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Case study

Formal representation of 3D structural geological models

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ABSTRACT

The development and widespread application of geological modeling methods has increased demands for the integration and sharing services of three dimensional (3D) geological data. However, theoretical research in the field of geological information sciences is limited despite the widespread use of Geographic Information Systems (GIS) in geology. In particular, fundamental research on the formal representations and standardized spatial descriptions of 3D structural models is required. This is necessary for accurate understanding and further applications of geological data in 3D space. In this paper, we propose a formal representation method for 3D structural models using the theory of point set topology, which produces a mathematical definition for the major types of geological objects. The spatial relationships between geologic boundaries, structures, and units are explained in detail using the 9-intersection model. Reasonable conditions for describing the topological space of 3D structural models are also provided. The results from this study can be used as potential support for the standardized representation and spatial quality evaluation of 3D structural models, as well as for specific needs related to model-based management, query, and analysis.

1. Introduction

In recent years, 3D geological modeling methods have evolved, and have been widely applied in various geological fields, such as petroleum and mining exploration, geological surveys, and research in academia (Lemon and Jones, 2003; Fernández et al., 2004; Perrin et al., 2005; Kaufman and Martin, 2008; Zanchi et al., 2009). In addition, 3D geological modeling has also become an effective way of providing a quantitative digital representation of the Earth's subsurface space. Most geological modeling systems already provide extensive capabilities of 3D structural and attribute modeling and such systems are usually based on topological models and mathematical interpolation (Mallet, 2002; Berg et al., 2011).

In addition to academic research and industrial applications, many countries are currently engaged in ambitious programs to build a Spatial Data Infrastructure (SDI). This will enable the integration and sharing of spatial information including 3D geological data (USGS, 2007; Berg et al., 2011). There are some ongoing plans to develop 3D geological mapping and modeling for the integration of domestic spatial data, such as the Glass Earth plan (CSIRO, 2012), the Digital Geo-Science Spatial Model project (DGSM) published by the British Geological Survey (Smith, 2005).

Continuous improvements in geological modeling methods and the strong demand for geodata integration and services have led to the emergence of an increasing number of problems. These issues go beyond modeling methods and visualization technologies, but rather, involve various issues mainly dealing with modeling results i.e., quality evaluation, data exchange and sharing, effective management (databases or file systems), and specialized applications (query and analysis). Therefore, 3D geological modeling systems need to evolve into 3D geological information systems (Apel, 2006); and the most important and fundamental problem that needs to be addressed is determining rational representation methods to describe a 3D geological model. Currently, in terms of research on geological data exchange and sharing, several international standards have been proposed to explore the description and definition of geological data structures and document formats, e.g., GeoSciML (Sen and Duffy, 2005), RESQML (King et al., 2012). These data exchange methods involve simple data models for a single geological object, which lacks the holistic 3D model representation and comprehensive descriptions of spatial relationships between geological objects.

The objective of this paper is to provide a formal, theoretical definition of 3D structural models and geological objects, which is the foundation for analyzing relationships between geological objects and the reasonable spatial conditions behind the 3D structural model.

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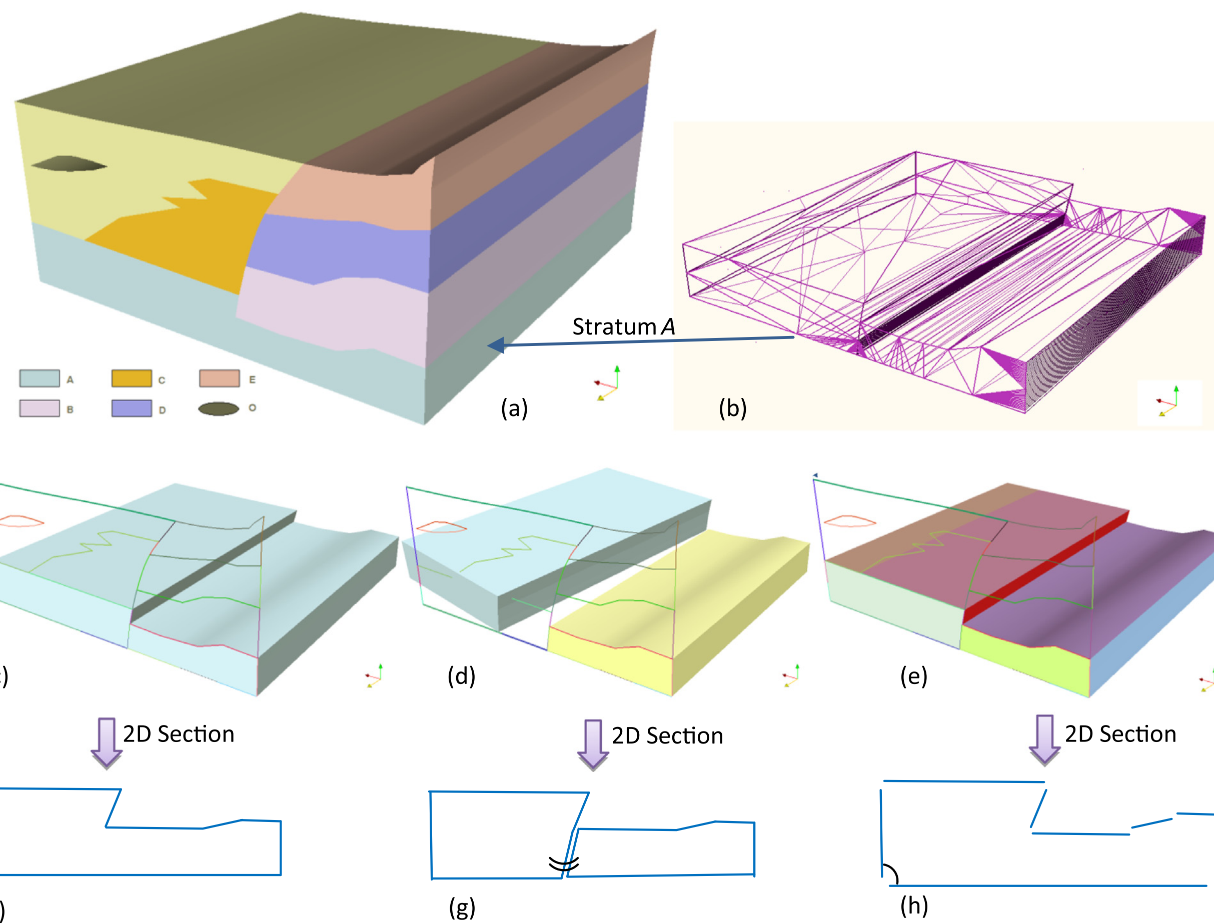


Fig. 1. Topological representation methods for a 3D structural model. (a) Snapshot of a 3D geological model; (b) Wireframe display of stratum *A* by triangulated surfaces; (c) and (f) The first type: stratum *A* is represented by a fully-closed surface; (d) and (g) The second type: stratum *A* is split into two components by the fault surface; and (e) and (h) The third type: stratum *A* is formed by combining various surfaces at the boundary.

2. Related works

There are two approaches to spatial phenomena in GIS (Schneider, 1997): entity-oriented/feature-based method and space-oriented/position-based method. These have developed into a complete theoretical system for representing spatial data and defining data types. The former uses a vector data model, while the latter uses a raster data model. More studies have been conducted on the former because of its advantage of being able to create abstract descriptions of geo-spatial relationships between objects. Numerous entity-oriented 3D spatial data models, as part of 3D GIS, have been proposed and a review can be found in Zlatanova's works (Zlatanova et al. 2004).

There are also two approaches to represent geological phenomena: geoobject-based models and grid/voxel-based models (Apel, 2006). The former involves the construction of a surface-based model to express the characteristics and relationships of geological structures and strata formations (Lienhardt 1994; Lévy and Mallet, 1999; Sprague and Kemp, 2005; Wu et al., 2005; Caumon et al., 2009; Caumon, 2010), known as a structural model. The latter involves the construction of a grid/voxel-based model, which uses mathematical interpolation methods to generate 3D distribution characteristics representing a geological attribute field (Mallet, 2002; Royer, 2004), known as an attribute model. The corresponding 3D geological data models have been adapted to portray complex geoobjects and are closely related to specific application requirements and different geological modeling methods. There are some studies conducted to define geomodes, but the focus was on building and editing geomodels, such as the Sealed Geological Models (Caumon et al., 2004) and G-maps (Lienhardt, 1994; Lévy and Mallet, 1999; Mallet, 2002). A summary of these models can be found in Wu and Caumon's works (Wu, 2004; Caumon et al., 2009; Caumon, 2010).

The essence of geoobject-based representation involves spatial partitioning under specific conditions by constructing a mapping relationship between the segmented spaces and geoobjects. During the process of 3D structural modeling, spatial partitioning of the subsurface space is generally carried out on the basis of various geological boundary conditions, such as the structural and stratigraphic/lithological boundaries. From this, the geoobjects and their spatial relationships using the 9-intersection model (9IM) can be defined resting on point set theory and point set topology (Egenhofer and Herring, 1990; Egenhofer and Franzosa, 1991; Schneider, 1997). A formal representation of data models is necessary for a better understanding of the complexity of geoobjects and their corresponding spatial operations, and represents a first step towards standardizing spatial data types and topological encoding of spatial relationships. Schetselaar and Kemp (2006) extract the subset essential to spatial reasoning in geology from inventories of the topological relationships existing in $\mathbb{R}^2$ and $\mathbb{R}^3$. They defined constraints for the 9-intersection of two geoobjects and enumerated the various topological encoding of spatial relationships to support geological modeling. There are two main constraints: the observation region is fully partitioned and the geomodel is essentially the treatment of simplified abstractions of geoobjects represented by simple points, simple lines, simple surfaces and simple bodies. However, geoobjects are complex and may have several components or holes, the described methods are inadequate abstractions for real applications. Moreover, clarity and consistency of spatial relationships between geoobjects in geomodel space is particularly important due to the range of different users, especially for data exchange and sharing services.

3. Topological representation for 3D structural models

In various 3D structural modeling methods, the topology and geometry of subsurface objects are constructed under specific partitioning constraints using surface-based boundary representation methods. Due to differences in recognition and concerns about geological structures, there are three possible types of topological representation for 3D structural models:

(i) Complete geometric objects are used to represent geoobjects. For example, a closed geometric surface represents a geological body and a normal geometric surface represents a fault (Fig. 1c and f).
(ii) Complete geometric objects are similarly used to represent geoobjects. However, when a geological body is partitioned by a geological structure (e.g. a fault), the resultant body is formed by combining various strata or geometric objects (Fig. 1d and g).
(iii) Various interfaces (such as that of horizons or fault faces) are used to form the boundaries of geoobjects that are mutually exclusive and do not intersect (Fig. 1e and h).

Among these types, the third type of presentation method is much closer to the construction process of 3D structural models. The core process of structural modeling involves partition of the subsurface space by boundary surfaces. The boundary surfaces are created from the chosen conditions, e.g., structures or stratigraphy. The resultant surfaces that belong to the same geoobject are then combined closely to form a complete geoobject. Hence, the first and second types of presentation methods are can be considered as further integrations of the third type, and are designed to facilitate the exclusive visualization and analysis of a separate geoobject. Moreover, the third type is more effective in terms of describing the spatial relationship between geoobjects. These methods emphasize both the geometric representation of each geoobject and descriptions of relationships between geoobjects.

4. Formal representation

4.1. Integration of a structural model

A structural model is also considered to be a spatial data model. The representation of a spatial model should focus on three aspects (Zlatanova et al., 2004): spatial partitioning, supported objects and primitives, and constructive rules.

As mentioned in the previous section, a 3D structural model assumes that the full partition of the subsurface space and the two classical types of geoobjects can be distinguished: (i) geological boundaries as partitioning conditions, which represent one kind of geological structural object with a geometric surface, and (ii) geological bodies, which are space filling, and represent one kind of geological unit with a geometric volume. The rules for

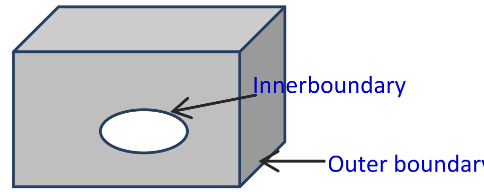


Fig. 2. Geoobject (3-cell) possessing inner and outer boundaries.

construction and the definitions of the interrelationships between objects, and so forth, are discussed in the next section.

On the same temporal and observational scale, a 3D structural model is formally defined as:

$$3\text{–GeoModel} = \{Gp, O, R\} \tag{1}$$

where, Gp represents the spatial geometry data, R represents the relationship between geoobjects, and $O=\{U, S\}$ represents geoobjects that include geological units (U) and, geological structures (S).

The geological boundary object is a special type of geological structure as part of the term S and is an abstract description of all boundary conditions for spatial partitioning. In an actual model, such an object may be a part of a horizon or a fault.

4.2. Formal definitions of geoobjects

The main theory for the formal definition of spatial objects and descriptions of their relationships is that of point set topology (Egenhofer and Herring, 1990; Egenhofer and Franzosa, 1991). In this study, we consider that geologic spaces are discretized into open sets of various dimensions, referred to as simple n-cells, where $n=dim(A)$ denotes the geometrical dimension of a cell A. For each cell A, A^+ and A^o are the exterior and interior of A, respectively, while ∂A is the boundary of A, and A^- is the closure containing A, with $A^-=\partial A \bigcup A^o$. The n-cells are essentially formal abstractions of simple geometric structures of spatial objects like simple surface (2-cell) and simple body (3-cell) without holes topologically equivalent to an n-ball without border. However, the simple geometric structures are insufficient to cope with the variety and complexity of geoobjects. These objects may have several components or holes, e.g., a *lenticular body* (Fig. 2).

The following Definitions 1 and 2 are needed for complex spatial objects and the main extensions for n-cells is to cope with the separations of the exterior (holes), using the concepts of complex spatial objects and topological predicates defined by Schneider and Thomas (2006). The simple n-cells and extensions of n-cells are collectively referred to as n-cells in this paper.

Definition 1. Let $\{F_0,\ldots,F_m\}$ be a set of simple n-cells in $\mathbb{R}^3$ ($n=2$ or 3), the regular set $F = F_0 - \bigcup_{i=1}^{m} F_i^o$ is called an n-cell with holes and $F_1,\ldots,F_m$ are called holes $\forall\, i, j > 1$ and $i \neq j$, F_i disjoints F_j and F_0 contains F_i and F_j. Then the boundary of F is given as $\partial F = \bigcup_{i=0}^{m} \partial F_i$ and the interior of F is given as $F^o = F_0^o - \bigcup_{i=1}^{m} F_i$.

According to Definition 1, an n-cell ($n=2$ or 3) is spatial connected where any two points in it can be joined by a path. Connectedness is one of the principal topological properties. A connected object cannot be represented as the union of two or more parts.

Definition 2. Let A be a simple 2-cell and F be a simple 3-cell. A is defined as a closed cell only if $A = \partial F$. Then the boundary of A is given as $\partial A = \varnothing$.

According to Definition 2, a closed line or surface has no borders.

Definition 3. Let $D=\{\mathbf{C},\mathbf{U}\}$ be a set of n-cells in $\mathbb{R}^3$ ($n=2,3$), where $\mathbf{C}$ is a set of 2-cells and where $\mathbf{U}$ is a set of 3-cells. D is called a 3D spatial partition in $\mathbb{R}^3$, only if the following constraints are observed:

(1) $\forall\, C_1, C_2 \in \mathbf{C}$, then $dim(C_1) = dim(C_2)=2$, C_1 and C_2 do not overlap and intersect, that is,

$$C_1^o \cap C_2^o = \varnothing$$
$$\partial C_1 \cap C_2^o = \varnothing$$
$$\partial C_2 \cap C_1^o = \varnothing$$

(2) $\forall\, U_1, U_2 \in \mathbf{U}$, then $dim(U_1) = dim(U_2)=3$, and U_1 and U_2 do not overlap and intersect, that is,

$$U_1^o \cap U_2^o = \varnothing$$
$$\partial U_1 \cap U_2^o = \varnothing$$
$$\partial U_2 \cap U_1^o = \varnothing$$

(3) Let $U_1 \in \mathbf{U}$, $C_1 \in \mathbf{C}$, then C_1 and U_1 do not overlap, that is,
if $C_1^o \cap U_1^o \neq \varnothing \Rightarrow C_1^o \cap U_1^+ = \varnothing$

(4) $\forall\, U_1 \in \mathbf{U}$, then $\partial U_1 \subseteq \mathbf{C}$ and if $\exists C_1 \in \mathbf{C}$ and $C_1^o \cap \partial U_1 \neq \varnothing$, $C_1 \in \partial U_1$, that is:
if $C_1^o \cap \partial U_1 \neq \varnothing \Longrightarrow C_1^o \cap U_1^+ = \varnothing$ and $C_1^o \cap U_1^o = \varnothing$

The first three conditions place a constraint on the 3D topological space having only two basic geometric types: the 2- and 3-cells. The former represents the basic partition surface and the latter represents the volume filling in subspace. The fourth condition shows that the boundaries of an object in $\mathbf{U}$ are closely composed of existing objects in $\mathbf{C}$ and that this is a combinational relationship. Let $C_1 \in U_1 \cap U_2 \neq \varnothing$, then $C_1 \in \partial U_1$ and $C_1 \in \partial U_2$.

Definition 4. 3-GeoModel$=\{O, R\}$ is a 3D structural model defined on the spatial partition $D=\{\mathbf{C},\mathbf{U}\}$, where the geoobject $o\in O$, the following definitions are given:

(1) $\forall\, o \in \mathbf{C}$, o is called the *GeologicBoundary*;
(2) $\forall\, o \in \mathbf{U}$, o is called the *basicGeologicUnit*;
(3) If $o = \bigcup_{C_i\in \mathbf{C}} C_i$, o is called the *compositionGeologicStructure*.

The *GeologicBoundary* and *compositionGeologicStructure* are collectively referred to as the *GeologicStructure*, whose $dim(o)=2$. The *basicGeologicUnit* is a simple type of *GeologicUnit*, whose $dim(o)=3$. These definitions and their relationships are similar to concepts in GeoSciML (Sen and Duffy, 2005). In a 3D structural model, n-cell represents the geometric characteristics of the geoobjects i.e., the geometric property. As shown in Fig. 3, the geological structures and units are presented by 2- and 3-cells, respectively.

Definitions 4 is the formal definition of the basic types of geological objects and, includes three basic types. A *basicGeologicUnit* is composed of both the interior storing 3-cell and a set of

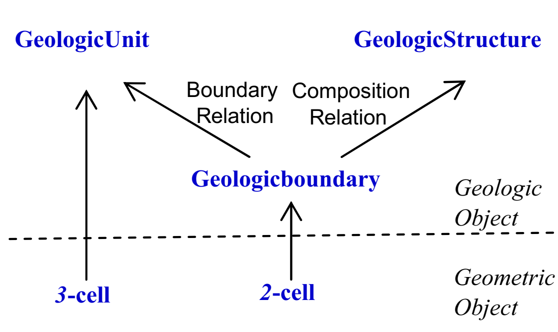


Fig. 3. Relationships between geoobjects.

boundaries provided by *GeologicBoundary*. A *GeologicStructure* is also composed of a set of *GeologicBoundaries*. Therefore, a *GeologicBoundary* provides an abstract description of all the conditions for spatial partitioning. These definitions are consistent with the existing construction process and representations of the 3D structural model mentioned in Section 3.

Consequently, the spatial relationships between the three basic types of geoobjects and the reasonable conditions behind the 3D structural model can be analyzed.

5. Spatial relationships based on the 9-intersection model

Assume A, B are two geo-objects in a 3-GeoModel. The 9-intersection model (9IM) describes the topological relationships between A and B and can be concisely represented by the matrix R_9 (Egenhofer and Herring, 1990):

$$R_9(A,B) = \begin{pmatrix} A^o \cap B^o & A^o \cap \partial B & A^o \cap B^+ \\ \partial A \cap B^o & \partial A \cap \partial B & \partial A \cap B^+ \\ A^+ \cap B^o & A^+ \cap \partial B & A^+ \cap B^+ \end{pmatrix} \tag{2}$$

The basic criterion of 9IM for distinguishing different relationships is the detection of empty and non-empty intersections. Eight relationships are possible, and they are given the semantic names: *Disjoint*, *Meet*, *Contain*, *Inside*, *Cover*, *Coverby*, *Equal* and *Overlap*.

Property 1. Let A and B are two different non-empty geoobjects, then:

$$A^+ \cap B^+ \neq \varnothing$$

Proof: According to Definition 1, the 3-GeoModel space is defined as G, which is in turn defined as a topological space in $\mathbb{R}^3$. For non-empty $A\subseteq G$, non-empty $B\subseteq G$, and $A^- \cup B^- \subseteq G$. Meanwhile, $A^+\cap B^+=(G-A^-)\cap(G-B^-)=G-A^- \cup B^- + A^-\cap B^-$.

If $A^+\cap B^+=\varnothing$ and $A^-\cap B^-\neq\varnothing$, then $A^- \cup B^-=G+A^-\cap B^- \supset G$, which contradicts $A^- \cup B^- \subseteq G$;

If $A^+ \cap B^+=\varnothing$ and $A^-\cap B^-=\varnothing$, then $A^- \cup B^-=G$, $A^-=G$, and $B^-=\varnothing$ (or $B^-=G$ and $A^-=\varnothing$), which contradicts $B^-\neq\varnothing$.

Thus, $A^+\cap B^+\neq\varnothing$.

Property 2. If C is a *compositionGeologicStructure*, C_i is the *GeologicBoundary* inside C. Then, C satisfies the following: $C = \bigcup_{C_i\subseteq C} C_i => \partial C = \bigcup_{C_i\subseteq C \wedge \partial C_i \cap C^o = \varnothing} \partial C_i$

Proof: From Definitions 3 and 4, if $C = \cup_{C_i\subseteq C} C_i$, then $C^o \subseteq \{C_i^o + \partial C_i\}$, $\partial C \subseteq \{\partial C_i\}$. Since $\partial C \cap C^o = \varnothing$, thus, $\partial C = \cup_{C_i\subseteq C\wedge \partial C_i \cap C^o=\varnothing} \partial C_i$.

This shows that the boundaries of a *compositionGeologicStructure* are composed of the boundaries of all the *GeologicBoundary* inside the object, but these boundaries cannot be inside or overlap the *GeologicStructure*.

The following discussion focuses on the relationships between the following geoobjects: *GeologicBoundary, GeologicUnit,* and *GeologicStructure.*

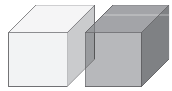
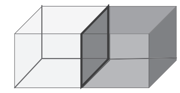
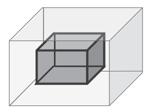
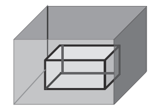
5.1. Relationships between basicGeologicUnits

According to Definition 3, *basicGeologicUnits* are fundamental geoobjects for filling in the geologic space and do not overlap or intersect. Hence, two non-empty different *basicGeologicUnits* given as A and B must satisfy the following relationships:

$A^+ \cap B^+ \neq \varnothing$

$A^o \cap B^o = \varnothing$

Table 1
The 4 topological relationships between two different *basicGeologicUnits*.

No.	Matrix R_9	Semantics	Graphic Description	No.	Matrix R_9	Semantics	Graphic Description
1	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	*Disjoint*		**2**	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	*Meet*	
3	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$	*Meet (B's boundary equal)*		**4**	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$	*Meet (A's boundary equal)*	

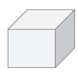
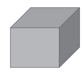
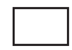
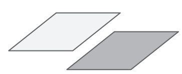
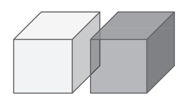
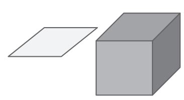
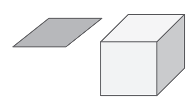
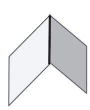
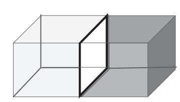
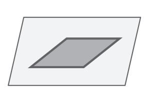
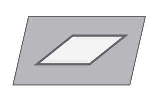
A  *B*  Adjacent boundary 

Table 2
The 8 topological relationships between two different *GeologicBoundaries*.

No.	Matrix R_9	Semantics	Graphic Description	No.	Matrix R_9	Semantics	Graphic Description
1	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	*Disjoint*		**2**	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$	*Disjoint (both A and B are closed)*	
3	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$	*Disjoint (B is closed)*		**4**	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$	*Disjoint (A is closed)*	
5	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	*Meet*		**6**	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$	*Meet*	
7	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$	*Meet (B's boundary equal)*		**8**	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$	*Meet (A's boundary equal)*	

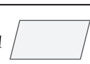
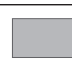
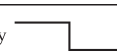
A  *B*  Adjacent boundary 

Table 3
The 2 cases of GeologicBoundary *A* equal to *B*.

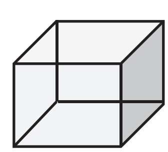
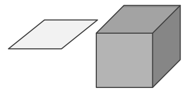
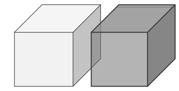
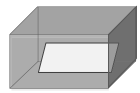
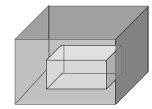
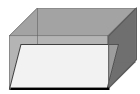
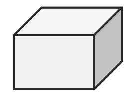
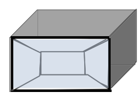
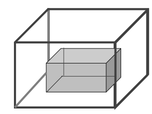
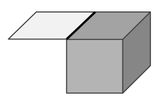
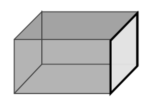
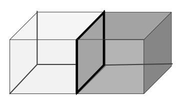
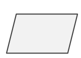
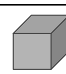
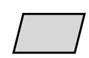
No.	Matrix R_9	Graphic Description	No.	Matrix R_9	Graphic Description
1	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		**2**	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	

Table 4
The 11 topological relationships between *basicGeologicUnit* and *GeologicBoundary*.

No.	Matrix R_9	Semantics	Graphic Description	No.	Matrix R_9	Semantics	Graphic Description
1	$\begin{pmatrix}0&0&1\\0&0&1\\1&1&1\end{pmatrix}$	*Disjoint*		2	$\begin{pmatrix}0&0&1\\0&0&0\\1&1&1\end{pmatrix}$	*Disjoint (A is closed)*	
3	$\begin{pmatrix}1&0&0\\1&0&0\\1&1&1\end{pmatrix}$	*Inside*		4	$\begin{pmatrix}1&0&0\\0&0&0\\1&1&1\end{pmatrix}$	*Inside (A is closed)*	
5	$\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}$	*Coverby (Inside)*		6	$\begin{pmatrix}0&1&0\\0&0&0\\1&0&1\end{pmatrix}$	*Meet (A is closed)*	
7	$\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix}$	*Coverby (Inside)*		8	$\begin{pmatrix}0&1&0\\0&0&0\\1&1&1\end{pmatrix}$	*Meet (A is closed)*	
9	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&1\end{pmatrix}$	*Meet*		10	$\begin{pmatrix}0&1&0\\0&1&0\\1&1&1\end{pmatrix}$	*Meet*	
11	$\begin{pmatrix}0&0&1\\0&1&0\\1&1&1\end{pmatrix}$	*Meet*					

A  *B*  Adjacent boundary 

$\partial A \cap B^o = \varnothing$

$A^o \cap \partial B = \varnothing$

This gives rise to four types of spatial relationships between *A* and *B* (Table 1).

5.2. Relationships between GeologicBoundaries

According to Definition 3, *GeologicBoundaries* are considered as spatial objects for space partitioning and do not overlap or intersect. 8 types of spatial relationships can be identified between two non-empty different *GeologicBoundaries* given as *A* and *B* (Table 2). Two cases can be identified between two equal *A* and *B* (Table 3).

5.3. Relationships between basicGeologicUnit and GeologicBoundary

Definitions 3 and 4 indicate that boundaries of a *basicGeologicUnit* should be composed of a set of *GeologicBoundaries*. The following conditions can be given to describe the relationships between a *GeologicBoundary A* and a *basicGeologicUnit B*.

- *A* is inside or outside of *B*. If *A* is inside *B*, *A* is never a boundary of any *basicGeologicUnit* and does not indeed participate in space partitioning and *A* is also called a hanging object.
- If $A = \partial B$, that is, *A* is the boundary of *B*. Then, $\partial B \cap A^o \neq \varnothing$, $\partial B \cap A^+ = \varnothing$ and $\partial A \cap \partial B = \varnothing$ (*A* must be closed).
- If $A \in \partial B$, that is, *A* is part of boundaries of *B*. Then, $\partial B \cap A^o \neq \varnothing$, $\partial B \cap A^+ \neq \varnothing$ and $\partial A \cap \partial B = \varnothing$ if *A* is closed or $\partial A \cap \partial B \neq \varnothing$ if *A* is not closed.
- *A* meets *B*. Then, $\partial B \cap A^o = \varnothing$ and $\partial A \cap \partial B \neq \varnothing$.

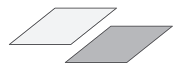
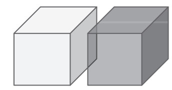
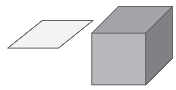
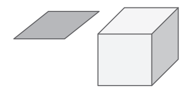
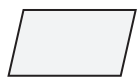
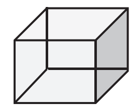
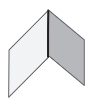
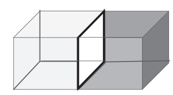
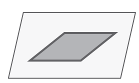
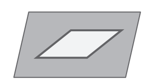
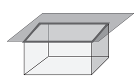
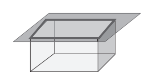
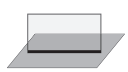
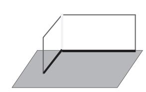
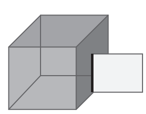
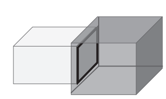
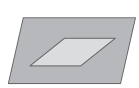
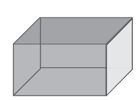
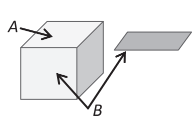
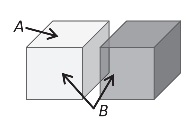
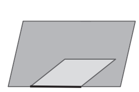
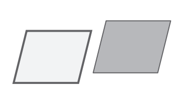
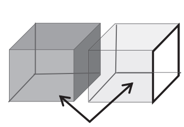
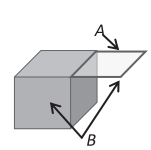
This gives rise to 11 types of spatial relationships between *A* and *B* (Table 4).

5.4. Relationships between GeologicBoundary and GeologicStructure

According to Definition 3, A *GeologicBoundary* is a part that forms the *GeologicStructure* in a part–whole combinational relationship. 24 topological relationships including *Disjoint*, *Meet*, *Inside*, *Coverby* and *Equal* can occur between a *GeologicBoundary A* and a *GeologicStructure B*, shown in Table 5. And, the following conditions are observed:

- If *A* is inside *B or B* contains *A*, then $B^o \cap A^o \neq \varnothing$, $B^o \cap A^+ \neq \varnothing$, $B^+ \cap A^o = \varnothing$ and $\partial A \cap \partial B = \varnothing$.
- If *A* meets *B* and *B* is a *compositionGeologicStructure*, then $\partial A \cap B^o \neq \varnothing$ can occur.
- If *A* is covered by *B*, then $B^o \cap A^o \neq \varnothing$, $B^o \cap A^+ \neq \varnothing$, $B^+ \cap A^o = \varnothing$ and $\partial A \cap \partial B \neq \varnothing$.

Table 5
The 24 topological relationships between *GeologicBoundary* and *GeologicStructure*.

No.	Matrix R_9	Semantics	Graphic Description	No.	Matrix R_9	Semantics	Graphic Description
1	$\begin{pmatrix}0&0&1\\0&0&1\\1&1&1\end{pmatrix}$	*Disjoint*		**2**	$\begin{pmatrix}0&0&1\\0&0&0\\1&0&1\end{pmatrix}$	*Disjoint (both A and B are closed)*	
3	$\begin{pmatrix}0&0&1\\0&0&1\\1&0&1\end{pmatrix}$	*Disjoint (B is closed)*		**4**	$\begin{pmatrix}0&0&1\\0&0&0\\1&1&1\end{pmatrix}$	*Disjoint (A is closed)*	
5	$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	Equal		**6**	$\begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix}$	Equal *(both A and B are closed)*	
7	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&1\end{pmatrix}$	*Meet*		**8**	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&1\end{pmatrix}$	*Meet*	
9	$\begin{pmatrix}0&0&1\\0&1&1\\1&0&1\end{pmatrix}$	*Meet (B's boundary equal)*		**10**	$\begin{pmatrix}0&0&1\\0&1&0\\1&1&1\end{pmatrix}$	*Meet (A's boundary equal)*	
11	$\begin{pmatrix}0&0&1\\1&1&0\\1&1&1\end{pmatrix}$	*Meet*		**12**	$\begin{pmatrix}0&0&1\\1&0&0\\1&1&1\end{pmatrix}$	*Meet*	
13	$\begin{pmatrix}0&0&1\\1&0&1\\1&1&1\end{pmatrix}$	*Meet*		**14**	$\begin{pmatrix}0&0&1\\1&1&1\\1&1&1\end{pmatrix}$	*Meet*	
15	$\begin{pmatrix}0&0&1\\1&0&1\\1&0&1\end{pmatrix}$	*Meet (B is closed)*		**16**	$\begin{pmatrix}0&0&1\\1&0&0\\1&0&1\end{pmatrix}$	*Meet (B is closed)*	
17	$\begin{pmatrix}1&0&0\\1&0&0\\1&1&1\end{pmatrix}$	*Inside*		**18**	$\begin{pmatrix}1&0&0\\1&0&0\\1&0&1\end{pmatrix}$	*Inside (B is closed)*	
19	$\begin{pmatrix}1&0&0\\0&0&0\\1&1&1\end{pmatrix}$	*Inside (A is closed)*		**20**	$\begin{pmatrix}1&0&0\\0&0&0\\1&0&1\end{pmatrix}$	*Inside (both A and B are closed)*	
21	$\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}$	*Coverby*		**22**	$\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix}$	*Coverby*	
23	$\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix}$	*Coverby*		**24**	$\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix}$	*Coverby*	

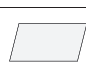
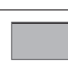
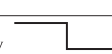
A  *B*  Adjacent boundary 

Table 6
The 82 topological relationships between *GeologicStructure* and *GeologicStructure*.

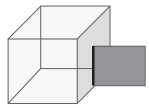
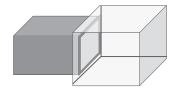
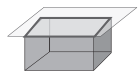
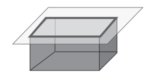
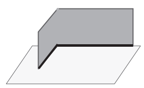
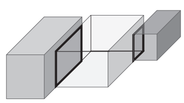
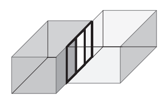
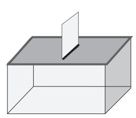
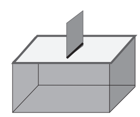
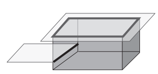
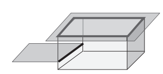
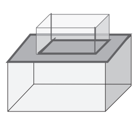
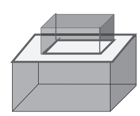
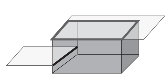
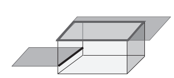
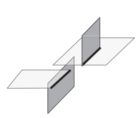
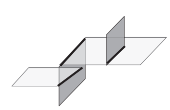
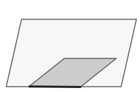
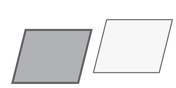
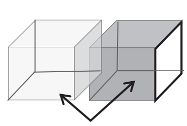
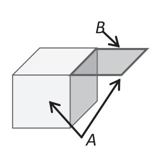
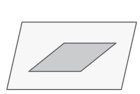
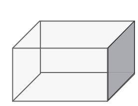
No.	Matrix R_9	Semantics	Graphic Description	No.	Matrix R_9	Semantics	Graphic Description
25	$\begin{pmatrix}0&1&1\\0&0&0\\1&1&1\end{pmatrix}$	*Meet (A is closed)*		**26**	$\begin{pmatrix}0&1&1\\0&0&0\\1&0&1\end{pmatrix}$	*Meet (A is closed)*	
27	$\begin{pmatrix}0&1&1\\0&1&1\\1&0&1\end{pmatrix}$	*Meet*		**28**	$\begin{pmatrix}0&1&1\\0&0&1\\1&0&1\end{pmatrix}$	*Meet*	
29	$\begin{pmatrix}0&1&1\\0&1&1\\1&1&1\end{pmatrix}$	*Meet*		**30**	$\begin{pmatrix}0&1&1\\0&0&1\\1&1&1\end{pmatrix}$	*Meet*	
31	$\begin{pmatrix}0&1&1\\1&0&0\\1&0&1\end{pmatrix}$	*Meet*		**32**	$\begin{pmatrix}0&1&1\\1&1&0\\1&0&1\end{pmatrix}$	*Meet*	
33	$\begin{pmatrix}0&0&1\\1&1&1\\1&0&1\end{pmatrix}$	*Meet*		**34**	$\begin{pmatrix}0&1&1\\0&1&0\\1&1&1\end{pmatrix}$	*Meet*	
35	$\begin{pmatrix}0&1&1\\1&0&1\\1&0&1\end{pmatrix}$	*Meet*		**36**	$\begin{pmatrix}0&1&1\\1&0&0\\1&1&1\end{pmatrix}$	*Meet*	
37	$\begin{pmatrix}0&0&1\\1&1&0\\1&0&1\end{pmatrix}$	*Meet*		**38**	$\begin{pmatrix}0&1&1\\0&1&0\\1&0&1\end{pmatrix}$	*Meet*	
39	$\begin{pmatrix}0&1&1\\1&1&1\\1&0&1\end{pmatrix}$	*Meet*		**40**	$\begin{pmatrix}0&1&1\\1&1&0\\1&1&1\end{pmatrix}$	*Meet*	
41	$\begin{pmatrix}0&1&1\\1&0&1\\1&1&1\end{pmatrix}$	*Meet*		**42**	$\begin{pmatrix}0&1&1\\1&1&1\\1&1&1\end{pmatrix}$	*Meet*	
43	$\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$	*Cover*		**44**	$\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix}$	*Cover*	
45	$\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}$	*Cover*		**46**	$\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix}$	*Cover*	
47	$\begin{pmatrix}1&1&1\\0&0&1\\0&0&1\end{pmatrix}$	*Contain*		**48**	$\begin{pmatrix}1&1&1\\0&0&0\\0&0&1\end{pmatrix}$	*Contain (B is closed)*	

Table 6 (*continued*)

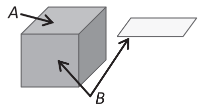
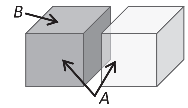
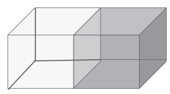
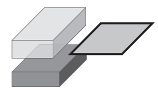
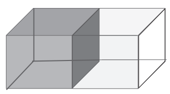
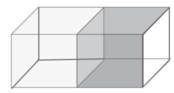
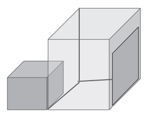
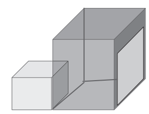
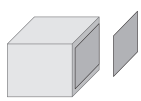
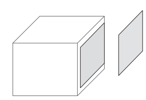
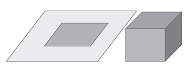
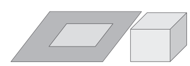
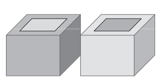
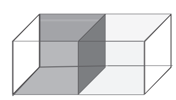
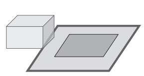
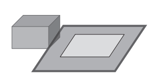
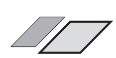
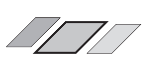
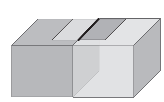
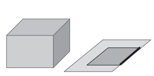
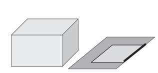
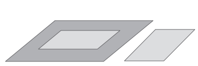
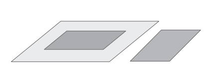
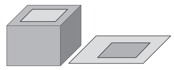
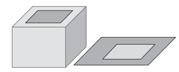
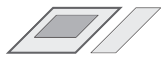
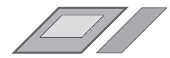
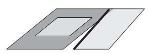
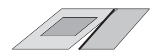
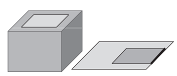
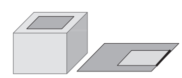
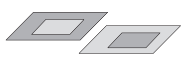
49	$\begin{pmatrix}1&0&1\\0&0&1\\0&0&1\end{pmatrix}$	*Contain (A is closed)*		**50**	$\begin{pmatrix}1&0&1\\0&0&0\\0&0&1\end{pmatrix}$	*Contain (both A and B are closed)*	
51	$\begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix}$	*Overlap (both A and B are closed)*		**52**	$\begin{pmatrix}1&0&1\\0&1&0\\1&0&1\end{pmatrix}$	*Overlap*	
53	$\begin{pmatrix}1&0&1\\0&0&1\\1&0&1\end{pmatrix}$	*Overlap (B is closed)*		**54**	$\begin{pmatrix}1&0&1\\0&0&0\\1&1&1\end{pmatrix}$	*Overlap (A is closed)*	
55	$\begin{pmatrix}1&1&1\\0&0&0\\1&0&1\end{pmatrix}$	*Overlap (A is closed)*		**56**	$\begin{pmatrix}1&0&1\\1&0&0\\1&0&1\end{pmatrix}$	*Overlap (B is closed)*	
57	$\begin{pmatrix}1&1&1\\0&0&0\\1&1&1\end{pmatrix}$	*Overlap (A is closed)*		**58**	$\begin{pmatrix}1&0&1\\1&0&1\\1&0&1\end{pmatrix}$	*Overlap (B is closed)*	
59	$\begin{pmatrix}1&1&1\\0&0&1\\1&0&1\end{pmatrix}$	*Overlap*		**60**	$\begin{pmatrix}1&0&1\\1&0&0\\1&1&1\end{pmatrix}$	*Overlap*	
61	$\begin{pmatrix}1&1&1\\1&0&0\\1&0&1\end{pmatrix}$	*Overlap*		**62**	$\begin{pmatrix}1&0&1\\0&0&1\\1&1&1\end{pmatrix}$	*Overlap*	
63	$\begin{pmatrix}1&0&1\\1&1&0\\1&0&1\end{pmatrix}$	*Overlap*		**64**	$\begin{pmatrix}1&1&1\\0&1&0\\1&0&1\end{pmatrix}$	*Overlap*	
65	$\begin{pmatrix}1&0&1\\0&1&1\\1&0&1\end{pmatrix}$	*Overlap*		**66**	$\begin{pmatrix}1&0&1\\0&1&0\\1&1&1\end{pmatrix}$	*Overlap*	
67	$\begin{pmatrix}1&0&1\\0&1&1\\1&1&1\end{pmatrix}$	*Overlap*		**68**	$\begin{pmatrix}1&1&1\\1&1&0\\1&0&1\end{pmatrix}$	*Overlap*	
69	$\begin{pmatrix}1&1&1\\0&1&1\\1&0&1\end{pmatrix}$	*Overlap*		**70**	$\begin{pmatrix}1&0&1\\1&1&0\\1&1&1\end{pmatrix}$	*Overlap*	
71	$\begin{pmatrix}1&0&1\\1&0&1\\1&1&1\end{pmatrix}$	*Overlap*		**72**	$\begin{pmatrix}1&1&1\\0&0&1\\1&1&1\end{pmatrix}$	*Overlap*	

Table 6 (*continued*)

73	$\begin{pmatrix}1&1&1\\1&0&1\\1&0&1\end{pmatrix}$	*Overlap*		**74**	$\begin{pmatrix}1&1&1\\1&0&0\\1&1&1\end{pmatrix}$	*Overlap*	
75	$\begin{pmatrix}1&0&1\\1&1&1\\1&0&1\end{pmatrix}$	*Overlap*		**76**	$\begin{pmatrix}1&1&1\\0&1&0\\1&1&1\end{pmatrix}$	*Overlap*	
77	$\begin{pmatrix}1&0&1\\1&1&1\\1&1&1\end{pmatrix}$	*Overlap*		**78**	$\begin{pmatrix}1&1&1\\0&1&1\\1&1&1\end{pmatrix}$	*Overlap*	
79	$\begin{pmatrix}1&1&1\\1&1&1\\1&0&1\end{pmatrix}$	*Overlap*		**80**	$\begin{pmatrix}1&1&1\\1&1&0\\1&1&1\end{pmatrix}$	*Overlap*	
81	$\begin{pmatrix}1&1&1\\1&0&1\\1&1&1\end{pmatrix}$	*Overlap*		**82**	$\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$	*Overlap*	

A  *B*  *Adjacent boundary* 

5.5. Relationships between GeologicStructures

According to Definition 3, A *GeologicStructure* is a complex geoobject consisting of both *GeologicBoundaries* and other *GeologicStructures.* Hence, a *GeologicStructure A* and a *GeologicStructure B* satisfy the following conditions:

- If *A* meets *B*, then $B^o \cap A^o \neq \varnothing$, $B^o \cap A^+ \neq \varnothing$ and $B^+ \cap A^o = \varnothing$.
- If *A* is covered by *B*, then $B^o \cap A^o \neq \varnothing$, $B^o \cap A^+ \neq \varnothing$, $B^+ \cap A^o = \varnothing$ and $\partial A \cap \partial B \neq \varnothing$.
- If *A* covers *B*, then $B^o \cap A^o \neq \varnothing$, $B^o \cap A^+ = \varnothing$, $B^+ \cap A^o \neq \varnothing$ and $\partial A \cap \partial B \neq \varnothing$.
- If *A* overlaps *B*, then $B^o \cap A^o \neq \varnothing$, $B^o \cap A^+ \neq \varnothing$, $B^+ \cap A^o \neq \varnothing$

82 topological relationships including *Disjoint*, *Meet*, *Inside*, *Coverby* and *Equal* can be found, shown in Table 6, where the former 24 relationships have been listed in in Table 5.

5.6. Relationships between GeologicStructure and basicGeologicUnit

Definitions 3 and 4 indicate that topological relationships between relationships between a *GeologicStructure A* and a *basicGeologicUnit B* are diverse and relationships including *Disjoint*, *Meet*, *Contain*, *Inside*, *Cover*, *Coverby*, *Equal* and *Overlap* are possible. 43 topological relationships can be found, shown in Table 7.

6. Spatial rationality for 3D structural models

Evaluating the quality of a 3D structural model is carried out to analyze the correctness of its digital characterization of the subsurface space (Caumon et al., 2004; Schetselaar and Kemp, 2006). The formal representation of 3D structural models in this study can be used to place some constraints on spatial partitioning and spatial relationships between geo-objects, thereby providing an evaluation method for the spatial reasonableness of structural models.

6.1. Reasonableness of spatial partitioning

According to Definitions 3 and 4, the subsurface space represented by a 3D structural model primarily partitioned by the *GeologicBoundary* and the corresponding spatial relationships between geoobjects are also determined by spatial partitioning. As such, partitioning using geological boundaries must comply with certain conditions. For a 3D structural model with reasonable partitioning, the spatial relationships between *GeologicBoundary* and *basicGeologicUnit* must also comply with certain conditions.

According to Definition 3, let *A* and *B* be the *GeologicBoundary* and *basicGeologicUnit* object, and the following conditions be met:

If $A^o \cap \partial B \neq \varnothing$, then $A^o \cap B^+ = \varnothing$ and $A^o \cap B^o = \varnothing$
If $A^o \cap B^o \neq \varnothing$, then $A^o \cap B^+ = \varnothing$
If $A^o \cap B^+ \neq \varnothing$, then $A^o \cap B^o = \varnothing$

These constraints indicate that a *GeologicBoundary* is not allowed to traverse (or leap across) multiple *GeologicUnits*. This is because a *GeologicBoundary* is actually an interface with different adjacent *GeologicUnits*. The spatial relationships between a *GeologicBoundary* and *GeologicUnit* are not allowed to occur in Table 8. In fact, these disallowed spatial relationships have been shown in Table 7.

6.2. Reasonableness of compositing geological objects

According to Definition 4, several geoobjects can be combined to form a new geoobject. For a geological structural object formed

Table 7
The 43 topological relationships between *GeologicStructure* and *basicGeologicUnit*.

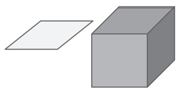
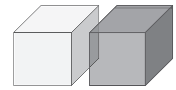
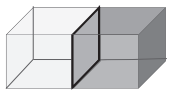
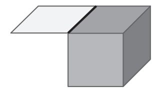
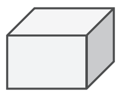
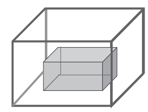
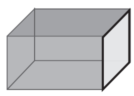
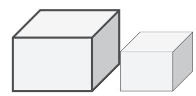
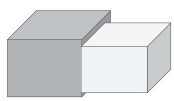
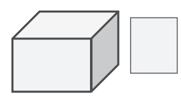
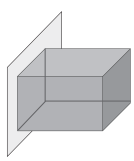
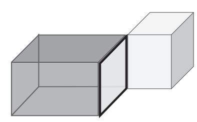
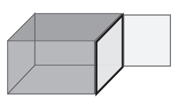
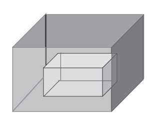
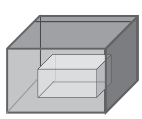
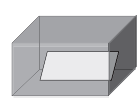
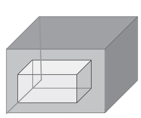
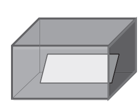
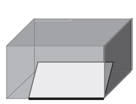
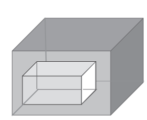
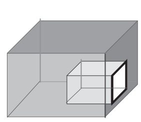
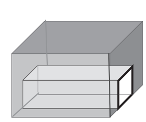
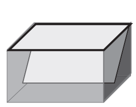
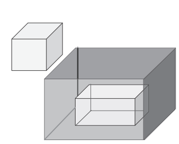
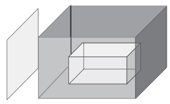
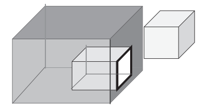
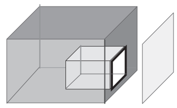
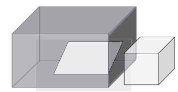
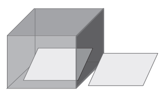
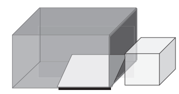
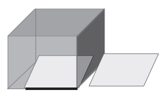
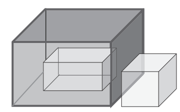
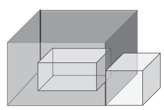
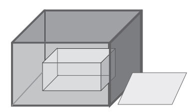
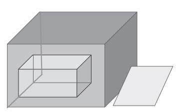
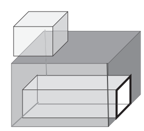
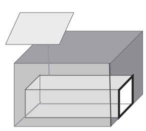
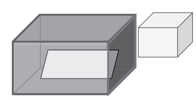
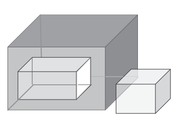
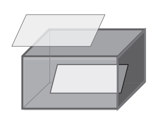
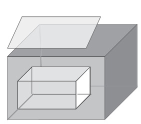
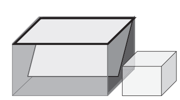
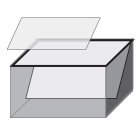
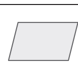
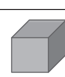
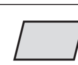
No.	Matrix R_9	Semantics	Graphic Description	No.	Matrix R_9	Semantics	Graphic Description
1	$\begin{pmatrix}0&0&1\\0&0&1\\1&1&1\end{pmatrix}$	*Disjoint*		**2**	$\begin{pmatrix}0&0&1\\0&0&0\\1&1&1\end{pmatrix}$	*Disjoint (A is closed)*	
3	$\begin{pmatrix}0&0&1\\0&1&0\\1&1&1\end{pmatrix}$	*Meet*		**4**	$\begin{pmatrix}0&0&1\\0&1&1\\1&1&1\end{pmatrix}$	*Meet*	
5	$\begin{pmatrix}0&1&0\\0&0&0\\1&0&1\end{pmatrix}$	*Meet (A is closed)*		**6**	$\begin{pmatrix}0&1&0\\0&0&0\\1&1&1\end{pmatrix}$	*Meet (A is closed)*	
7	$\begin{pmatrix}0&1&0\\0&1&0\\1&1&1\end{pmatrix}$	*Meet*		**8**	$\begin{pmatrix}0&1&1\\0&0&0\\1&0&1\end{pmatrix}$	*Meet (A is closed)*	
9	$\begin{pmatrix}0&1&1\\0&0&0\\1&1&1\end{pmatrix}$	*Meet (A is closed)*		**10**	$\begin{pmatrix}0&1&1\\0&0&1\\1&0&1\end{pmatrix}$	*Meet*	
11	$\begin{pmatrix}0&1&1\\0&0&1\\1&1&1\end{pmatrix}$	*Meet*		**12**	$\begin{pmatrix}0&1&1\\0&1&0\\1&1&1\end{pmatrix}$	*Meet*	
13	$\begin{pmatrix}0&1&1\\0&1&1\\1&1&1\end{pmatrix}$	*Meet*		**14**	$\begin{pmatrix}1&0&0\\0&0&0\\1&1&1\end{pmatrix}$	*Inside (A is closed)*	
15	$\begin{pmatrix}1&1&0\\0&0&0\\1&0&1\end{pmatrix}$	*Inside (A is closed)*		**16**	$\begin{pmatrix}1&0&0\\1&0&0\\1&1&1\end{pmatrix}$	*Inside*	
17	$\begin{pmatrix}1&1&0\\0&0&0\\1&1&1\end{pmatrix}$	*Inside (A is closed)*		**18**	$\begin{pmatrix}1&1&0\\1&0&0\\1&0&1\end{pmatrix}$	*Inside*	
19	$\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}$	*Coverby*		**20**	$\begin{pmatrix}1&1&0\\1&0&0\\1&1&1\end{pmatrix}$	*Inside*	
21	$\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix}$	*Coverby*		**22**	$\begin{pmatrix}1&1&0\\0&1&0\\1&1&1\end{pmatrix}$	*Coverby*	
23	$\begin{pmatrix}1&1&0\\1&1&0\\1&1&1\end{pmatrix}$	*Coverby*		**24**	$\begin{pmatrix}1&0&1\\0&0&0\\1&1&1\end{pmatrix}$	*Overlap*	

Table 7 (*continued*)

25 $\begin{pmatrix}1&0&1\\0&0&1\\1&1&1\end{pmatrix}$	*Overlap*		**26** $\begin{pmatrix}1&0&1\\0&1&0\\1&1&1\end{pmatrix}$	*Overlap*	
27 $\begin{pmatrix}1&0&1\\0&1&1\\1&1&1\end{pmatrix}$	*Overlap*		**28** $\begin{pmatrix}1&0&1\\1&0&0\\1&1&1\end{pmatrix}$	*Overlap*	
29 $\begin{pmatrix}1&0&1\\1&0&1\\1&1&1\end{pmatrix}$	*Overlap*		**30** $\begin{pmatrix}1&0&1\\1&1&0\\1&1&1\end{pmatrix}$	*Overlap*	
31 $\begin{pmatrix}1&0&1\\1&1&1\\1&1&1\end{pmatrix}$	*Overlap*		**32** $\begin{pmatrix}1&1&1\\0&0&0\\1&0&1\end{pmatrix}$	*Overlap (A is closed)*	
33 $\begin{pmatrix}1&1&1\\0&0&0\\1&1&1\end{pmatrix}$	*Overlap (A is closed)*		**34** $\begin{pmatrix}1&1&1\\0&0&1\\1&0&1\end{pmatrix}$	*Overlap*	
35 $\begin{pmatrix}1&1&1\\0&0&1\\1&1&1\end{pmatrix}$	*Overlap*		**36** $\begin{pmatrix}1&1&1\\0&1&0\\1&1&1\end{pmatrix}$	*Overlap*	
37 $\begin{pmatrix}1&1&1\\0&1&1\\1&1&1\end{pmatrix}$	*Overlap*		**38** $\begin{pmatrix}1&1&1\\1&0&0\\1&0&1\end{pmatrix}$	*Overlap*	
39 $\begin{pmatrix}1&1&1\\1&0&0\\1&1&1\end{pmatrix}$	*Overlap*		**40** $\begin{pmatrix}1&1&1\\1&0&1\\1&0&1\end{pmatrix}$	*Overlap*	
41 $\begin{pmatrix}1&1&1\\1&0&1\\1&1&1\end{pmatrix}$	*Overlap*		**42** $\begin{pmatrix}1&1&1\\1&1&0\\1&1&1\end{pmatrix}$	*Overlap*	
43 $\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$	*Overlap*				

A  *B*  Adjacent boundary 

with multiple *GeologicBoundaries*, their spatial relationships must comply with the composite conditions stated below.

Let *A* and *B* represent the *GeologicBoundary* and *GeologicStructure*, then the following conditions be satisfied:

Let $A^o \cap B^o \neq \varnothing$, then $A^o \cap B^+ = \varnothing$ and $\partial B \cap A^o = \varnothing$

According to the conditions described above, when a relationship defined as *Contain* exists between a *GeologicStructure* and *GeologicBoundary*, then *GeologicBoundary A* is one of the parts forming *GeologicStructure B* in a part–whole combinational relationship. Therefore, it is impossible for *B* to contain only a part of *A*, or for *A* to contain a part of *B*. Some cases depicted in Fig. 4 are not allowed.

6.3. Reasonableness for adjacent geological objects

In a structural model, the boundaries of two adjacent *basicGeologicUnits* inevitably overlap at some common *GeologicBoundary* objects. According to Definitions 3 and 4, a *GeologicBoundary* is

Table 8
Disallowed spatial relationships between *GeologicBoundary* and *baisicGeologicUnit*.

Semantics	Disallowed relationship no. in Table 7
Meet	8,9,10,11,12,13
Inside	*15,17,18,20*
Coverby	*22,23*
Overlap	*24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43*

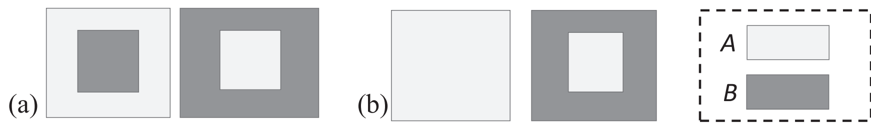


Fig. 4. Some cases of unreasonable spatial relationships between *GeologicBoundary A* and *GeologicStructure B*. (a) *A* and *B* mutually but partially contain each other; (b) *B* contains part of *A*.

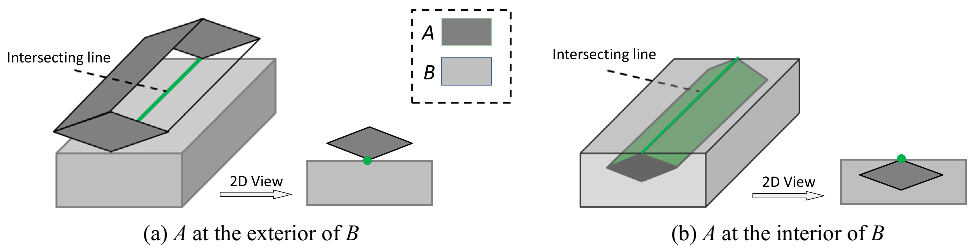


Fig. 5. Unreasonable adjacent relationships between *basicGeologicUnits* (a) *A* at the exterior of *B* (b) *A* at the interior of *B*.

expressed by a *2*-cell. Thus, if U_1 and U_2 are both *basicGeologicUnits* in a *3*-GeoModel, then the following conditions must be met:

$$C = U_1 \cap U_2 \text{ and } C \neq \varnothing \Longrightarrow dim(U_1) = dim(U_2) = dim(C) + 1$$

This condition shows that the common boundaries of two adjacent *basicGeologicUnits* are in fact formed by *GeologicBoundary* objects, whose geometrical dimensions comply with the conditions mentioned above. This also shows that in a reasonable *3*-GeoModel, the common boundaries (overlapping parts or intersecting parts) are not allowed to be expressed by *0*-cells and *1*-cells. Thus, the spatial relationships depicted in Fig. 5 are not allowed. If the types of spatial relationships depicted in Fig. 5 exist, then Fig. 5a can only be interpreted as a type of *Disjoint* relationship, while Fig. 5b is treated as a *Contain* or *Meet (inner-boundary equal)* relationship.

7. Conclusion

In this paper, we presented a formal representation method for 3D structural models that produces different mathematical definitions for various geoobjects. The spatial relationships between these geoobjects were also described in detail. This theoretical representation is based on existing conclusions from 3D geological models, and provides additional aspects of the mathematical descriptions and definitions. This representation allows the improved analysis of the rationality and integrity of geological models, which leads to a clearer and more accurate understanding of the standardized construction and utility of 3D models. The proposed method lays the theoretical groundwork for studying the different data types, the specifications of geoobjects, querying, and analysis. The research in this study is conducive to building a complete geological information theoretical framework, and thereby developing Geological Information Sciences.

Future work will be based on the formal representation of 3D structural model to improve the representation and management of geological models in multi-dimensional spaces. The aim is to build a unified theoretical framework, and to use this to conduct research on multi-scale, spatio-temporal and dynamic representations of geological models to meet the demand for the integration and sharing of geological data.

Acknowledgments

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