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Case study

First-order uncertainty analysis using Algorithmic Differentiation of morphodynamic models

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ABSTRACT

We present here an efficient first-order second moment method using Algorithmic Differentiation (FOSM/AD) which can be applied to quantify uncertainty/sensitivities in morphodynamic models. Changes with respect to variable flow and sediment input parameters are estimated with machine accuracy using the technique of Algorithmic Differentiation (AD). This method is particularly attractive for process-based morphodynamic models like the Telemac-2D/Sisyphe model considering the large number of input parameters and CPU time associated to each simulation.

The FOSM/AD method is applied to identify the relevant processes in a trench migration experiment (van Rijn, 1987). A Tangent Linear Model (TLM) of the Telemac-2D/Sisyphe morphodynamic model (release 6.2) was generated using the AD-enabled NAG Fortran compiler. One single run of the TLM is required per variable input parameter and results are then combined to calculate the total uncertainty.

The limits of the FOSM/AD method have been assessed by comparison with Monte Carlo (MC) simulations. Similar results were obtained assuming small standard deviation of the variable input parameters. Both settling velocity and grain size have been identified as the most sensitive input parameters and the uncertainty as measured by the standard deviation of the calculated bed evolution increases with time.

1. Introduction

Morphodynamic models of increasing complexity have been developed in the past 30 years and are now widely applied by the engineering community to predict the natural or anthropogenic bed evolution in rivers, estuaries and seas.

In process-based models, the interactions between hydrodynamic forcing and sediment are described in detail, and sediment transport rates, generally decomposed into bed-load and suspended load, are calculated as a function of the local wave and current conditions. Thanks to recent progress in the use of parallel processors and efficient numerical methods, models can now be used as powerful engineering tools to represent the bed evolution in estuarine, fluvial and coastal environments at medium to large time and spatial scales.

Despite progress in the description of physical processes, morphodynamic modelling is still considered to be a difficult task. Morphodynamic models are generally seen much less accurate than hydrodynamic models due to the accumulation of errors which are difficult to quantify (Stansby, 2013). The high number of processes involved in the description of the flow-sediment interactions requires a large amount of input data and empirical parameters which are difficult to measure in-situ. Morphodynamic models rely on the use of highly empirical sediment transport predictors which are based on small scale experiments and their up-scaling for application in the field is questionable (Haff, 1996). The choice of empirical model parameters therefore requires a large degree of expertise from end-users to properly adapt the model to their application, leading to greater use of CPU resources in order to calibrate the most sensitive model parameters and improve the model predictability.

Uncertainty estimation is becoming common practice in many environmental problems, for example in flood risk and hydrological modelling. Despite its importance, the problem of uncertainty in morphodynamic models has been little addressed due to both practical as well as philosophical reasons. This reluctance

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may be attributed to unrealistic faith in the ability of physical-based models to represent natural processes in a deterministic way and may also be due to the fact that uncertainty is difficult to quantify given the large number of input parameters with limited time and CPU resources (see the discussion in Pappenberger and Beven (2006)).

There are a number of studies on the uncertainty in the sediment transport predictions. The effect of variability in the input conditions (sediment grain size and flow velocity) on various sediment transport predictors has been addressed for example by Pinto et al. (2006) using Monte Carlo sampling. The sensitivity of sediment transport predictors to the bed roughness parameter has been highlighted in Davies and Villaret (2003). The uncertainty in the sediment transport predictions is generally admitted to be a factor 2–5 to account for variability in the sediment and flow parameters (Davies et al., 2002).

Much less is known on the uncertainty of the calculated bed evolution in morphodynamic models. Monte Carlo analysis which has been previously applied in flood modelling and risk assesment (Wyncoll and Gouldby, 2015; Apel et al., 2004), involves a large number of simulations and becomes prohibitively too expensive in most in-situ morphodynamic applications. Among the few studies on uncertainty in morphodynamics, the effect of initial conditions was addressed by van der Wegen et al. (2011) who propose a method to generate the bed composition. Ensemble averaged simulations have been presented in Fortunato et al. (2009) and van der Wegen and Jaffe (2013) to investigate the uncertainty in process-based coastal models. The uncertainty as measured by the standard deviation in the model output, was found to increase in time and to be also larger when the transport rates are larger.

Our objective here is to present an efficient tool for uncertainty analysis which can be applied to estimate the effect of uncertainty in various model input parameters on the morphodynamic model output. Our approach, FOSM/AD, utilizes Algorithmic Differentiation (AD) to perform a first-order second-moment uncertainty analysis (Melching, 1992). AD was previously applied by Vogel et al. (2006) for a sensitivity analysis in environmental flow models. More information about AD methods can be found (Griewank and Walther, 2008; Naumann, 2012, www.autodiff.org). The AD-enabled NAG Fortran compiler (Naumann and Riehme, 2005; dco/fortran/adnag, 2013) has been used to create a tangent-linear model (TLM) of the Telemac-2D/Sisyphe morphodynamic model for the 6.2 release of the code (see Riehme et al. (2010) and Kopmann et al. (2012) for details).

The objective of this paper is (1) to gain confidence in the results of FOSM/AD by comparison with a classical Monte Carlo analysis (MC), and (2) to achieve more insight in the relative contributions of various input parameters to the global uncertainty of the morphodynamic model results.

The two different methods for uncertainty analysis are presented in Section 2. We show in Section 3 a simple 1D application of both methods, in order to estimate the uncertainty associated to the prediction of a trench evolution. The results of MC and FOSM/AD are compared in Section 4 for an uncertainty analysis and in Section 5 for a local sensitivity analysis. In conclusion we give an outlook of further applications of AD in field conditions and for model calibration using an adjoint model of the Telemac model also generated by the AD-enabled NAG Fortran compiler.

2. Different methods for uncertainty analysis

2.1. Objective and assumptions

The aim of an uncertainty analysis is to quantify the variability in the model outputs due to prescribed uncertainty in the input parameters.

For a set of input parameters represented by a vector of model inputs X , a single model output may be represented by

$$Y = F(X) \tag{1}$$

where the function $F()$ represents the deterministic model. In a probabilistic framework, the inputs X are considered a random variable with a chosen joint distribution which results in a random distribution for the output Y. With an arbitrary non-linear model function $F()$, the exact distribution of Y is usually intractable so methods such as Monte Carlo and FOSM/AD seek to estimate properties of the unknown distribution such as the mean $\mathrm{E}(Y)$, the variance $Var(Y)$ and confidence intervals.

In our analysis, we consider the effect of various flow and sediment input parameters on the calculated bed evolution (Z_b). The method presented below can be generalized to multiple output variables, but for simplicity we will consider below only one single output variable.

The effect of other sources of uncertainty in the model, like the variability in the initialization and boundary conditions (bathymetry, hydrodynamic forcing etc…) is out of scope of the present work.

This analysis is restricted to the effect of the bed roughness parameter k_s and sediment grain size d_{50} which have been identified as the most sensitive parameters regarding sediment transport predictions (Pinto et al., 2006; Davies and Villaret, 2003). The effect of variability in the settling velocity W_s has also been included when suspended load is the dominant mode (as in the application below). The effect of the sediment transport predictor itself has been included by varying the empirical factor (M_{PM} factor) in the Meyer-Peter and Müller (1948) bed load formula.

2.2. Monte Carlo analysis

In a Monte Carlo analysis, a large number of potential input parameters are randomly sampled according to their probability distribution and the numerical model is run for each. This produces a Monte Carlo sample of potential model output values which are used to approximate the distribution of Y. Summary statistics such as the mean and variance can be estimated by the corresponding sample moments, for example:

$$Var(Y) \simeq \frac{1}{N-1} \sum_{j=1}^{N} (y^{(j)} - \bar{y})^2 \tag{2}$$

where $y^{(j)}$ is the jth output sample of N and $\bar{y}$ is the sample mean. The accuracy of these approximations increases with the number of samples. The samples can be used to approximate any other property of the output without the need to make any assumptions on its distribution.

Monte Carlo sampling avoids making any simplifying assumptions on the model function $F()$ or on the distributions of the inputs and outputs. However, its main disadvantage is that a large number of model runs are required which, for complex models such as Telemac-2D/Sisyphe, can consume significant CPU resources.

Stratified sampling techniques such as Latin Hypercube Sampling can reduce the error in the Monte Carlo approximation (Stein, 1987). This means that fewer samples are required for the same level of accuracy, thus saving computing time. Meta-models such as Gaussian process emulators can also be used to further reduce the number of model runs by using a small number of runs to build an approximate model with which to conduct the uncertainty analysis (Oakley and O'Hagan, 2002).

2.3. FOSM/AD: first-order uncertainty analysis using AD with a tangent-linear model (TLM)

When the unknown inputs X are all real-valued variables, a Taylor expansion of the model function about a best estimate such as the mean $E(X)$ can be used to approximate moments of the output variable Y. When applied to the mean, a first-order Taylor expansion gives:

$$E(Y) \simeq F(E(X)) \tag{3}$$

which justifies approximating the output mean by a single run evaluated at the input means. Applied to the variance, a first-order expansion gives:

$$Var(Y) \simeq \nabla F(E(X))^T \bullet Var(X) \bullet \nabla F(E(X)) \tag{4}$$

where $\nabla F()$ is the vector of partial derivatives with respect to each element of X. This is often known as the delta method. Under the common assumption of independence between each of the inputs in X, the variance matrix $Var(X)$ becomes diagonal which simplifies the equation to:

$$Var(Y) \simeq \sum_{i=1}^{n} [\frac{\partial F}{\partial X_i}(E(X))]^2 \bullet Var(X_i) \tag{5}$$

where n is the number of variable inputs. Partial derivatives in Eq. (5) can be estimated using finite differences approximations or calculated exactly up to machine accuracy using Algorithmic Differentiation (AD).

For a model function F given as a computer program, AD allows to generate an annotated model $(Y,\dot{Y}) = \dot{F}(X,\dot{X})$ the so-called tangent-linear model (TLM) of F. The TLM computes alongside with $Y = F(X)$ a projection of the Jacobian (matrix of partial derivatives) ∇F in the direction $\dot{X}$:

$$\dot{Y} = \nabla F(E(X)) \cdot \dot{X} \tag{6}$$

see Naumann (2012) and Griewank and Walther (2008) for more details about AD or visit the communities web portal www.autodiff.org. The TLM of the model function $F()$ can be applied for an uncertainty analysis by first computing and storing the partial derivatives required in Eq. (5). The partial derivatives $\frac{\partial F}{\partial X_i}(E(X))$ of $F()$ with respect to the individual uncertain variables for $1 \le i \le n$ are obtained by evaluating the TLM repeatedly as

$$(F(E(X)), \frac{\partial F}{\partial X_i}(E(X))) = F(E(X), \hat{e}_i) \text{ for } 1 \le i \le n \tag{7}$$

with $\hat{e}_i = (0, \ldots, 1, \ldots 0) \in R^n$ being the i-th Cartesian basis vector.

Eq. (5) can then be evaluated easily from the stored partial derivatives to obtain the variance $Var(Y)$. With AD computing the mean and standard deviation of the output, any further property of the output variable such as confidence intervals can be estimated by approximating the output by a Gaussian distribution with these parameters. Compared with a Monte Carlo analysis, Algorithmic Differentiation has the advantage of requiring just a single run of the TLM for each input parameter which will often be significantly faster than a large number of runs of the standard model. Being based on a first-order Taylor expansion, the method is likely to work best for smooth models that are close to linear. By relying only on means and variances, FOSM/AD is also likely to work best when the inputs and outputs are both Gaussian.

2.4. Further applications

Further applications of AD include field applications and automatic model calibration. For example, different methods of sensitivity analysis (MC, FOSM and Meta-modelling) have been applied in a 10 km long Telemac-2D/Sisyphe morphodynamic model of the Danube River (Kopmann and Schmidt, 2010; Clees et al. 2012). The efficient calculation of the partial derivatives with AD can also be used to analyse the dependencies between input and output parameters in the context of structures optimization (Merkel et al. 2013). More recently, an adjoint model of Telemac-2D/Sisyphe has been developed using the AD-enabled NAG Fortran compiler and successfully applied for an automatic calibration of input model parameters in laboratory tests (Schäfer, 2014).

3. The trench evolution test case

3.1. Description of the test case

The numerical model set up is based on the laboratory experiments, conducted by van Rijn (1987). Experiments were performed in a straight channel at Delft Hydraulics, and the geometry of the experimental facility was as follows: 30 m long and 0.50 m wide with vertical side walls. The channel was filled with a 0.20 m thick layer of sand with median grain size $d_{50} = 0.160\ 10^{-3}$ m. The average velocity was 0.51 m/s and the water depth was approximately equal to 0.39 m at the channel inlet. The experiment (Test 3) considered in this work involved a trench with side slope 1:3. Measurements of bed level after 15 h of experiment as well as estimates of the bed-load and suspended load ($Q_b = 0.1$ kg/m^2/s and $Q_s = 0.3$ kg/m^2/s) have been provided. Bed ripples dimensions have been also measured in the range 0.015–0.035 m, with corresponding mean bed roughness coefficient ($k_s = 0.025 \pm 0.01$ m).

3.2. Numerical model set up

We use the Telemac-2D finite element flow model internally coupled to the 2D sediment transport and morphodynamic model Sisyphe: at each time step the flow model sends the flow field (mean velocity, water depth and bed shear stress) to the sediment transport model, which calculates the sediment transport decomposed into bed load and suspended load and sends back the updated bed level to the flow model (Villaret et al. 2011). For the

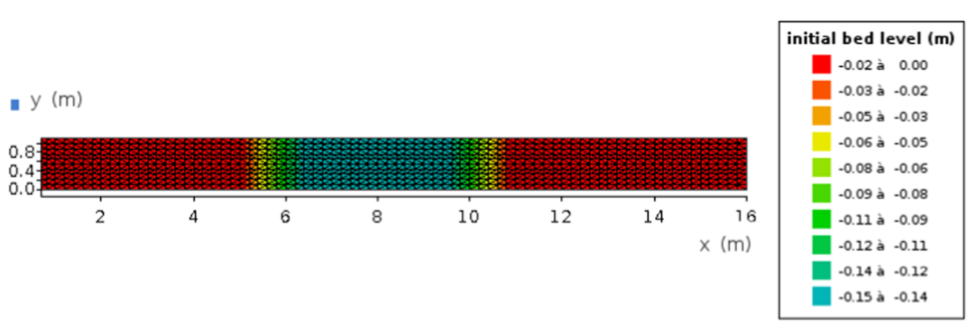

Fig. 1. Mesh and computational domain.

bed geometry we use a relatively coarse grid in order to lower CPU time (Monte Carlo simulations being time consuming). The triangular mesh elements of $0.20 \times 0.05\ \mathrm{m}^2$ are shown on Fig. 1. The length of the computational channel has been conveniently reduced down to an active length of 16 m and the mesh encompasses less than 1000 nodes. A more refined grid was also used and shown to give similar results. To mimic the laboratory conditions with the model, a constant water depth was imposed at the downstream outlet, and a constant discharge was specified at the upstream inlet. Flow was computed with a fixed bed until steady flow conditions were reached in order to initialize the flow velocity, and with a movable bed afterwards in which the trench propagates in the direction of the flow. For the hydrodynamic model, a Nikuradse friction law is applied. Preliminary testing of the choice of numerical methods (characteristics, SUPG, distributive schemes…) shows little effects, and the flow model is run here using the method of characteristics (Hervouet, 2007).

The time step is set to 1 s, and the 2D model takes only 2 min for 15 h of bed evolution on a HP Zbook Linux Workstation (in scalar version).

3.3. Bed-load transport

The bed shear stress is corrected for skin friction assuming $k_{sp}=3\,d_{50}$. The bed-load transport rate is then calculated as a function of the excess bed shear stress (corrected for skin friction) above its critical value using an empirical formula (Meyer-Peter and Müller, 1948).

The Meyer-Peter and Müller (MPM) formula which has been used to calculate the bed-load transport rate is given below:

$$\frac{Q_b}{\sqrt{g(s-1)d_{50}^3}} = M_{PM}(\theta' - \theta_c)^{3/2} \tag{8}$$

where g is the gravity (m^2/s), s is the relative grain density, θ' is the adimensional skin friction and θ_c is the Shields parameter, which is calculated in Sisyphe as a function of grain size. Only the empirical factor in Eq. (8) ($M_{PM}=8$ by default) will be considered uncertain.

The effect of a longitudinal bed slope on the magnitude of the sand transport rate can be accounted for by the approach of Koch and Flokstra (1981), which involves an additional empirical coefficient for sloping bed effect with default setting ($\beta=1.3$).

3.4. Suspended load

In Sisyphe, the suspended sediment concentration is determined by solving a depth-averaged transport/diffusion equation, where the source term represents the net erosion (E) minus deposition (D) flux in m/s. Different numerical methods are available to solve the advection terms. Here we used the method of characteristics.

The erosion flux is expressed in terms of an 'equilibrium' reference concentration, and the deposition flux is calculated as the product of settling velocity W_s and near bed concentration. In the 2D model, the advection term is corrected to account for the vertical distribution of velocity and concentration, leading to a global reduction in the convection velocity (Huybrechts et al., 2010).

3.5. Bed evolution

The variation of bed elevation can be derived by solving the Exner equation:

$$(1-p)\frac{\partial Z_b}{\partial t}+\mathrm{Div}(\vec{Q}_b) + (\mathrm{E}-D)=0 \tag{9}$$

where p is the bed porosity (X_{3it}=lnH_{it}0.4 for non-cohesive sediment), Z_b (m) is the bottom elevation and Q_b (m^2/s) is the solid volume transport rate (bed load) per unit width.

The Exner equation can be solved by using finite-element or finite-volume techniques. The method we use is based on a flux calculation per segment. The procedure fully ensures mass continuity as well as a positive sediment bed thickness, as explained by Hervouet et al. (2011).

3.6. Model calibration

Preliminary runs were performed in order to test the sensitivity of the morphodynamic model results to the choice of sediment transport formula. The sediment and flow input parameters (e.g. grain size and bed roughness) were imposed based on experimental measurements (d_{50}=0.160 10^{-3} m, $k_s=0.025$ m). The mean settling velocity was first estimated from the mean grain size, and then allowed to vary in order to account for sorting effects.

Best fit results were obtained using the MPM bed load and van Rijn (2007) reference concentration. After calibration of the settling velocity ($W_s=0.0175$ m/s), the accuracy of the calculated bed evolution after 15 h is about 2.10^{-3} m (2–3% of the bed evolution).

In Fig. 2, the black line shows the initial 0.15 m-deep and 3 m wide trench longitudinal profile, with its centre originally located at a distance $x=8$ m from the channel entrance. The blue spots represent the trench position measured after 15 h of bed evolution to be compared with the results of the calibrated model (in red).

4. Uncertainty analysis – comparison of FOSM/AD and MC

In this section, we estimate the uncertainty associated with the morphodynamic model predictions for the trench evolution test case. We select a set of input variables and assume small deviations around their mean reference values. The objective is to compare the results obtained by both MC and FOSM/AD methods.

The TLM of Telemac-2D/Sisyphe required for FOSM/AD was generated by the AD-enabled NAG Fortran compiler (Naumann and Riehme, 2005; dco/fortran/adnag, 2013).

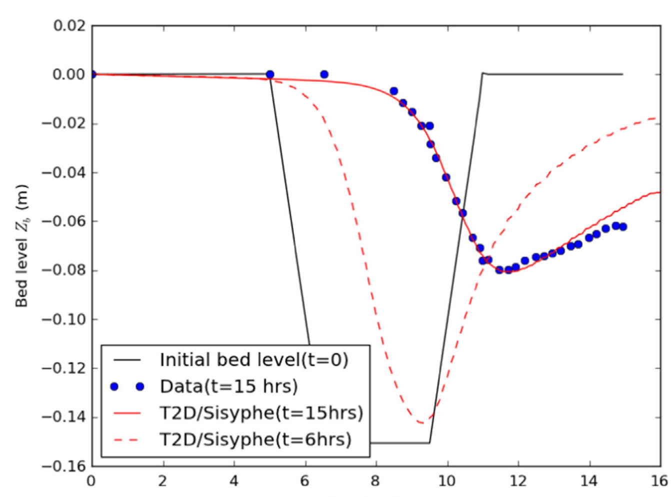

Fig. 2. Trench evolution after 6 h (dotted line) and 15 h (full line) – comparison between the calibrated model results in red ($W_s=0.0175$ m/s, $k_s=0.05$ m, $d_{50}=0.160$ 10^{-3} m, $M_{PM}=8$) and experimental data (blue circles). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Table 1
Mean and standard deviation of the Gaussian distributions specified for the four variable input parameters.

Variable input parameters	Mean value $E(X_i)$	Standard deviation σ_i
Bed roughness k_s (m)	0.025	0.005
Grain size d_{50} (m)	0.160 10^{-3}	0.015 10^{-3} (set 1) 0.030 10^{-3} (set 2)
Settling velocity W_s (m/s)	0.0175	0.0015
M_{PM} factor	8	1

4.1. Uncertain input parameters

The bed roughness and sediment grain size have been selected as the most sensitive input parameters regarding sediment transport predictions. The settling velocity has also a large influence on the results in this suspended load regime. In addition, the empirical factor in the Meyer-Peter and Müller formula (Eq. (8)) is also allowed to vary around its default value ($M_{PM}=8$). The other model parameters – skin friction k_{sp}, Shields parameter, β factor for sloping bed effect – are calculated by the model as a function of grain size, using empirical expressions or default values (k_{sp}=3d_{50} and $\beta=1.3$).

Each of the four inputs described in Table 1 are assumed to be mutually independent and each represented by a Gaussian distribution. The means and standard deviations of these distributions are tabulated in Table 1. Based on measurement error, we assume for the bed roughness $\sigma_{k_s}=0.005$ m (20% of the mean value) which corresponds to half the estimated confidence interval of the measured bed roughness. For both grain diameter and settling velocity, we assume the standard deviation to be roughly 10% of the mean value ($\sigma_{d_{50}}=0.015\ 10^{-3}$ m and $\sigma_{W_s}=0.0015$ m/s). In order to assess the limitation of the first-order approach, we later assume $\sigma_{d_{50}}=0.030\ 10^{-3}$ m (20% of the mean value) in Section 5.

4.2. Uncertainty analysis

The Tangent Linear Model (TLM) of the Telemac-2D/Sisyphe model can be used to evaluate the first-order derivative associated with each input variable at the input means, in a single run per input parameter. Since all input variables are assumed to be independent, Eq. (5) can be applied to calculate the variance of the calculated bed evolution using only four TLM runs.

For the Monte Carlo estimates, the statistical software R (Core Team, 2014) was used with the lhs package (Carnell, 2012) to randomly generate 100 samples of the input vector via optimal Latin hypercube sampling. Each of these was run through the standard model to produce MC samples of the output variables of interest, e.g. the bed evolution. The number of samples of the MC simulations was initially varied between 50 and 500. 100 stratified samples were considered sufficient to represent accurately the mean and standard deviation of the bed evolution.

The CPU time of each TLM run is approximately a factor 3 compared to the standard model. Therefore, the FOSM/AD uncertainty analysis is approximately 8 times faster than a MC analysis of 100 samples when four random inputs are considered.

4.3. Comparison between MC and FOSM/AD

The results obtained by the FOSM/AD and MC uncertainty analysis using the first d_{50} standard deviation are compared on Fig. 3. FOSM/AD computes a higher uncertainty but gives overall similar results to the MC approach. The deviation between FOSM/AD and MC increases with time, most likely as the result of non-linearity.

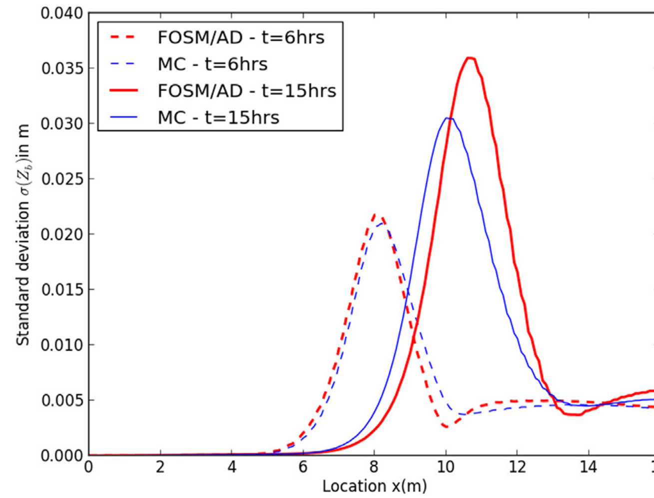

Fig. 3. Uncertainty analysis – comparison between MC (in blue thin) and FSOM/AD (in red thick). Standard deviation of bed level $\sigma(Z_b)$ due to variable grain size for $\sigma(d_{50})=0.015\ 10^{-3}$ m, settling velocity for $\sigma(W_s)=1.5\ 10^{-3}$ m/s, bed roughness for $\sigma(k_s)=5\ 10^{-3}$ m and M_{PM} factor for $\sigma(M_{PM})=1$ after 6 and 15 h of bed evolution.

Based on this analysis, the uncertainty associated with input parameters in morphodynamic model is large (as much 15–25% of the bed evolution). As shown in Fig. 3, the uncertainty varies spatially and the peak increases with time. The standard deviation is maximum in the down-sloping part of the trench where the flow is decelerating and the bathymetry changes more rapidly. These results are in qualitative agreement with Fortunato et al. (2009) uncertainty analysis in a tidal inlet.

The 95% confidence intervals estimated by both FOSM/AD and MC are shown in Fig. 4. The skewness of the bed evolution distribution is captured by the MC analysis, as shown by the asymmetry between the upper and lower confidence limit relative to the mean simulation. An additional Gaussian assumption was necessary to estimate the confidence intervals for FOSM/AD so these will always be symmetrical about the mean. The deposit obtained by FOSM/AD for the upper limit is overestimated in comparison to MC. However, FOSM/AD is able to reproduce the main features of interest for engineering purpose. According to

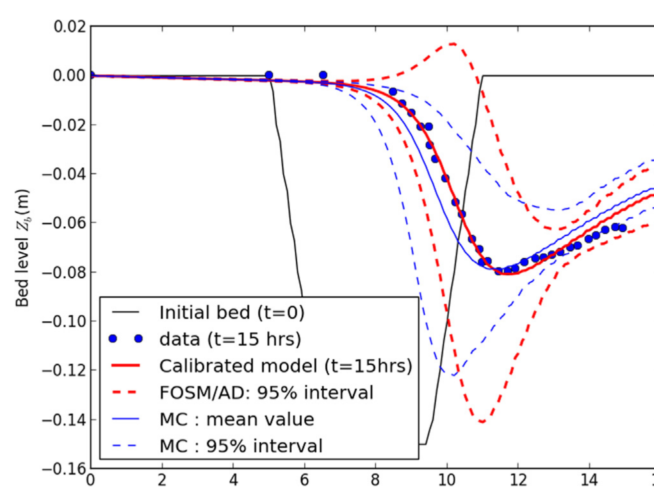

Fig. 4. Uncertainty analysis after 15 h of bed evolution. The 95% confidence interval calculated based on percentile of the MC is shown in blue dotted line and the envelope calculated based on the standard deviation for FOSM/AD in red thick dotted line. The blue thin line (full) is the ensemble mean MC simulations, the red full line shows the calibrated model results.

both methods, the rate of in-fill of the trench can be overestimated by 15% and the migration rate of the trench by about 30% as a result of variability in the grain size, settling velocity, bed roughness and empirical transport law parameters.

5. Sensitivity analysis – comparison of FOSM/AD and MC

5.1. Objective

Sensitivity analysis aims to quantify the degree by which each of the uncertain inputs contributes towards uncertainties in the outputs of interest. A sensitivity analysis provides valuable information for the calibration process to determine which input parameters need to be informed more accurately.

Sensitivity analysis methods tend to be either global or local: local analyses address the sensitivity relative to point estimates of input values such as the mean; global analyses examine the sensitivity with respect to the entire input distribution. Here we perform a local sensitivity analysis by varying each input parameter One-At-a-Time (OAT). A global analysis such as a full Variance Based Sensitivity Analysis (VBSA) would be required to look at interactions between input variables but requires a larger number of MC simulations and cannot be done with FOSM/AD (Campolongo et al., 2007).

The OAT sensitivity analysis consists of varying each of the input variables (d_{50}, W_s, k_s and M_{PM} factor) one at a time keeping the others fixed at their mean values. The output variances due to a single input are each calculated as with the main uncertainty analysis. These can be compared to each other or to the main output variance when all inputs were uncertain. For FOSM/AD, Eq. (5) can be applied to estimate these variances using the same partial derivatives produced for the global uncertainty analysis. However, for MC estimates a new set of 100 simulations are required for each of the input parameters to estimate the new variances.

5.2. Effect of grain size

We consider two values of the grain size standard deviation: $\sigma_{d_{50}}=0.015\ 10^{-3}$ m and $\sigma_{d_{50}}=0.030\ 10^{-3}$ m. The standard deviation of the 15 h bed evolution, with all variables except grain size fixed at their mean values, is estimated by both MC and FOSM/AD and compared. The results of all MC simulations are shown in Fig. 5 for $\sigma_{d_{50}}=0.015\ 10^{-3}$ m.

The standard deviation estimates are compared in Fig. 6 for both values of the grain size standard deviation.

Fig. 7 compares mean and 95% confidence interval estimates for the trench evolution. Differences between the AD/FOSM and MC confidence limits are again attributed to the non-Gaussian behaviour of the bed evolution.

For the smaller value of the standard deviation ($\sigma_{d_{50}}=0.015\ 10^{-3}$ m), both FOSM/AD and MC are found to give similar results. For the larger value of the standard deviation ($\sigma_{d_{50}}=0.030\ 10^{-3}$ m), the estimated standard deviation in the model output is very large. FOSM/AD overestimates the standard deviation of the bed evolution in comparison to the MC method. Most likely this is due to the first-order Taylor expansion used by FOSM/AD. This approximation is correct only when the model response is linear which does not appear to be the case here.

As expected, FOSM/AD gives best estimates when deviations are small and higher order moments will be investigated in future work.

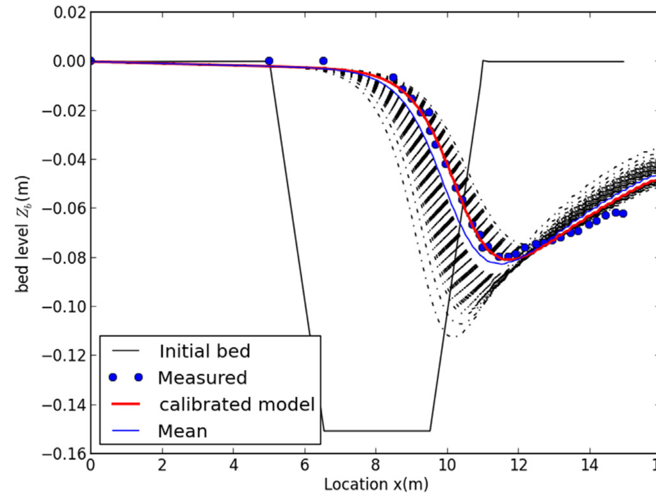

Fig. 5. Sensitivity analysis to the grain size for $\sigma(d_{50})=0.015\ 10^{-3}$ m. All input variables are kept constant except the grain size ($W_s=0.0175$ m/s, $k_s=0.05$ m, $M_{PM}=8$). Results obtained for the 100 MC simulations (in black dotted line) after 15 h of bed evolution. The full line (in red thick) shows the calibrated model results. The blue thin line shows the mean (ensemble averaged) of the 100 MC runs.

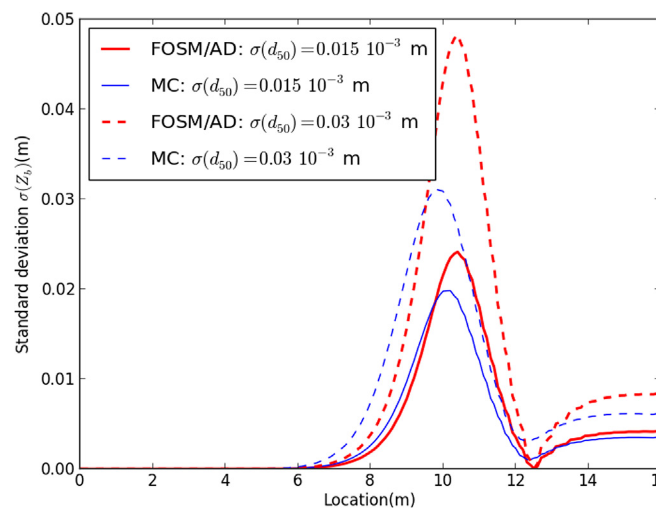

Fig. 6. Sensitivity analysis to the grain size for $\sigma(d_{50})=0.015\ 10^{-3}$ m (full lines) and for $\sigma(d_{50})=0.030\ 10^{-3}$ m (dotted lines). Standard deviation of the bed level $\sigma(Z_b)$ after 15 h, calculated by FOSM/AD (in red thick) and Monte Carlo simulations (in blue thin). Here all input variables are assumed constant except the grain size.

5.3. Local sensitivity analysis of selected input parameters

The effects of each individual input variable on the bed evolution (after 15 h of simulation) are compared in Fig. 8. The standard deviations obtained by MC and FOSM/AD are similar. In this case, with dominant suspended load, the settling velocity is found to be the most sensitive parameter, followed by the grain size. Both W_s and d_{50} are found to contribute to as much as 20–25% of variability in the morphodynamic model results. The other input parameters (M_{PM} factor, bed roughness) are found to play a minor role in the present application.

6. Conclusions

The first order second moment method using Algorithmic Differentiation (FOSM/AD) presented in this paper, provides an

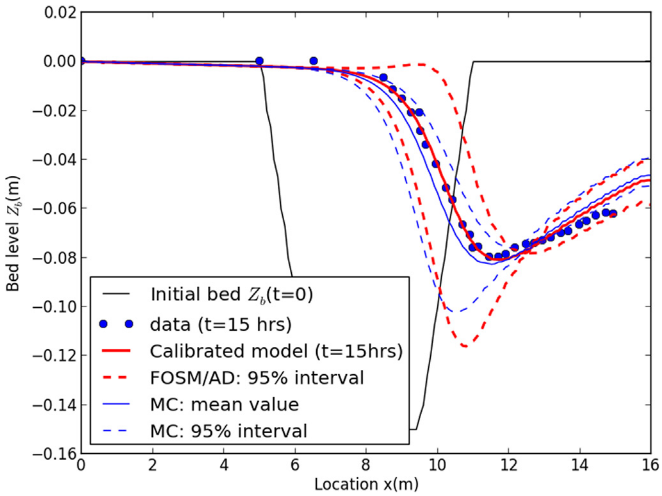

Fig. 7. Sensitivity analysis to the grain size for $\sigma(d_{50})=0.015\ 10^{-3}$ m. The 95% confidence interval calculated based on percentile of the MC is shown in blue thin dotted line and the envelope calculated based on the standard deviation for FOSM/AD in red thick dotted line.

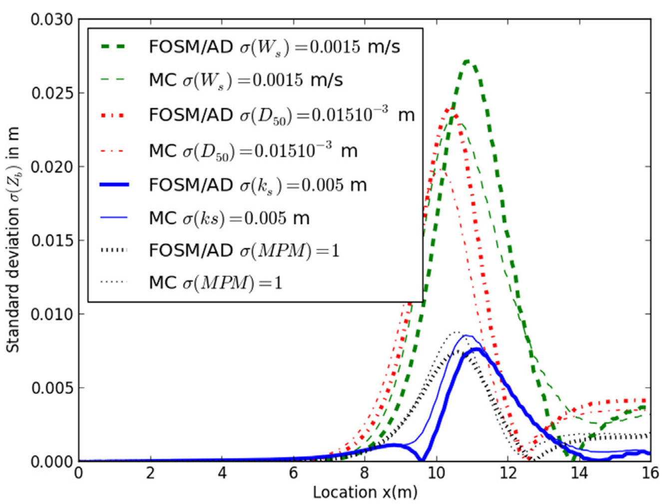

Fig. 8. Contribution of the 4 variable input parameters to the standard deviation of the bed evolution after 15 h of simulation. The effect of variable grain size is shown in red dash dotted for $\sigma(d_{50})=0.015\ 10^{-3}$ m, the effect of bed roughness in blue full line for $\sigma(k_s)=5\ 10^{-3}$ m, the effect of settling velocity in green dashed line for $\sigma(W_s)=1.5\ 10^{-3}$ m/s, and the effect of the M_{PM} factor in black dotted line for $\sigma(M_{PM})=1$.

efficient and reliable tool for both uncertainty and sensitivity analysis in morphodynamic models.

Both FOSM/AD and Monte Carlo simulations (MC) have been applied to estimate the uncertainty of the bed evolution as a result of variability in 4 input parameters (W_s,k_s, d_{50}, M_{PM} factor) using an AD-generated TLM model of Telemac-2D/Sisyphe. The advantage of FOSM/AD is CPU time and efficiency over MC. In the example above involving 100 MC runs and 4 uncertain input variables, FOSM/AD is approximately a factor 10 faster for the uncertainty analysis. The advantage of MC is its simplicity, robustness and flexibility, allowing to account for the morphodynamic model non-linear response.

With Gaussian input distributions and small standard deviations, FOSM/AD gives a reliable first-order estimate of the sensitivity to all input variables. For larger values of the standard deviations, the non-linearities in the model response become more significant and FOSM/AD is found to overestimate the uncertainty in comparison to the MC method.

Given the number of input parameters and CPU cost associated with morphodynamic model simulations, the FOSM/AD approach is well adapted, for both sensitivity and uncertainty analysis in large scale models. The method can be applied to more complex 2D/3D applications, although a parallel version of the TLM would need to be applied.

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