categorized into variance-based and simulation based groups. The first type involves integrating of the grade variance in traditional deterministic algorithms. Albach considering grade variance, developed a linear programming to design a lignite mine (Albach, 1967). A similar approach based on stochastic integer programming model has been suggested by Gangwar (1973). Denby and Schofield (1995) used genetic algorithm to integrate the grade variability in planning process.

The second uncertainty-based approach is based on using alternative scenarios of the ore body called “Realization” that are

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provided by conditional simulation methods. Initially Ravenscroft discussed the risk analysis in mine production planning based on the realizations (Ravenscroft, 1992). Dowd (1994) integrated uncertainties of the commodity price, mining costs, and processing costs in a risk based optimization framework. Dimitrakopoulos and Ramazan (2004) considering grade uncertainty, equipment access, and mobility constraints suggested an LP approach that was based on the expected ore block grades and the probabilities of being above cutoffs. Godoy and Dimitrakopoulos (2004) presented a realization based meta-heuristic approach. They generated production schedules for all realizations and then, using Simulated Annealing algorithm, combined the mining sequences in order to produce a single schedule. Ramazan and Dimitrakopoulos (2004) suggested an MIP model that starts with generating production schedules for each realization and then, calculating the extraction probability of each blocks in a given period. The blocks with probability between zero and one have been used in a new optimization model to generate a schedule. The same research has been reported by Menabde et al. (2004). Dimitrakopoulos and Abdel Sabour (2007) using real options valuation (ROV) method attempted to handle multiple uncertainties such as grade and economic parameters in production planning. Gholamnejad et al. (2008) presented a stochastic programming based model that grade uncertainty is integrated explicitly in the mathematical programming model by applying chance constrained programming approach to approximate it into a linear format. Lamghari and Dimitrakopoulos (2012) considering the metal uncertainty, utilized the tabu search procedure to solve the open pit optimization problem. Two different diversification strategies were used to search the feasible domain in order to generate several initial solutions which will be improved later by the tabu search procedure.

The further researches led to multi-stage modeling methodologies in order to minimize the deviations from production targets in addition to the NPV maximization (Benndrof and Dimitrakopoulos, 2009; Consuegra and Dimitrakopoulos, 2010; Leite and Dimitrakopoulos, 2009; Ramazan and Dimitrakopoulos, 2007; Smith, 2001). Ramazan and Dimitrakopoulos (2007) presented a stochastic integer programing (SIP) model to generate production schedules. The geological risk discounting concept (Dimitrakopoulos and Ramazan, 2004) was used in order to control the risk distribution between production periods and minimize the deviations from targets. Another similar SIP model was developed by Leite and Dimitrakopoulos (2009). Benndorf improved the SIP model by adding a third part to the objective function termed "smooth mining controller" in order to create a safe operational condition (Benndrof and Dimitrakopoulos, 2009). Consuegra and Dimitrakopoulos (2010) developed a SIP model to integrate the grade uncertainty in pushbacks design. Later on, Ramazan and Dimitrakopoulos (2012) established a SIP model to integrate the uncertainty of product supply in the optimization model.

Despite the development of numerous approaches to integrating the geological uncertainty, however, the solving methodologies have been received relatively less attention. It has been shown that the single stage models are unable to integrating the grade uncertainty explicitly and creating an optimal single solution. In fact they are a series of repeated implementations of the traditional approaches on ore body simulations. On the other hand, the multi-stage stochastic models which have to be solved by available mixed integer programming packages, are limited to relatively small size instances.

This paper proposes an efficient solution methodology based on Ant Colony Optimization (ACO) to solve the real scale planning problems in presence of the geological uncertainty. The procedure has the capability to simultaneously optimize the UPL and production scheduling. Paper outlines the modeling procedure, two different strategies and discusses the difference between obtained solutions and provided deterministic solution by traditional approach.

2. Formulation of the long-term production planning

Open pit production planning could be effectively modeled as an Integer Programming (IP) formulation with the objective of NPV maximization subject to a set of technical and operational constrains. It can be expressed as following:

$$\text{maximize } Z = \sum_{n=1}^{N} \sum_{t=1}^{T} \frac{V_n}{(1+d)^t} x_{n,t} \tag{1}$$

Subject to:

$$x_{n,t} \epsilon \ (0, 1), \qquad \text{for } n = 1 \text{ to } N \ , \ t = 1 \text{ to } T \tag{2}$$

Slope constraint: each block can only be mined if its predecessors are already mined before.

$$x_{n,t} - \sum_{\tau=1}^{t} x_{m,\tau} \le 0, \qquad \text{for } m = 1 \text{ to } N \ , t = 1 \text{ to } T \tag{3}$$

where $m\epsilon$ (set of predecessors blocks of block n)

Reserve constraint: a block cannot be mined more than once.

$$\sum_{t=1}^{T} x_{n,t} \le 1, \qquad \text{for } n = 1 \text{ to } N \tag{4}$$

Processing capacity: the total ore processed during each period should be within the predefined upper and lower limits.

$$\sum_{n=1}^{N} o_n \times w_n \times x_{n,t} \ge \underline{O}, \qquad \text{for } t = 1 \text{ to } T \tag{5}$$

$$\sum_{n=1}^{N} o_n \times w_n \times x_{n,t} \le \bar{O}, \qquad \text{for } t = 1 \text{ to } T \tag{6}$$

Mining capacity: the total material mined during each period should be within the predefined upper and lower limits.

$$\sum_{n=1}^{N} w_n \times x_{n,t} \ge \underline{M}, \qquad \text{for } t = 1 \text{ to } T \tag{7}$$

$$\sum_{n=1}^{N} w_n \times x_{n,t} \le \bar{M}, \qquad \text{for } t = 1 \text{ to } T \tag{8}$$

Average grade constraint: the average grade of material mind during each period should be more than predefined value.

$$\frac{1}{N_t} \sum_{n=1}^{N} g_n x_{n,t} \ge \underline{G}, \qquad \text{for } t = 1 \text{ to } T \tag{9}$$

Where

- N, is the total number of blocks,
- n, is the block index,
- T, is number of periods,
- t, is the period index,
- V_n, is value of n^{th} block,
- $x_{n,t}$, is a binary variable associated to n^{th} block that mined in t^{th} period,

$$x_{n,t} = \begin{cases} 1 & \text{if block is mined in period } t \\ 0 & \text{otherwise} \end{cases}$$

- o_n, is a parameter indicating that the n^{th} block is an ore block or not,

$$o_n = \begin{cases} 1 & \text{if } n^{th} \text{ block is an ore block} \\ 0 & \text{otherwise} \end{cases}$$

- w_n, is the weight of n^{th} block,

- $\bar{O}$ and $\underline{O}$, are the upper and lower bound of processing capacity,
- $\bar{M}$ and $\underline{M}$, are the upper and lower bound of mining capacity,
- N_t reflects the number of blocks located in t^{th} period,
- g_n, is the average grade of n^{th} block,
- $\underline{G}$, is the lower bound of average grade.

In the most real cases, the block model contains thousands to millions of blocks that make an IP model with millions of integer variables and constraints, which can be extremely difficult or expensive to solve. So, meta-heuristics like ACO are well suited for performing the optimization and are able to simplify the formulation by implicitly obeying slopes and various other constraints.

3. Ant colony optimization procedure

The methodology of ACO procedure was inspired by the foraging behavior of the ants which developed by Dorigo and Stützle (2004). In nature, ants wander randomly for seeking food and return to their colony after finding it while laying down some chemical trails called "pheromone". The pheromone trails transmit a message to others to follow the trails instead of traveling randomly. Over the time, a path with more passing ants gets more deposited pheromone. On the other hand, the pheromone trails start to evaporate and lose its attraction. Obviously, magnitude of the evaporation in longer paths is higher than deposition in compare of the shorter paths. Thus, intensity of laid pheromone on the shortest paths increases up gradually since exceeds the evaporation rate and it's what makes it to be attracted by other ants.

ACO approach is applied successfully to solve well-known optimization problems such as Travelling sales man, Vehicle routing, and Assignment problem (Dorigo and Stützle, 2004). The application of ACO approach in open pit mine production planning has been introduced by Sattarvand (2009) and Sattarvand and Niemann-Delius (2013). Fig. 1 shows the general flowchart of this procedure.

3.1. Schedule encoding and decoding

A given open pit schedule can be considered as the superposition of a series of the pits related to the different mining

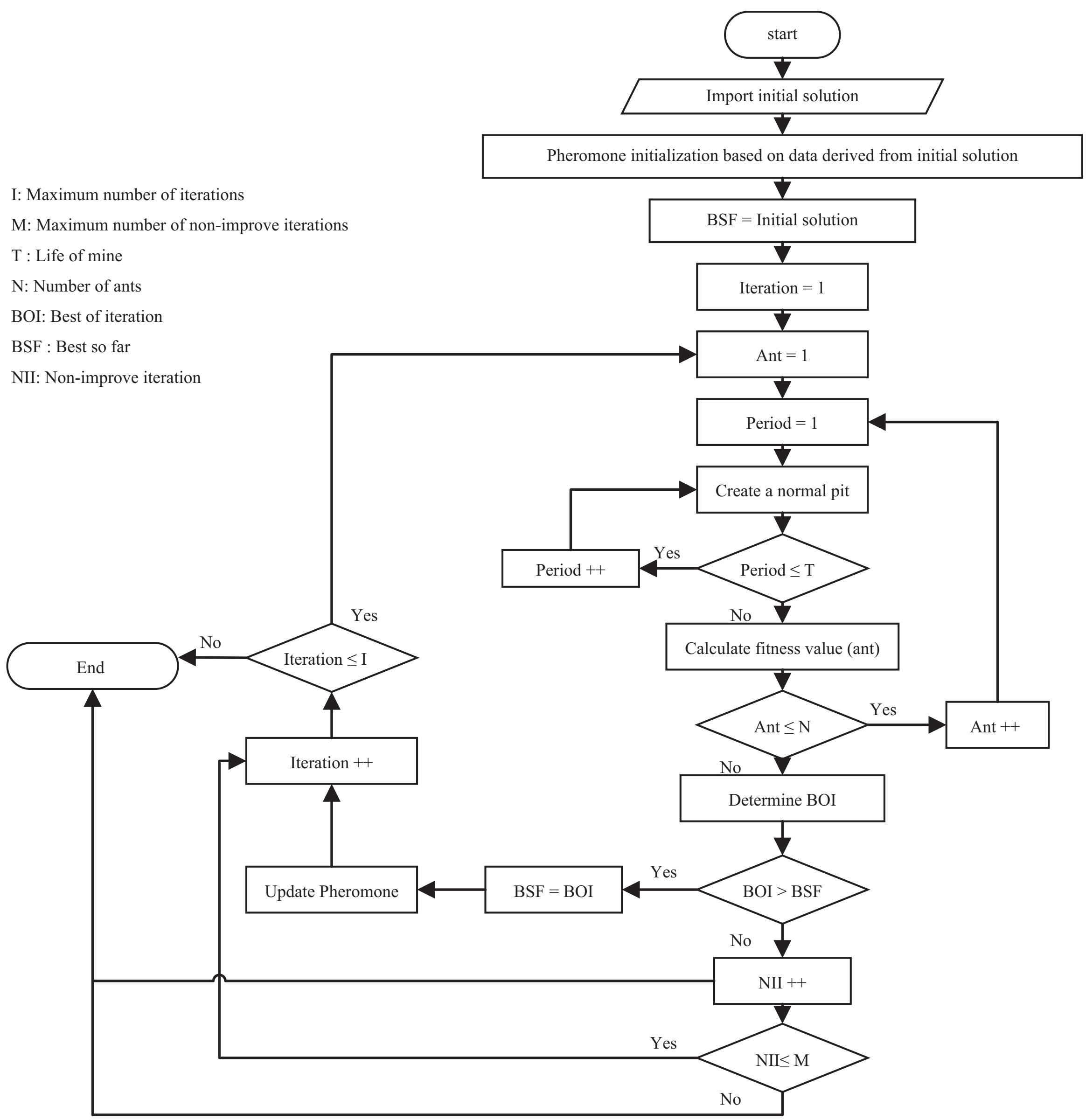

Fig. 1. ACO approach for open-pit production planning.

periods. In turn, any given pit could be represented by a series of block model columns and determining the pit depth in each column. Consequently, any 3D mine schedule (a solution) can be represented by $pitdepth_{(i,j,t)}$, which shows the depth of t^{th} period along column(i, j). It could be stored as a two dimensional array of integers denoting the depth of the pits related to different mining periods.

It should be noted that the data stored in $pitdepth_{(i,j,t)}$, just represents the surface of t^{th} period, it is necessary to determine the blocks scheduled in t^{th} period. This is done during a back transform procedure in which by starting from the first period, all blocks of column(i, j) that have been located between $pitdepth_{(i,j,t)}$ and $pitdepth_{(i,j,t-1)}$ will be scheduled in t^{th} period. Note that topography surface has been considered as the prior pit depth for the first period.

3.2. Initial solution and pheromone initialization

Initially, the algorithm needs to assign some initial pheromone to blocks in order to start the main process. For this purpose a sub-optimal solution can be used to initialize the pheromone value (τ_n^t) which represents the attractiveness of the n^{th} block to be the deepest point of the mine in the t^{th} period. The initial sub-optimal solution is represented as an array $\boldsymbol{X}_N^0$ where each component $n \in \{1, \ldots, N\}$, $x_n^0 = t$, represents the t^{th} period that block n is initially assigned to. The assignment of the initial pheromone value (τ_0) would be done in a way that a higher values are assigned to the ore blocks that located around the period surfaces or to an imaginary layer right above the topography surface (Sattarvand, 2009; Sattarvand and Niemann-Delius, 2013).

3.3. Iterations

During each ACO iteration, a set of new schedules are constructed using the current pheromone configuration and during the during the perturbation process in the solution of last iteration. Perturbation process as the core of ACO methodology is just a depth determination process along block columns. It considers a set of variables (pheromone values) for each block that represents its attractiveness to be the deepest point of its containing column related to different periods. As mentioned, these values are initialized by assigning of higher pheromones to a few numbers of the blocks around the initial sub-optimal pit depth and then updated to new values after each iteration based on the quality of the founded solutions. Three main steps in each iteration consist of depth determination, normalization and pheromone update that have been explained next.

3.3.1. Depth determination

Considering the pheromone value of blocks and some heuristic information such as economic value of blocks, the depth determination routine is performed on each column in order to make a non-normalized schedule surface. Any ant k utilizes a probabilistic choice rule named "random proportional rule" in order to select the pit depth in each column. The probability of choosing n^{th} block as the pit floor by k^{th} ant could be expressed as:

$$P_n^k = \frac{[\tau_n]^\alpha [\eta_n]^\beta}{\sum_{l \epsilon N_n^k} [\tau_l]^\alpha [\eta_l]^\beta} \tag{10}$$

where τ_n is the pheromone value of the block n, η_n is the heuristic information such as block value or any information, α and β are two parameters which represent the relative influence of the pheromone trail and the heuristic information, and N_n^k is a set of feasible selections for the ant k.

All different variants of ACO attempt to find the pit depth based on the calculated probabilities. In ant colony system (ACS) method which is selected in this study, ant k uses a "pseudorandom proportional rule" to select the deepest block of pit in each column as following:

$$j = \begin{cases} \text{argmax}_{l \in N_n^k} \{[\tau_n]^\alpha [\eta_n]^\beta\}, & \text{if} \quad \dfrac{q \le q_0}{\text{otherwise}} \\ J, & \end{cases} \tag{11}$$

where q is a random variable uniformly distributed in $[0, 1]$, q_0 is a parameter $(0 \le q_0 \le 1)$, and J is a random variable selected according Eq. (10) (with $\alpha = 1$).

The process is applied to all columns containing an ore block at least. Depth of totally waste columns will be defined based on the adjacent columns depths during normalization process. Experience shows that restricting the process to the upper and lower bounds helps to be more efficient. So, the bounds are set to the depths of largest possible pit and depth of earlier schedule surfaces (or initial topography), respectively. Finally, a normalization process is required to convert the constructed non-normal solution to a feasible configuration.

3.3.2. Constraint handling

There is not any explicit mechanism for constraint handling in ACO. However, slope constraint needs to be applied on the independently selected pit depths in order to adapt them to a feasible pit configuration from slope angles point of view. ACO uses a special normalization procedure in order to alter the selected depths and convert them to a feasible shape in a way that the new pit shape covers all of the determined depths as well as the outline of the earlier pit (or original topography for the first pit) (Gilani and Sattarvand, in press). Rests of the constraints are handled in a manner at which they are allowed to violate at the expense of a penalty cost added to the objective function. In other words the objective function can be written as following:

$$maximize\ \ Z = \sum_{n=1}^{N} \sum_{t=1}^{T} \frac{V_n}{(1+d)^t} . x_{n,t} - P(x) \tag{12}$$

subject to:

$$P(x) = (C_t^{m-}.\, d_t^{m-} + C_t^{m+}.\, d_t^{m+} + C_t^{o-}.\, d_t^{o-} + C_t^{o+}.\, d_t^{o+} + C_t^{g-}.\, d_t^{g-}), \quad \forall \quad t = 1, \ldots, \quad T \tag{13}$$

and also subject to constraint (2)–(4),
where:

- $C_t^{m+} = c^{m+}/(1+d)^t$: Unit surplus cost incurred if the total weight of rock mined during period t exceeds $\bar{M}$ (c^{m+} is the undiscounted unit surplus cost).
- $C_t^{m-} = c^{m-}/(1+d)^t$: Unit shortage cost associated with the failure to meet $\underline{M}$during period t (c^{m-} is the undiscounted unit shortage cost).
- $C_t^{o+} = c^{o+}/(1+d)^t$: Unit surplus cost incurred if the total weight of ore mined during period t exceeds $\bar{O}$.
- $C_t^{o-} = c^{o-}/(1+d)^t$: Unit shortage cost associated with the failure to meet $\underline{O}$ during period t.
- $C_t^{g-} = c^{g-}/(1+d)^t$: Unit shortage cost associated with the failure to meet $\underline{G}$ during period t.
- $d_t^{m-}$and d_t^{m+}, denote the shortage and the surplus in the amount of rock mined during period t, respectively.
- $d_t^{o-}$and d_t^{o+}, denote the shortage and the surplus in the amount of ore mined during period t, respectively.
- d_t^{g-}, denote the shortage in the average ore grade sending to plant during period t.

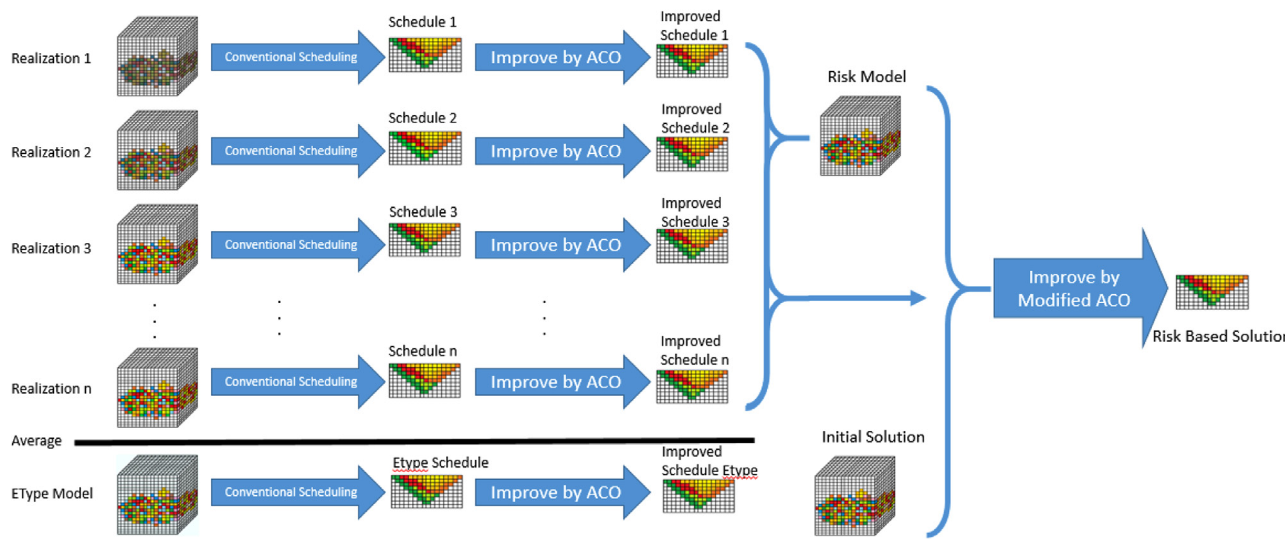

Fig. 2. Flowchart of the grade uncertainty integration in ACO pit optimizer.

3.3.3. Pheromone update

Pheromone update procedure consists of two steps. The first step, called "pheromone evaporation", involves a uniform reduction in the value of all pheromones in order to help the ACO model disregarding the bad solutions. The next step, called "pheromone deposition", consists of adding additional pheromone to the blocks that have participated in construction of the schedules. Different strategies of pheromone update such as ant system (AS), elitist ant system (EAS), ranked based ant system (AS_{rank}), max-min ant system (MMAS) and ant colony system (ACS) have been investigated. The main differences in these strategies are the manner of block selection for pheromone update and the amount of pheromones to be added. AS is the first and simplest method, where all of its' constructed schedules are allowed to be contributed in pheromone deposition. EAS allows the best-so-far schedule to deposit extra pheromone. In AS_{rank} only a few good schedules are allowed to add pheromones. MMAS allows only the best-so-far schedule to deposit pheromones and utilizes special pheromone limitations in order to prevent the process from stagnation in local optimums. Research continued by ACS as the best variant from running speed and required computational resources points of view (Soleymani Shishvan and Sattarvand, 2015).

Ant Colony System differs from AS in three main points. First, it exploits the search experience accumulated by the ants more strongly than other system. Second, pheromone evaporation and pheromone deposition take place only on the best-so-far solution. Finally, passing through any path leads to remove some of its pheromone in order to increase the exploration of alternative paths.

Generally, two local and global pheromone update rule has been done which the first one is applied immediately after each pit construction and the second one is done after each iteration only by the best-so-far ant.

$$\textit{local update}: \qquad \tau_n^t \leftarrow (1-\xi)\tau_n^t + \tau_0, \qquad (0 < \xi < 1) \tag{14}$$

$$\textit{global update}: \qquad \tau_n^t \leftarrow (1-\rho)\tau_n^t + \rho\Delta\tau_n^{best}, \qquad (0 < \rho < 1) \tag{15}$$

Where ξ is the local evaporation rate, τ_0 is the initial value of pheromone trails, ρ is the evaporation rate and $\Delta\tau_n^{best}$ is the amount of pheromone deposition by the best-so-far ant on the n^{th} block.

3.4. Termination of the algorithm

Algorithm terminates after a certain number of iteration or when it does not catch any improvement.

4. Incorporation of the grade uncertainty

Like the methods used for long term production planning, the proposed approach herein has considered constant meet production targets for each period, maximize the overall discounted cash flows, minimize the stripping ratio and guarantee safety slope requirements. An initial solution (mine schedule) is required to specify initial targets for each period, which can be generated by traditional algorithms. For this purpose, the current commercial package has been used in order to create initial solutions. Then, the optimal solution design is defined by the uncertainty based ACO algorithm that yields the best performance in terms of the requirements stated above. For this purpose, two additional block models termed "EType block model" and "Risk block model" have been used in order to incorporate the geological uncertainty. The EType block model is simply generated by averaging the geological simulations such that the metal grade of any block is the average of that block's grade in all simulations. The risk block model is created by assigning the probability of each block to exploit in any period. Considering the variability in metal grade that strongly linked to the variability in the economic value of the material within the pits, the general integer formulation has been modified a little that would be solved by the uncertainty based ACO algorithm. Fig. 2 illustrates the general procedure of the uncertainty based ACO approach for the open pit mine production planning.

The modified formulation aims to minimize the average absolute deviation (difference) from a target tonnage over all geological simulations by adding the probabilistic factor to the objective function of general formulation discussed Section 3.3.2 as following:

$$\textit{maximize } Z = \frac{1}{S}\left\{ \sum\nolimits_{s=1}^{S} \sum\nolimits_{n=1}^{N} \sum\nolimits_{t=1}^{T} \frac{V_{ns}}{(1+d)^t}.\, x_{nt} - S.\, C_p Prob_n^t - P(x) \right\} \tag{16}$$

subject to:

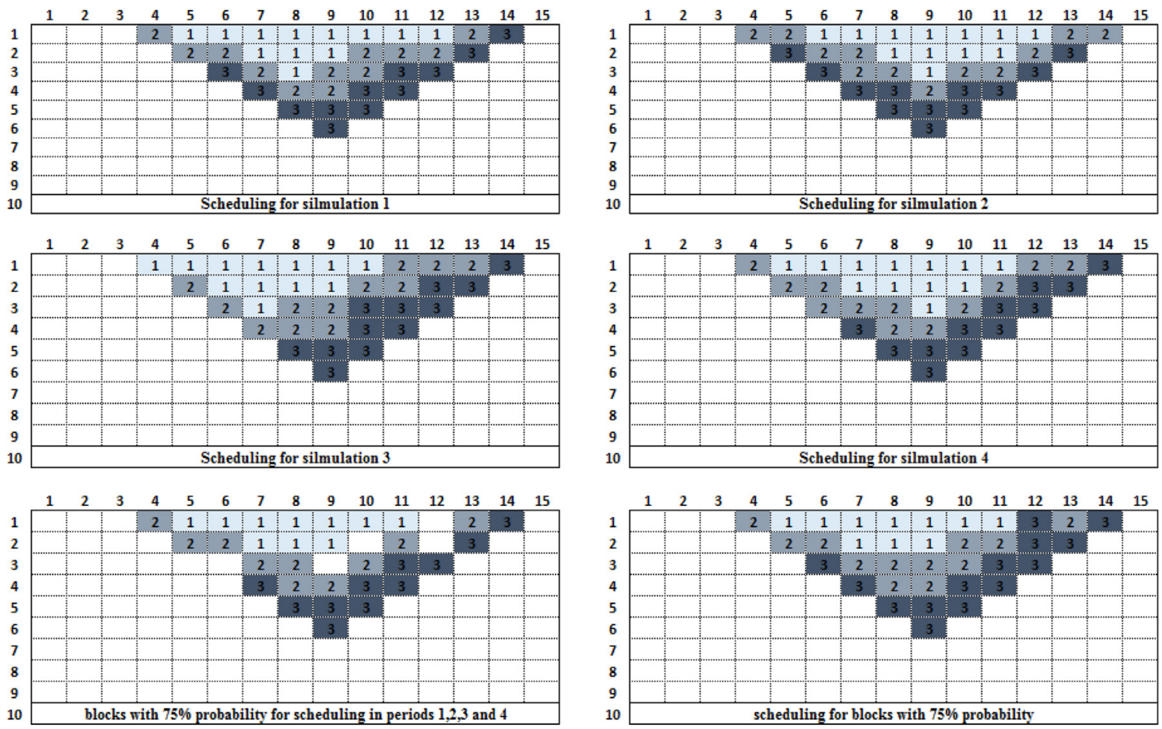

Fig. 3. Two dimensional scheduling problem with three periods and $\text{Prob}_n^t = 75$.

$$P(x) = (C_t^{m-}.\ d_{t,s}^{m-} + C_t^{m+}.\ d_{t,s}^{m+} + C_t^{o-}.\ d_{t,s}^{o-} + C_t^{o+}.\ d_{t,s}^{o+} + C_t^{g-}.\ d_{t,s}^{g-}),\ \ \forall\ \ s = 1, \ldots, S\ \ \text{and}\ \ \forall\ \ t = 1, \ldots, T \tag{17}$$

and also subject to constraint (2)–(4), where:

- $d_{t,s}^{m-}$ and $d_{t,s}^{m+}$, denote the shortage and the surplus in the amount of rock mined during period t if scenario s occurs, respectively.
- $d_{t,s}^{o-}$ and $d_{t,s}^{o+}$, denote the shortage and the surplus in the amount of ore mined during period t if scenario s occurs, respectively.
- $d_{t,s}^{g-}$, denote the shortage in the average ore grade sending to plant during period t if scenario s occurs.
- $Prob_n^t$: represents the probability of locating n^{th} block in p^{th} period,
- $C_p = 100 - Prob_n^t,\ \ p = 1, \ldots, 100$ *and* $(C_p < C_{p-1})$: Coefficient cost adjusted to $Prob_n^t$.
- S, is the total number of simulations.
- s, is the simulation index.

The values of parameter $Prob_n^t$ are extracted directly from mentioned risk block model created using all geological simulations and their generated mine schedules via traditional algorithms which its elements are calculated by counting the number of times that n^{th} block has been scheduled in t^{th} period along all simulations as following:

$$Prob_n^t = \frac{100}{S} \sum_{s=1}^{S} x_{n,s}^t \tag{18}$$

where $x_{n,s}^t$ is the decision variable which takes 1 if the n^{th} block of s^{th} simulation is scheduled in t^{th} period and takes 0 if else.

Considering the geological uncertainty, another additional rule was carried out in pheromone initialization process based on the values of $Prob_n^t$ extracted from the risk block model instead of assigning a fix value (τ_0) as following:

$$\tau_{n_{initial}}^t = C_\tau \tau_0 \frac{Prob_n^t}{100} \tag{19}$$

where, C_τ is a coefficient to control the intensity of $Prob_n^t$ on pheromone initialization.

As mentioned before the pheromone initialization is carried out according to the initial solution that can be generated by traditional approach for EType block model in order to consider the geological uncertainty. The experiments shows the uniform pheromone initialization led to increase the running time (Dorigo and Stützle, 2004). Thus, the initial solution was improved using deterministic version of ACO (Dtrm-ACO) and its output was considered as the initial solution for the main algorithm.

Considering the main aim of the uncertainty based ACO procedure to scheduling the low-risk blocks in early periods, the pheromone update should be carried out proportional to the risk related to blocks. For this purpose, according to the risk block model, the blocks with high probability for scheduling in period p or mathematically with high value of $Prob_n^t$ would get more pheromone value and have the greater chance of scheduling in early periods. Hence, a new pheromone deposition rule is used after each iteration as following:

$$\tau_n^t \leftarrow \tau_n^t + C_{prob} \tau_0 \frac{Prob_n^t}{100} \tag{20}$$

Two distinct strategies have been used to update the pheromone value based on the value of $Prob_n^t$ in the proposed procedure. The first strategy, called "ACO-SRB",[1] is based on a fixed single probability value. While the second strategy, called "ACO-MRB"[2] utilizes all of the probability values of $Prob_n^t$. In other words, in the first strategy, the n^{th} block is allowed to get pheromone in t^{th} period if and only if $Prob_n^t \geq Prob$. This forces the algorithm to provide a scheduling design including the blocks with the risk less than threshold ($Prob_n^t$ higher than predefined $Prob$). Fig. 3

[1] Single risk based version of ACO.
[2] Multiple risk based version of ACO.

illustrates a two dimensional scheduling example for $Prob_n^t \geq 75$ constructed by four geological simulations. On the other hand, in the second strategy, all blocks contribute in the pheromone update process proportional to their $Prob_n^t$. This will increase the chance of block with higher value of $Prob_n^t$ or in other word the low-risk blocks to being scheduled in early periods and vice versa.

5. Numerical results and discussion

The proposed framework for long term production planning that account the geological uncertainty is applied to the Sungun copper mine located in the northwest of Iran. Sungun is a traditional open pit operation with truck-and-loader mining system which having a mining rate of 46Mt per year, makes it as one of the largest open pit operation in Iran. For this case study, the whole of mine is being considered, which contains $(156 \times 168 \times 96 = 2{,}515{,}968)$ blocks that are $25 \times 25 \times 12.5\ m^3$ in the x, y and z directions respectively. Considering that conditional simulation procedure is very onerous in terms of computer resources and processing time, so one of the first questions to arise is the number of required simulations. The answer is related to the specific requirements of the analysis and the degree of the required precision (Ravenscroft, 1992). On the other hand, the limitation of the number of simulations depends on the size and detail of the model. In our study, because of the requirement for confidence limits and accurate probabilistic valuations, large numbers of simulations may be needed in order to produce practical outcomes. But, it must be noted that considerable computing resource will be required for large number of simulations. Based on previous studies (Godoy and Dimitrakopoulos, 2004; Lamghari and Dimitrakopoulos, 2012), mine has provided 20 conditional simulations using the computationally SGSim[3] approach that contain simulated copper grades, recoveries, tonnage and simulated material types. The geotechnical studies specify five zones representing areas with separate slope angles, which should be satisfied in all generated solutions (Table 1). The average grade of copper is 0.661%. The ore deposit is categorized into two categories, supergene and hypogene. Ore tonnage and average grade of supergene zone are 182 Mt and 0.62% respectively and it is 12% of the total tonnage of the orebody. Due to the higher grade, supergene is the major ore type feeding the concentrator in the early years of operation but because it contains copper oxide, the recovery of metal is lower and also leaching as a recovery method is not environment friendly. Ore tonnage in hypogene zone is 1300 Mt with an average grade of 0.44%.

Some initial scheduling designs has been provided using a commercial software for all simulations and the EType block models in order to create the risk block model and initial solution. The ore/waste distribution and average grade of the created initial mining plan for all simulations and the EType block models has been illustrated in Figs. 4,5 and 6.

As mentioned before the initial solution would be improved by the proposed approach while simultaneously reducing the variability and its related risk. The economic parameters such as copper price, unit costs, unit revenues and discount factor are summarized in Table 2 which are inspired by the real data.

As mentioned before, constraint handing has been done using penalty functions that the related penalties values or undiscounted shortage and surplus costs are considered as following:

- The annual mining rate less than $\underline{M} = 37\ Mt$, leads to production shortage and increasing the overhead and mining costs by 10%.

[3] Sequential Gaussian simulation

Table 1
Economical parameters for long-term production scheduling of Sungun copper mine.

Parameter	Value	Unit
Metal price	5,500	\$/ton (metal)
Selling cost	20	\$/ton (metal)
Mining cost (waste)	1.56	\$/ton
Mining cost (ore)	1.75	\$/ton
Processing cost	11.85	\$/ton
Additional cost	2.57	\$/ton
Dilution	8	%
Mining recovery	95	%
Discount rate	10	%

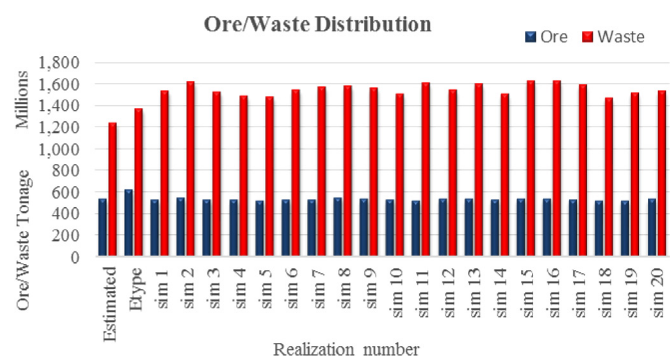

Fig. 4. Ore/waste distribution in ultimate pit limits related to each simulation.

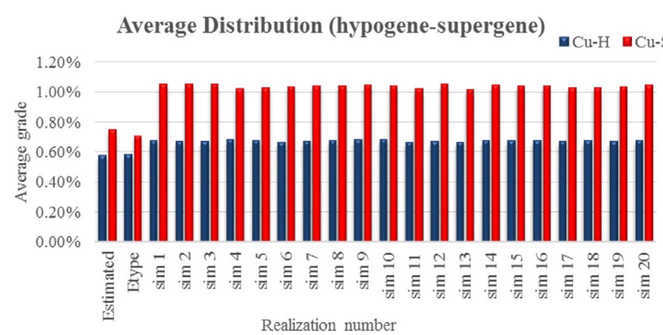

Fig. 5. Average grade of ultimate pit limits related to each simulation.

- The annual mining rate more than $\bar{M} = 46\ Mt$, leads to production surplus, getting auxiliary services from contractors and increasing the mining cost by 20%.
- The annual milling rate less than $\underline{Q} = 12\ Mt$, leads to production shortage and increasing the overhead and mining costs by 10% because of reduction in the metal recovery.
- The annual ore production rate more than $\bar{O} = 14\ Mt$, leads to production surplus, overflowing in beneficiation plant, damaging the factory and more important having to waste the extracted ore.
- If the yearly average grade of the copper is less than the $\underline{G} = 0.55\%$, plant recovery changes as following:

$$Recovery(\%) = G_{cu} \times \frac{R(\%)}{\underline{G}(\%)} \tag{21}$$

The numerical tests were implemented on an Intel® Core™ i7-4470 computer (3.4 GHz) with 16 gigabytes of RAM running under Windows 7. A series of preliminary tests were conducted in order to determine the appropriate ACO parameter as following which led to reach a good performance in this special problem:

- Maximum number of iterations = 1000;

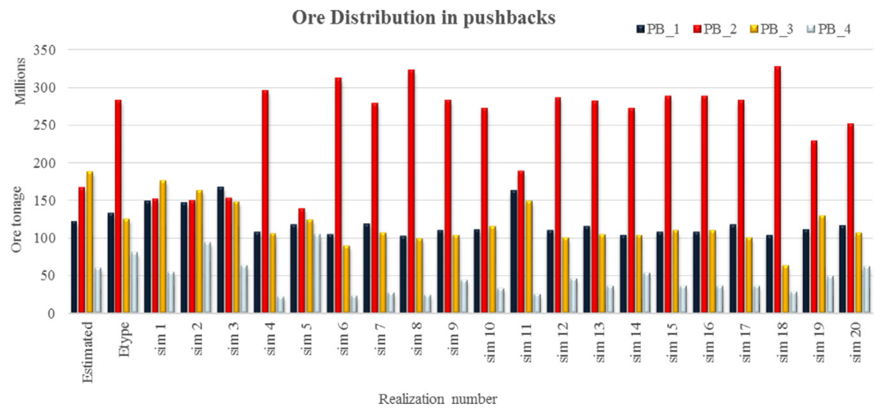

Fig. 6. Ore distribution in the initial designed pushbacks for all simulations.

Table 2
Different geotechnical regions.

Region	Azimuth (deg)	Slop angle (deg)
1	0	38
2	90	38
3	130	30
4	235	30
5	275	36

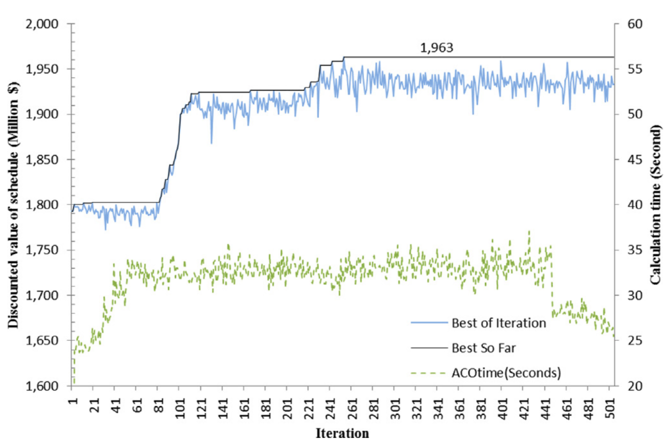

Fig. 7. Efficiency of the ACO-SRB on yearly production planning with *Prob* = 100%.

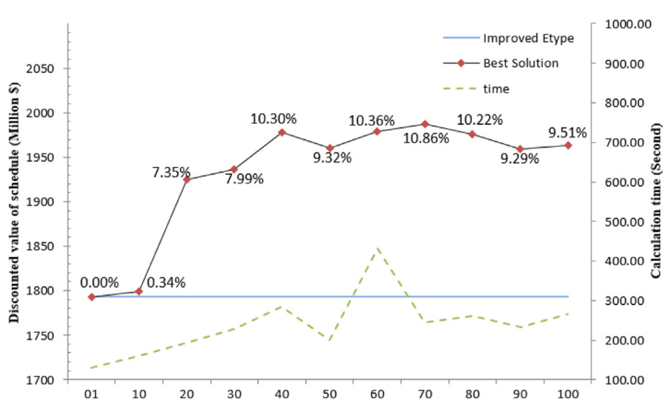

Fig. 8. Efficiency of the ACO-SRB on yearly production planning with different probabilities.

- Maximum number of sequential non-improving iterations = 250;
- Number of the ants = 10;

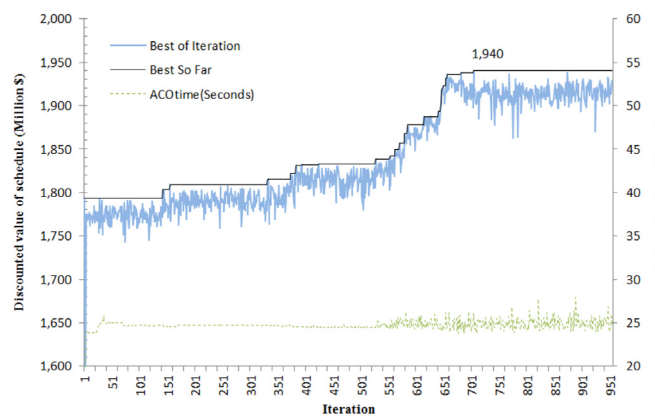

Fig. 9. Efficiency of the ACO-MRB on yearly production planning.

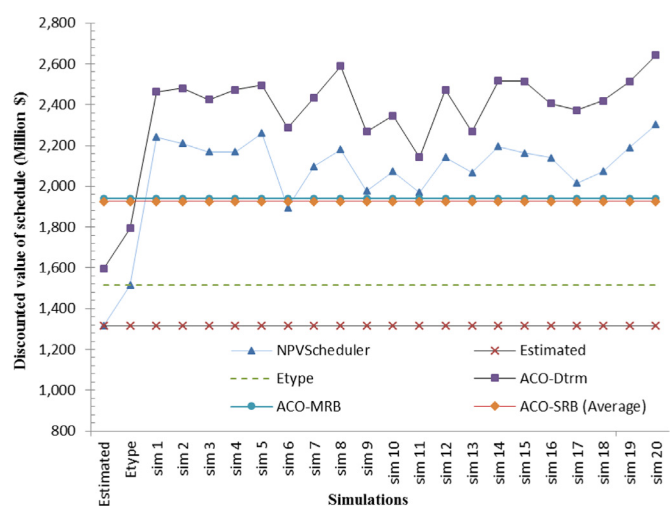

Fig. 10. Comparison between traditional and proposed approaches.

- Global evaporation rate (ρ) = 0.1;
- Local evaporation rate (ξ) = 0.15;
- The upper and lower perturbation distance is considered as 3 and 0, respectively;
- α and β are set to 1 and 0.15, respectively;
- C_{prob} = 5;
- Initial pheromone (τ_o) = 0.1;
- Minimum pheromone (τ_{min}) = 0.001;
- Pseudorandom factor (q_0) = 0.7.

Deterministic version of ACO (ACO-Dtrm) was applied on 21 initial sub-optimal scheduling designs provided by a traditional

Table 3
Comparison between ACO-MRB and ACO-SRB.

Improvement (%)	ACO-MRB	ACO-SRB (%)										
		100	90	80	70	60	50	40	30	20	10	1
Related to Estimated	47.40	49.18	48.88	50.15	51.01	50.34	48.92	50.25	47.11	46.24	36.69	36.23
Related to EType	28.04	29.58	29.32	30.43	31.17	30.59	29.36	30.51	27.79	27.03	18.74	18.33
Related to improved EType	8.20	9.51	9.29	10.22	10.86	10.36	9.32	10.30	7.99	7.35	0.34	0.00
Time (Hour)	6.56	4.43	3.88	4.34	4.09	7.20	3.34	4.73	3.81	3.23	2.69	2.16

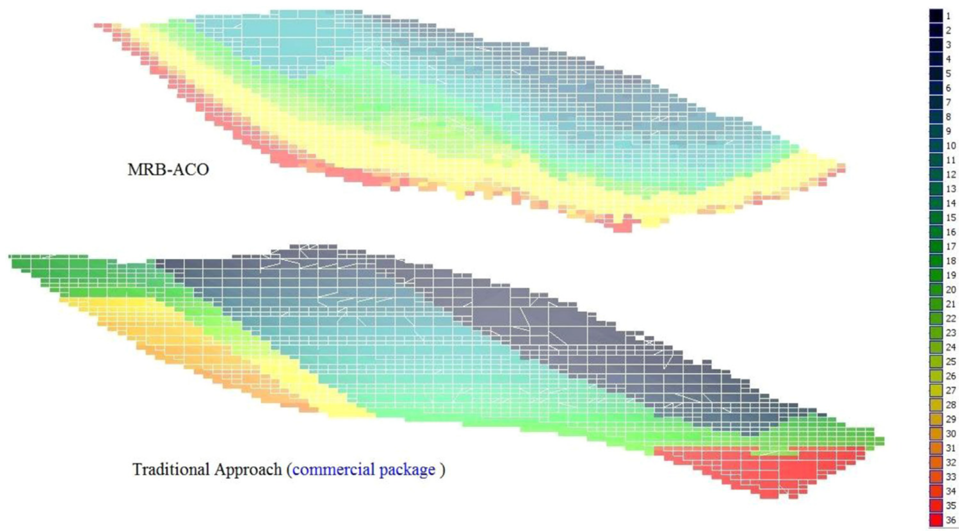

Fig. 11. North–South cross sections of obtained solution by traditional and MRB-ACO approaches.

method for 20 geological simulations and EType block model in order to create the risk block model and initial solution. Both proposed strategies (ACO-SRB and ACO-MRB) are test on the resulted initial solution. In terms of improvement in objective function values (evaluating the fitness values obtained before and after modifying the initial solution), ACO-SRB indicates an increase of 7.78% and ACO-MRB indicates an increase of 8.20%. In ACO-MRB, it takes 390 min to obtain the near optimum solution while 240 min has been spent by ACO-SRB; however, it is noted that it takes,250 and 130 min to converge on a scheduling design within 10% of the final solution respectively by ACO-MRB and ACO-SRB. Figs. 7–9 show the occurred perturbations mentioned above using both proposed strategies. It can be observed that ACO-SRB leads to better results in some cases and appears to be more effective in cases with higher value of *Prob*. However, it seems that ACO-SRB has more potential of falling into local optima and fails to explore the whole domain of solutions. By the way, it is a good practice when companies look forward to planning with fixed risk or probability. On the other hand, ACO-MRB generates solutions from the unexplored or less explored feasible solution space with low *Prob* and thus has a higher chance to find better solutions.

A general comparison based on obtained fitness value has been done between the provided scheduling designs using traditional approach and the improved ones by ACO-Dtrm, ACO-SRB and ACO-MRB for the estimated, all simulations and EType block models. Fig. 10 shows that all the three version of ACO are able to substantially increase the fitness value when compared to the original scheduling designs. ACO-Dtrm led to increase the fitness value of the provided initial solution for the estimated, all simulations and EType block models by 21.1%, 14.14% and 18.33%, respectively.

A more detailed comparison between ACO-SRB and ACO-MRB is illustrated in Table 3 in order to show their efficiency in providing final solution based on the initial solution created by traditional approach for estimated and EType block model and also the created one based on the initial solution for EType block model improved by ACO-Dtrm approach. Generally, this evaluation demonstrates that both strategies lead to improve the feasible sub-optimal solutions in term of NPV and also minimize the risk of deviating from the production targets. The clear outcome is that ACO-MRB is more effective than ACO-SRB (in 40% of the cases) especially when the values of *Prob* are low. For example ACO-SRB increases the fitness value of the improved solution for EType block model by 10.21% when *Prob* = 40~80% while its improvement is very little for *Prob* ≤ 20. Considering all *Prob* = 1~100%, an average improvement of 7.78% was achieved in the fitness value when ACO-SRB is used, which is less than that of ACO-MRB as 8.20%. Fig. 11 shows the section views of the initial solution provided by traditional approach and final schedule obtained by MRB-ACO.

6. Conclusions

A new stochastic optimization algorithm based on Ant Colony Optimization approach, which considers geological uncertainty in open pit mine production scheduling problem, is presented and successfully applied to production scheduling at the Sungun copper mine in the northwest of Iran. Two different strategies were developed based on a single predefined probability value (*Prob*) and multiple probability values ($Prob_n^t$), in order to improve the initial solutions that created by deterministic version of ACO procedure.

Results demonstrate the abilities of the stochastic approach to create a single schedule and control the risk of deviating from production targets which diminishes an overall project risk and also increase the project value. A comparison between two strategies and traditional approach illustrates that the multiple probability based strategy produces better results, however, the single predefined probability based approach is more practical in situations with high degree of flexibility. For instance, Estimated, EType, and improved EType solutions were improved by 47.40%, 28.04%, and 8.20%, respectively by the second strategy.

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